Circle the relevant digits	Scotch Student ID #			
	0	0	0	0
	1	1	1	1
	2	2	2	2
	3	3	3	3
	4	4	4	4
	5	5	5	5
	6	6	6	6
	7	7	7	7
	8	8	8	8
	9	9	9	9

Teacher's Name

Scotch College

MATHEMATICAL METHODS

U4-SAC 1b – Application Task: Test

Monday 15th August 2022

Reading Time	none
Writing Time	45 minutes

Task Sections	Marks	Your Marks
Extended Response Questions	30	
Total Marks	**30**	

Remote Declaration

I declare that any work I have submitted for this VCE assessment is wholly my own, unless properly referenced or authorised for use by my teacher. I have had no assistance from any person in my home nor have I been assisted by, or given assistance to, a boy in my class or cohort unless specifically permitted to do so by my teacher. I have not used the internet or other sources to assist me in my responses unless specifically permitted by my teacher. I acknowledge my work may be reproduced, communicated, compared and archived for the purposes of detecting plagiarism and collusion.

Signature: ____

General Instructions

- Answer all questions in the spaces provided.
- In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working must be shown.
- Unless otherwise indicated, the diagrams in this task are not drawn to scale.

Allowed Materials

- Calculators are not allowed.
- Notes and/or references are not allowed.

At the end of the task

- Ensure you cease writing upon request.

Electronic Devices

Students are **not** allowed to have a mobile phone, smart watch and/or any other unauthorised electronic device in the SAC, unless it is TURNED OFF and is placed on the front teacher desk.

Question 1 (6 marks)

a. Evaluate $\int_0^{\frac{\pi}{6}} 1-\sin(3x)\,dx$ 2 marks

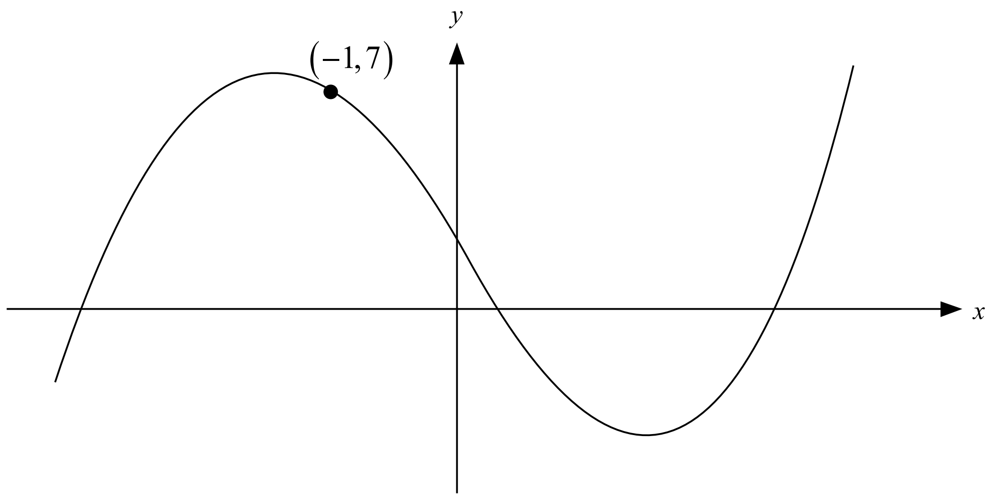
b. Evaluate $\int_1^4 \sqrt{x}+2\,dx$ 2 marks

c. The gradient of a curve is given by the formula $\frac{dy}{dx}=\frac{18}{x^3}+2$. The curve passes through point $(-3,6)$. Find the rule of the curve. 2 marks

Question 2 (5 marks)

Function f is defined as $f:\mathbb{R}\to\mathbb{R},\ f(x)=x^3-6x+2$.

The graph of $y=f(x)$ is shown below:



a. Show that the equation of the tangent to the curve $y=f(x)$ at point $(-1,7)$ has the rule $y=-3x+4$. 2 marks

b. The tangent to $y=f(x)$ at point $(-1,7)$ meets the curve again at point $(2,-2)$.

Find the area enclosed by the tangent to the curve at $(-1,7)$ and the graph of $y=f(x)$. 3 marks

Question 3 (7 marks)

A function g is defined as $g:[0,a]\to\mathbb{R},\ g(x)=2\cos\left(\frac{\pi}{2}x\right)+\sqrt{2}$

a. If g completes one full cycle on domain $[0,a]$, find the value of a. 1 mark

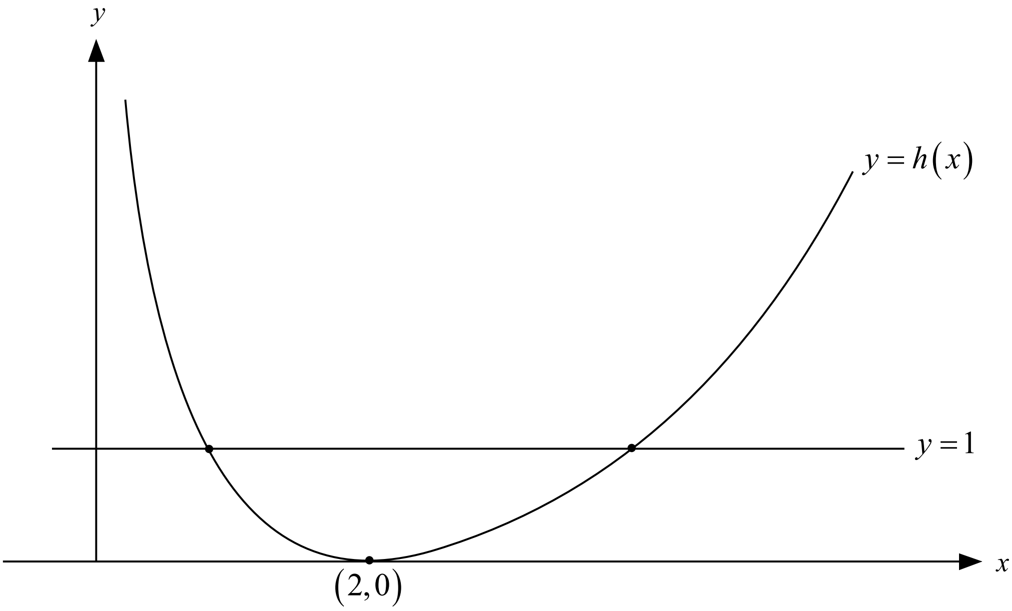
b. Write down the range of function g. 1 mark

c. Sketch the function g on the axes below, labelling the co-ordinates of all endpoints, intercepts and turning points. 3 marks

d. Find the total area enclosed by the curve, the x-axis, the y-axis and the line $x = 1$. 2 marks

Question 4 (9 marks)

The function $h:(0,\infty)\to\mathbb{R},\ h(x)=\dfrac{(x-2)^2}{x}$ and the line $y=1$ are shown below:



a. Show that the line $y=1$ meets the curve $y=h(x)$ at $(1,1)$ and $(4,1)$. 2 marks

b. Hence find the area of the region enclosed by the line $y=1$ and the curve $y=h(x)$.

Give your answer in the form $a+b\log_e(4)$, where a and b are rational numbers. 3 marks

c. The curve $y = h(x)$ is translated one unit in the negative x-direction and one unit in the negative y-direction. The resultant curve has the rule $y = g(x)$.

Using your answer to **part b**, or otherwise, evaluate the following:

i. $\int_0^3 g(x)\,dx$ 1 mark

ii. $\int_0^{\frac{3}{2}} g(2x)\,dx$ 1 mark

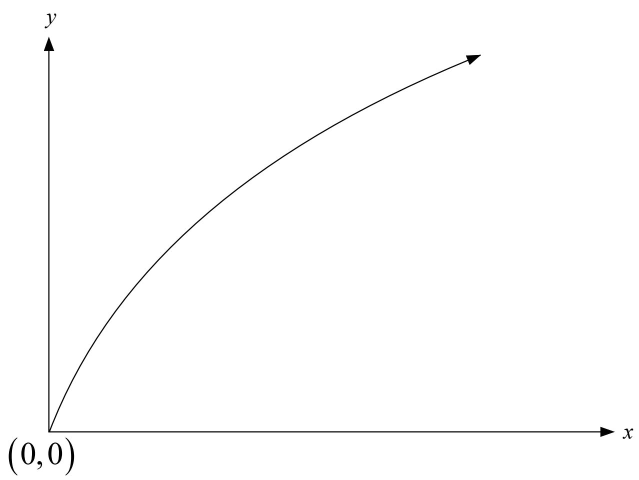
iii. $\int_0^{3k} -2g\left(\frac{x}{k}\right)+k\,dx$, giving your answer in terms of k. 2 marks

Question 5 (3 marks)

Function f has the rule $f(x) = \left(2 + 3x^2\right)^{\frac{1}{2}}$

a. Find $f'(x)$ 1 mark

b. The diagram below shows the graph of $y = \dfrac{3x}{\left(4 + 6x^2\right)^{\frac{1}{2}}} + x.$



Using your answer to **part a**, or otherwise, find the exact area of the region bounded by the curve $y = \dfrac{3x}{\left(4 + 6x^2\right)^{\frac{1}{2}}} + x,$ the x-axis and the line $x = 1$. 2 marks

END OF SAC 1b

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, \ n \neq -1$
$\frac{d}{dx}\left((ax+b)^n\right) = an(ax+b)^{n-1}$	$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, n \neq -1$
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e(x) + c, \ x > 0$
$\frac{d}{dx}\left(\sin(ax)\right) = a \cos(ax)$	$\int \sin(ax) dx = -\frac{1}{a}\cos(ax) + c$
$\frac{d}{dx}\left(\cos(ax)\right) = -a \sin(ax)$	$\int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$
$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a \sec^2(ax)$	

product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$		