	Scotch Student ID #			
Circle the relevant digits	0	0	0	0
	1	1	1	1
	2	2	2	2
	3	3	3	3
	4	4	4	4
	5	5	5	5
	6	6	6	6
	7	7	7	7
	8	8	8	8
	9	9	9	9

Teacher's Name

Scotch College

MATHEMATICAL METHODS

U4-SAC 2b – Application Task: Test

Thursday 8th September 2022

Reading Time	none
Writing Time	45 minutes

Task Sections	Marks	Your Marks
Extended Response Questions	30	
Total Marks	**30**	

Remote Declaration

I declare that any work I have submitted for this VCE assessment is wholly my own, unless properly referenced or authorised for use by my teacher. I have had no assistance from any person in my home nor have I been assisted by, or given assistance to, a boy in my class or cohort unless specifically permitted to do so by my teacher. I have not used the internet or other sources to assist me in my responses unless specifically permitted by my teacher. I acknowledge my work may be reproduced, communicated, compared and archived for the purposes of detecting plagiarism and collusion.

Signature: _____

General Instructions

- Answer all questions in the spaces provided.
- In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working must be shown.
- Unless otherwise indicated, the diagrams in this task are not drawn to scale.

Allowed Materials

- Calculators are allowed.
- Notes and/or references are not allowed.

At the end of the task

- Ensure you cease writing upon request.

Electronic Devices

Students are **not** allowed to have a mobile phone, smart watch and/or any other unauthorised electronic device in the SAC, unless it is TURNED OFF and is placed on the front teacher desk.

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Question 1 (7 marks)

The number of boats sold by a salesman on a typical weekend, X, is found to have the following probability distribution:

x	0	1	2	3	4	5
$\Pr(X = x)$	0.15	0.2	0.3	a	0.1	0.05

a. Show that $a = 0.2$. 1 mark

b. What is the expected number of boats sold by a salesman on a typical weekend? 1 mark

c. Find $E(X^2)$ and hence determine the standard deviation of X, correct to three decimal places. 2 marks

d. Find $\Pr(X \geq 3 | X < 5)$. 1 mark

e. The salesman receives a commission, $\$C$, of \$200 per boat for the first two boats he sells over the weekend and \$300 per boat for each additional boat. What is the expected value of the commission, $E(C)$? 2 marks

Question 2 (6 marks)

A student is completing a Mathematical Methods examination but has not completed enough study for the subject. Therefore, he decides to guess the answers to all of the multiple choice questions. Multiple choice questions have five options, only one of which is correct. There are 20 multiple choice questions on the examination. Let X be the number of multiple choice questions that the student guesses correctly.

a. What is the probability that the student guesses the answers to the first three questions correctly? 1 mark

b. What is the probability that he guesses exactly five of the correct answers? Give your answer correct to four decimal places. 1 mark

c. What is the probability that he guesses 12 or more correct answers, given that he guesses the correct answer to more than half of the questions? Give your answer correct to four decimal places. 2 marks

Another student is particularly lazy and only guesses the answers to some questions, leaving the rest blank.

d. What is the smallest number of guesses that this student must make to ensure the probability of obtaining at least five correct answers is more than 0.1? 2 marks

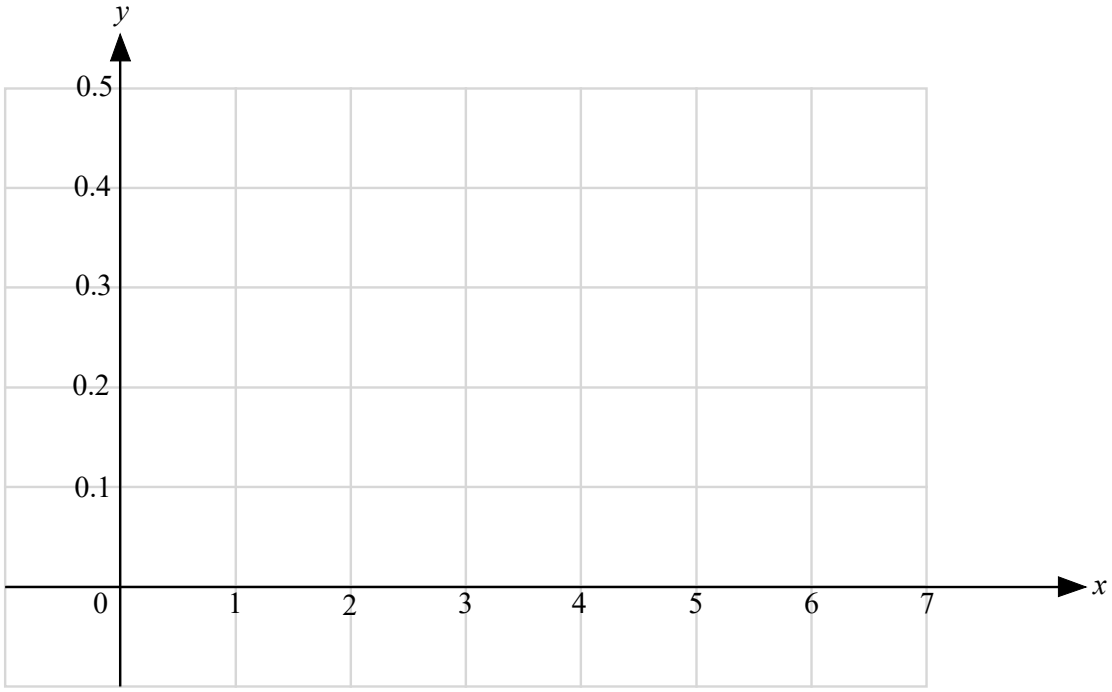
Question 3 (9 marks)

A random variable X has the probability density function with rule

$$f(x)=\begin{cases}\dfrac{1}{10} & 0\le x<4\\ \dfrac{1}{10}+k\sin\left(\dfrac{\pi}{2}(x-4)\right) & 4\le x\le 6\\ 0 & \text{elsewhere}\end{cases}$$

a. Show that the value of k is $\dfrac{\pi}{10}$. 3 marks

b. Sketch the graph of the probability density function $y=f(x)$ on the axes below giving the coordinates of any intercepts, endpoints and turning points as exact values. 3 marks



c. Determine $E(X)$. 1 mark

d. Calculate the median of X, correct to three decimal places. 2 marks

Question 4 (8 marks)

The length in centimetres of brown trout in Victorian rivers, X, is normally distributed with a mean of 39 centimetres and a standard deviation of 5 centimetres.

a. Find the probability that a randomly caught brown trout has a length which is between 37 and 45 centimetres. Give your answer correct to four decimal places. 1 mark

b. From a catch of eight brown trout, find the probability that less than three are between 37 and 45 centimetres. Give your answer correct to four decimal places. 2 marks

Another species of fish, the rainbow trout, is also found in Victorian rivers and has a normally distributed length. 35% of rainbow trout caught are under the legal length of 25 centimetres and must be thrown back.

c. If 88% of all rainbow trout of legal length are also under 50 cm in length, find the mean and standard deviation of all rainbow trout caught. Give both answers correct to four decimal places. 2 marks

In one particular river in Victoria, 40% of the trout are rainbow trout and the remainder are brown trout. To catch a brown trout legally, they must have a length of at least 34 centimetres. Correct to four decimal places, the probability of catching a brown trout under the legal length of 34 centimetres is 0.1587.

d. What is the probability that a fish of legal length caught in this river is a brown trout? Give your answer correct to two decimal places. 2 marks

e. Given a person catches exactly three brown trout out of their first seven fish caught, calculate the probability that they caught three brown trout in a row. 1 mark

END OF SAC 2b

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$		$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, \ n \neq -1$	
$\frac{d}{dx}\left((ax+b)^n\right) = an(ax+b)^{n-1}$		$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, n \neq -1$	
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$		$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$	
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$		$\int \frac{1}{x} dx = \log_e(x) + c, \ x > 0$	
$\frac{d}{dx}\left(\sin(ax)\right) = a\ \cos(ax)$		$\int \sin(ax) dx = -\frac{1}{a}\cos(ax) + c$	
$\frac{d}{dx}\left(\cos(ax)\right) = -a\ \sin(ax)$		$\int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$	
$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\ \sec^2(ax)$			
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A\|B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = \mathrm{E}(X)$	variance	$\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

Probability distribution		**Mean**	**Variance**
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x-\mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$\mathrm{E}(\hat{P}) = p$
standard deviation	$\mathrm{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

END OF FORMULA SHEET