| | Scotch Student ID # | | | |
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| Circle the relevant digits | 0 | 0 | 0 | 0 |
| | 1 | 1 | 1 | 1 |
| | 2 | 2 | 2 | 2 |
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**Scotch College**

| Teacher's Name |
|---|
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# MATHEMATICAL METHODS

## Unit 3-SAC 1b – Application Task: Test

**June 2023**

| | |
|---|---|
| **Reading Time** | none |
| **Writing Time** | 45 minutes |

| Task Sections | Marks | Your Marks |
|---|---|---|
| Extended Response Questions | 30 | |
| **Total Marks** | **30** | |

**Declaration**

*I declare that any work I have submitted for this VCE assessment is wholly my own, unless properly referenced or authorised for use by my teacher. I have had no assistance from any person in my home nor have I been assisted by, or given assistance to, a boy in my class or cohort unless specifically permitted to do so by my teacher. I have not used the internet or other sources to assist me in my responses unless specifically permitted by my teacher. I acknowledge my work may be reproduced, communicated, compared and archived for the purposes of detecting plagiarism and collusion.*

*Signature:* _____

**General Instructions**

- Answer all questions in the spaces provided.
- In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working must be shown.
- Unless otherwise indicated, the diagrams in this task are not drawn to scale.

**Allowed Materials**

- Calculators are not allowed
- Notes and/or references are not allowed.

**At the end of the task**

- Ensure you cease writing upon request.

**Electronic Devices**

Students are **not** allowed to have a mobile phone, smart watch and/or any other unauthorised electronic device in the SAC, unless it is TURNED OFF and is placed on the front teacher desk.

**Question 1** (4 marks)

**a.** If $y = \dfrac{\log_e(3x)}{2x}$, show that $\dfrac{dy}{dx} = \dfrac{1-\log_e(3x)}{2x^2}$. 2 marks

**b.** Let $f(x) = x(2x+1)^3$. Find $f'(1)$. 2 marks

**Question 2** (7 marks)

Consider the polynomial function $p$ with rule $p(x) = x^3 - 3x^2 - 8x + 10$ over the domain $[m, n)$ where $m, n \in \mathbb{R}$.

**a.** **i.** Show that $(x-1)$ is a factor of $p(x)$. 1 mark

**ii.** Hence, express $p(x)$ in the form $p(x) = f(x) \times g(x)$, where $f(x) = x - 1$ and $g(x) = x^2 - ax - b$ where $a, b \in \mathbb{Z}$. 1 mark

**b.** The domain of $f$ is $(-10, 10)$ and the domain of $g$ is $[1, \infty)$. Using this information, find the domain of $p$, in the form $[m, n)$. 1 mark

**c.** The function $f$ undergoes the following sequence of transformations to produce the function $f_t$.

- Dilation of factor 2 from the $x$-axis
- Reflection in the $x$-axis
- Translation of 3 units in the positive direction of the $x$-axis.

Write down the rule for the transformed function $f_t$. 2 marks

**d.** Find the domain and range for the transformed function $f_t$. 2 marks

**Question 3** (4 marks)

Let $f(x) = \sqrt{x-3}$ and $g(x) = x^2 - 13,$ which are both defined over their maximal domains.

**a.** State the maximal domain and range of $g \circ f$. 2 marks

**b.** Find the maximal domain of $g$ such that $f \circ g$ is defined. 2 marks

**Question 4** (4 marks)

A hybrid function $h$ is defined as follows:

$$h(x)=\begin{cases}\log_e(x)+a & \text{for } 1\le x<e\\ e^{x-(e+b)} & \text{for } e\le x\le 2e\end{cases}$$

where $a,b\in\mathbb{Z}$.

**a.** Express $a$ in terms of $b$ if $h$ is a continuous function. 2 marks

**b.** Assuming $h$ is a continuous function, find the value of $b$ such that $h$ is differentiable for $x\in(1,2e)$. 2 marks

**Question 5** (2 marks)

Let $T_1 : \mathbb{R}^2 \to \mathbb{R}^2,\ T_1(x, y) = (x - 2,\ 2y)$ and $T_2 : \mathbb{R}^2 \to \mathbb{R}^2,\ T_2(x, y) = (3x, -y)$.

Find the image of the curve $y = \frac{1}{x},$ under the transformation $T_1 \circ T_2$

**Question 6** (6 marks)

Consider the function $f:(a,\infty)\to\mathbb{R},\ f(x)=-4\log_e\left(\sqrt{2x+3}\right)$, where $a$ is the smallest real number such that $f$ is defined.

**a.** What is the value of $a$? 1 mark

**b.** The function $f(x)=-4\log_e\left(\sqrt{2x+3}\right)$ can be written as $f(x)=k\log_e(2x+3)$.

Show that $k=-2$. 1 mark

**c.** List the sequence of transformations which maps the graph $y=f(x)$ to the graph $y=6\log_e(x-5)$. 2 marks

**d.** The gradient of the normal to the graph of $f$ at the point $\big(b, f(b)\big)$ is $\frac{2}{3}$.

Find the value of $b$. 2 marks

**Question 7** (3 marks)

The function $g(x) = 25 - x^2$ has a tangent at the point $(p,\ g(p))$ which has the equation $y = -2px + p^2 + 25,$ where $p > 0.$

Find the value of $p$ for which the area enclosed by the tangent at the point $(p,\ g(p))$, the $x$-axis and the $y$-axis is a minimum and find this minimum area.

**END OF SAC 1b**