	Scotch Student ID #			
Circle the relevant digits	0	0	0	0
	1	1	1	1
	2	2	2	2
	3	3	3	3
	4	4	4	4
	5	5	5	5
	6	6	6	6
	7	7	7	7
	8	8	8	8
	9	9	9	9

Teacher's Name

Scotch College

MATHEMATICAL METHODS

Unit 3-SAC 1c – Application Task: Test

June 2023

Reading Time	none
Writing Time	45 minutes

Task Sections	Marks	Your Marks
Extended Response Questions	30	
Total Marks	**30**	

Declaration

I declare that any work I have submitted for this VCE assessment is wholly my own, unless properly referenced or authorised for use by my teacher. I have had no assistance from any person in my home nor have I been assisted by, or given assistance to, a boy in my class or cohort unless specifically permitted to do so by my teacher. I have not used the internet or other sources to assist me in my responses unless specifically permitted by my teacher. I acknowledge my work may be reproduced, communicated, compared and archived for the purposes of detecting plagiarism and collusion.

Signature: ______________________________

General Instructions

- Answer all questions in the spaces provided.
- In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working must be shown.
- Unless otherwise indicated, the diagrams in this task are not drawn to scale.

Allowed Materials

- Calculators are allowed
- Notes and/or references are not allowed.

At the end of the task

- Ensure you cease writing upon request.

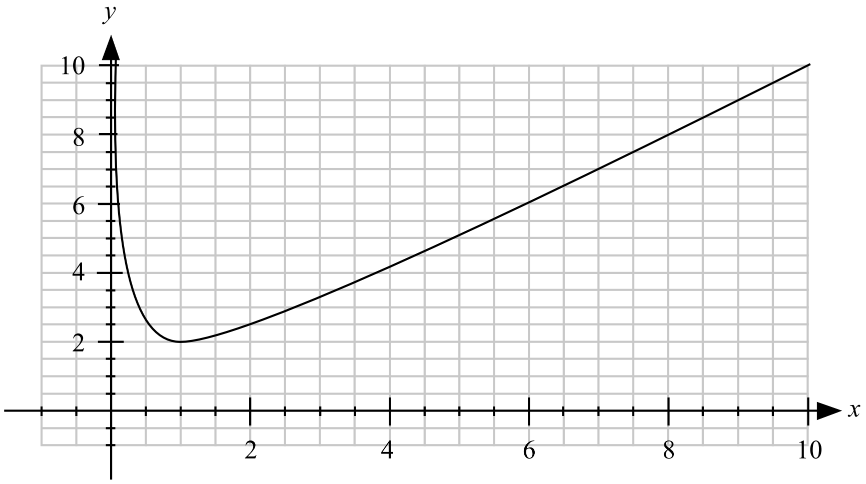
Electronic Devices

Students are **not** allowed to have a mobile phone, smart watch and/or any other unauthorised electronic device in the SAC, unless it is TURNED OFF and is placed on the front teacher desk.

Question 1 (6 marks)

Functions *f* and *g* are defined as $f:\mathbb{R}\setminus\{0\}\to\mathbb{R},\ f(x)=\frac{1}{x}$ and $g:[0,\infty)\to\mathbb{R},\ g(x)=x$.

Let $k(x)=f(x)+g(x)$. Part of the graph of $y=k(x)$ is shown below.



a. Show that $k(x)=\frac{x^2+1}{x}$ 1 mark

b. Determine the domain of *k*. 1 mark

c. Show that $k'(x)=\frac{(x-1)(x+1)}{x^2}$ 2 marks

d. Determine the coordinates of the stationary point on the graph of $y = k(x)$. 1 mark

e. Determine the values of x for which $k(x)$ is strictly increasing. 1 mark

Question 2 (12 marks)

Consider the function given by $f(x) = 3(x-1)^3 e^{(1-x)} + 3$.

a. The graph of f has a horizontal asymptote at $y = a$. State the value of a. 1 mark

b. The graph of f passes through the point $(0, b)$. Find the exact value of b. 1 mark

c. Consider the function $g_1 : \mathbb{R} \to \mathbb{R}, g_1(x) = f(2x - h) + k,$ where h and k are positive real numbers. List the sequence of transformations which maps the graph of $y = f(x)$ to the graph of $y = g_1(x)$. 3 marks

d. Consider the function $g_2 : \mathbb{R} \to \mathbb{R}, g_2(x) = f(2x - p) + q,$ where p and q are real numbers.

i. Determine the coordinates of the y-intercept and the stationary points of g_2, giving your answers in terms of p and q. 2 marks

ii. Hence or otherwise, find the value(s) of q for which g_2 has exactly one x-intercept. 2 marks

iii. If $p = q$, find the non zero value(s) of q for which the graph of g_2 cuts the y-axis at $y = \frac{1 - e^q}{2}$, giving your answer(s) correct to two decimal places. 1 mark

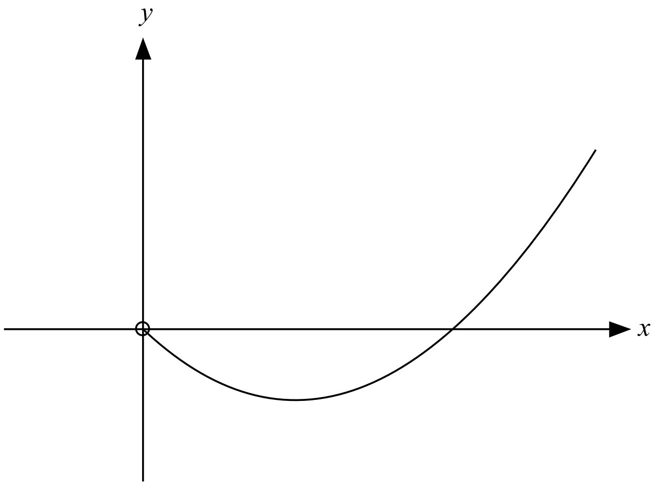
e. Let $g_3 : \mathbb{R} \to \mathbb{R}$, where $g_3(x) = (x - a)^3 e^{(1-x)}$, where $a \in \mathbb{R}$.

The coordinates of the stationary points of g_3 are $A(a, 0)$ and $B\left(a + r + s, 27e^{-(a+r)}\right)$, where r and s are positive integers.

Find the values of r and s. 2 marks

Question 3 (4 marks)

Let $f:(0,\infty)\rightarrow\mathbb{R}, f(x)=x\log_e(x)-x$. Part of the graph of f is shown below.



a. Find the values of x for which $f(x)>0$. 1 mark

b. The equation of the tangent to the graph of $y=f(x)$ at the point $(\sqrt{n}, f(\sqrt{n}))$ is $y=\frac{\log_e(n)}{2}x+b$. Find b in terms of n. 1 mark

c. Find the value(s) of n for which the tangents to the graph of $y=f(x)$ at the points with coordinates $(n, f(n))$ and $(\frac{1}{n}, f(\frac{1}{n}))$ are perpendicular, giving your answer(s) correct to two decimal places. 2 marks

Question 4 (8 marks)

Functions f and g are defined as $f:(-14,\infty)\to\mathbb{R}, f(x)=\log_e(\frac{x}{2}+7)$ and $g:(0,\infty)\to\mathbb{R}, g(x)=\log_e(\frac{x}{2})$.

Let $h(x)=f(x)+g(x)$, for $x\in(0,\infty)$.

a. The x-intercept of function h is $x=\sqrt{a}-b$. Find the value of a and b. 1 mark

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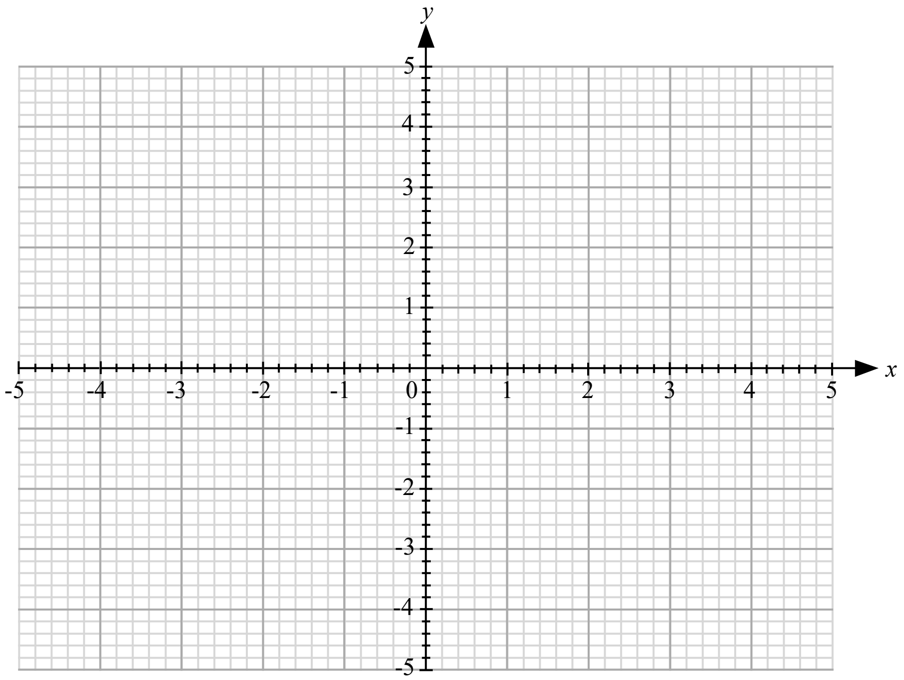
b. Sketch the graph $y=h(x)$ on the set of axes below. Label the asymptote with its equation and the axial intercept with its coordinates correct to two decimal places. 2 marks



c. **i.** Find the rule for the inverse function h^{-1}. 1 mark

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ii. Find the coordinates of the point(s) of intersection of the graphs of $y = h(x)$ and $y = h^{-1}(x)$, giving your answer(s) correct to two decimal places. 1 mark

Consider the function $h_2(x) = \log_e\left(\frac{x}{k} + 7\right) + \log_e\left(\frac{x}{k}\right)$ where k is a positive real constant.

d. Find the value of x in terms of k such that $h_2'(x) = 1$. 1 mark

e. Hence or otherwise, find the value of k so that the graphs of h_2 *and* h_2^{-1} have only one point of intersection. Give your answer correct to two decimal places. 2 marks

END OF SAC 1c

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, \ n \neq -1$
$\frac{d}{dx}\left((ax+b)^n\right) = an\left(ax+b\right)^{n-1}$	$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, n \neq -1$
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e(x) + c, \ x > 0$
$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$	$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$
$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$	$\int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$
$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$	

product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$	Newton's method	$x_{n+1} = x_n - \frac{f\left(x_n\right)}{f'\left(x_n\right)}$
trapezium rule approximation	$Area \approx \frac{x_n - x_0}{2n}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + 2f(x_{n-2}) + 2f(x_{n-1}) + f(x_n)\right]$		