	Scotch Student ID #			
Circle the relevant digits	0	0	0	0
	1	1	1	1
	2	2	2	2
	3	3	3	3
	4	4	4	4
	5	5	5	5
	6	6	6	6
	7	7	7	7
	8	8	8	8
	9	9	9	9

Teacher's Name

Scotch College

MATHEMATICAL METHODS

U4-SAC 1 – Application Task

2023

Task Sections	Marks	Your Marks
Investigation	25	
Total Marks	**25**	

Declaration

I declare that any work I have submitted for this VCE assessment is wholly my own, unless properly referenced or authorised for use by my teacher. I have had no assistance from any person in my home nor have I been assisted by, or given assistance to, a boy in my class or cohort unless specifically permitted to do so by my teacher. I have not used the internet or other sources to assist me in my responses unless specifically permitted by my teacher. I acknowledge my work may be reproduced, communicated, compared and archived for the purposes of detecting plagiarism and collusion.

Signature: _____

General Instructions

- Answer all questions in the spaces provided.
- In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

Allowed Materials

- A scientific calculator and a CAS calculator.
- One double-sided A4 page of notes to be handed in at the end of each session.

At the end of the task

- Submit the task to your teacher.

Component I

Consider the family of functions of the form $f:[0,2\pi]\to\mathbb{R}$, $f(x)=a\cos(nx)$ where a and n are positive integers.

a. Explain how the values of a and n affect the graph of $f(x)$.

b. Consider the case where $a=1$ and $n=1$. Calculate the value of the area bounded by the graph of $f(x)$, the x-axis, and the lines $x=0$ and $x=2\pi$. Show the steps of your calculations.

c. Graph the function $f(x)$ for different values of a and n. Include at least three different cases, each demonstrating a unique characteristic of the graph. For each case, describe how the values of a and n influence the shape of the graph.

d. By calculating the area bounded by the function, the x-axis and the lines $x = 0$ and $x = 2\pi$, investigate the impact of changing a and n on this bounded area.

Discuss any patterns or relationships you observe between the values of a, n and the resulting area.

Component II

Consider functions of the form $f:[0,2\pi]\to\mathbb{R},\ f(x)=a\cos(nx)$

and $g:[0,2\pi]\to\mathbb{R},\ g(x)=a\sin(nx)$

where *a* and *n* are positive integers.

a. Consider the case where $a=1$ and $n=1$. Calculate the total area of the regions bounded by the graph of $f(x)$ and $g(x)$ and the lines $x=0$ and $x=2\pi$. Show the steps of your calculations.

b. Consider the case where $a = 2$ and $n = 2$. Calculate the total area of the regions bounded by the graph of $f(x)$ and $g(x)$ and the lines $x = 0$ and $x = 2\pi$. Show the steps of your calculations.

c. Calculate the value of the total area of the regions bounded by the graph of $f(x)$ and $g(x)$ and the lines $x = 0$ and $x = 2\pi$ in terms of *a* and *n* and thus explain the significance of this result when analysing the area as *a* and *n* vary.

Component III

a. Consider the function $f:[0,4]\to\mathbb{R},\ f(x)=\sqrt{x}-x.$

i. Calculate the average value of the function f from $x=0$ to $x=4.$

ii. Calculate the signed area of function f from $x=0$ to $x=4.$

iii. Comment on the values of the average value and the signed area of the function f from $x=0$ to $x=4.$

b. Consider the function $g(x) = a\,f(x) + c$ over the domain $x \in [0, 4]$, where a is a positive constant and c is a real constant.

i. Explain how the values of a and c affect the graph of $g(x)$.

ii. Investigate the impact of changing a and c on the average value of the function g and the signed area of function g from $x = 0$ to $x = 4$. Discuss any patterns or relationships you observe as a and c change.

c. Consider the function $h(x)=f\left(n(x+b)\right)$ over the domain $x\in\left[-b,\frac{1}{n}-b\right]$ where n is a positive constant and b is a real constant.

i. Explain how the values of n and b affect the graph of $h(x)$.

ii. Investigate the impact of changing b and n on the average value of the function h and the signed area of function h over the domain $x\in\left[-b,\frac{1}{n}-b\right]$. Discuss any patterns or relationships you observe as b and n change.

END OF SAC 1

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$		$\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$	
$\frac{d}{dx}\left((ax+b)^n\right) = an(ax+b)^{n-1}$		$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, n \neq -1$	
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$		$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$	
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$		$\int \frac{1}{x} dx = \log_e(x) + c,\ x > 0$	
$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$		$\int \sin(ax) dx = -\frac{1}{a}\cos(ax) + c$	
$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$		$\int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$	
$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$			
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$	Newton's method	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
trapezium rule approximation	$Area \approx \frac{x_n - x_0}{2n}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + 2f(x_{n-2}) + 2f(x_{n-1}) + f(x_n)\right]$		