	Scotch Student ID #			
Circle the relevant digits	0	0	0	0
	1	1	1	1
	2	2	2	2
	3	3	3	3
	4	4	4	4
	5	5	5	5
	6	6	6	6
	7	7	7	7
	8	8	8	8
	9	9	9	9

Teacher's Name

Scotch College

MATHEMATICAL METHODS

Unit 4-SAC 2b – Investigation Task: Test

September 2023

Reading Time	none
Writing Time	45 minutes

Task Sections	Marks	Your Marks
Extended Response Questions	30	
Total Marks	**30**	

Declaration
I declare that any work I have submitted for this VCE assessment is wholly my own, unless properly referenced or authorised for use by my teacher. I have had no assistance from any person in my home nor have I been assisted by, or given assistance to, a boy in my class or cohort unless specifically permitted to do so by my teacher. I have not used the internet or other sources to assist me in my responses unless specifically permitted by my teacher. I acknowledge my work may be reproduced, communicated, compared and archived for the purposes of detecting plagiarism and collusion.   *Signature:* ______________________________

General Instructions

- Answer all questions in the spaces provided.
- In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working must be shown.
- Unless otherwise indicated, the diagrams in this task are not drawn to scale.

Allowed Materials

- Calculators are allowed
- Notes and/or references are allowed.

At the end of the task

- Ensure you cease writing upon request.

Electronic Devices

Students are **not** allowed to have a mobile phone, smart watch and/or any other unauthorised electronic device in the SAC, unless it is TURNED OFF and is placed on the front teacher desk.

Question 1 (8 marks)

Let X represent the number of books borrowed by a student at the school library each month.

The probability distribution of X is shown below:

x	0	1	2	3	4
$\Pr(X = x)$	0.05	0.12	0.46	0.27	k

a. Find the value of k. 1 mark

b. Given a student borrows less than four books from the library in a certain month, calculate the probability they borrow at least one book that month. 2 marks

c. If a student borrows at least two books in a certain month, they receive a sticker.

i. Calculate the probability a student receives a sticker in a certain month. 1 mark

ii. Let Y represent the number of stickers a student receives in a certain year.
Calculate $\Pr(Y \geq 9)$, giving your answer correct to four decimal places. 2 marks

d. Calculate the smallest number of months required so that the probability of receiving at least 20 stickers exceeds 0.9. 2 marks

Question 2 (4 marks)

Random variable X is normally distributed with a mean of 30 and a standard deviation of 6.

Let Z be the random variable with the standard normal distribution.

a. Given $\Pr(X \geq a) = \Pr(Z \geq 2)$, find the exact value of a. 1 mark

b. Given $\Pr(X \geq 34) = \Pr(Z \leq b)$, find the exact value of b. 1 mark

c. Given $\Pr(X \leq 20) = \Pr(-c \leq Z \leq c)$, find the value of c, giving your answer correct to four decimal places. 2 marks

Question 3 (4 marks)

The time taken, T minutes, for Scotch students to travel to school was found to be normally distributed. 14% of students take less than 8 minutes to travel to school and 21% of students take more than 30 minutes to travel to school.

a. Given a student takes longer than 8 minutes to get to school, calculate the probability they take less than 30 minutes. Give your answer correct to four decimal places. 1 mark

b. Calculate the mean and standard deviation of the time taken to get to school, giving your answers correct to four decimal places. 3 marks

Question 4 (12 marks)

The time, T minutes, a student will spend waiting in the queue for the Senior School tuckshop on a given day is a random variable with a probability density function given by:

$$f(t)=\begin{cases} 0 & t<0 \\ k & 0\le t\le 5 \\ \dfrac{2}{11}e^{-2(t-5)} & t>5 \end{cases}$$

a. Show that $k=\dfrac{2}{11}$, showing all steps of your answer. 2 marks

b. Calculate the probability that a student will spend less than 6 minutes in the Senior School tuckshop queue, giving your answer correct to four decimal places. 1 mark

c. Calculate the expected time a student will spend in the Senior School tuckshop queue on a given day, giving your answer correct to the nearest second. 2 marks

The time, Y minutes, spent in the Junior School tuckshop queue is found to be normally distributed with a mean of 2 minutes and a standard deviation of 0.8.

d. Write down the variance of Y. 1 mark

e. Calculate the probability that a student will spend more than 3 minutes in the Junior School tuckshop queue, giving your answer correct to four decimal places. 1 mark

f. 10 students attend the Junior School tuckshop. Let K represent the number of these students who spent over 3 minutes in the queue. Calculate $\Pr(K \geq 3)$, giving your answer correct to four decimal places. 2 marks

g. An equal number of students attend the Senior School and Junior School tuckshops on a certain day. Given a student waited less than 1 minute in the tuckshop queue, find the probability the student attended the Senior School tuckshop. Give your answer correct to four decimal places. 3 marks

Question 5 (2 marks)

The probability a Scotch student plays Football is 0.2.

The probability a Scotch student **does not** play Cricket is 0.6.

Given a Scotch student plays Cricket, the probability they **do not** play Football is p^2.

Find the set of all possible values of p, giving your answer in the form $p \in [a, b] \cup [c, d]$

where a, b, c and d are real numbers.

END OF SAC 2b

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$		$\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$	
$\frac{d}{dx}\left((ax+b)^n\right) = an\left(ax+b\right)^{n-1}$		$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, n \neq -1$	
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$		$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$	
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$		$\int \frac{1}{x} dx = \log_e(x) + c,\ x > 0$	
$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$		$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$	
$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$		$\int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$	
$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$			
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$	Newton's method	$x_{n+1} = x_n - \frac{f\left(x_n\right)}{f'\left(x_n\right)}$
trapezium rule approximation	$Area \approx \frac{x_n - x_0}{2n}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + 2f(x_{n-2}) + 2f(x_{n-1}) + f(x_n)\right]$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A \mid B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = \mathrm{E}(X)$	variance	$\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$
binomial coefficient	$\binom{n}{x} = \dfrac{n!}{x!(n-x)!}$		

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x-\mu)^2\, p(x)$
binomial	$\Pr(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$	$\mu = np$	$\sigma^2 = np(1-p)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)\,dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)\,dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$\mathrm{E}(\hat{P}) = p$
standard deviation	$\mathrm{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

END OF FORMULA SHEET