	Scotch Student ID #			
Circle the relevant digits	0	0	0	0
	1	1	1	1
	2	2	2	2
	3	3	3	3
	4	4	4	4
	5	5	5	5
	6	6	6	6
	7	7	7	7
	8	8	8	8
	9	9	9	9

Teacher's Name

Scotch College

MATHEMATICAL METHODS

U3-SAC 1 – Application Task

2023

Task Sections	Marks	Your Marks
Investigation	25	
Total Marks	**25**	

Declaration
I declare that any work I have submitted for this VCE assessment is wholly my own, unless properly referenced or authorised for use by my teacher. I have had no assistance from any person in my home nor have I been assisted by, or given assistance to, a boy in my class or cohort unless specifically permitted to do so by my teacher. I have not used the internet or other sources to assist me in my responses unless specifically permitted by my teacher. I acknowledge my work may be reproduced, communicated, compared and archived for the purposes of detecting plagiarism and collusion.   *Signature:* ______________________________________

General Instructions

- Answer all questions in the spaces provided.
- In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

Allowed Materials

- A scientific calculator and a CAS calculator.
- Any notes or references.

At the end of the task

- Submit the task to your teacher.

Component I

a. Consider functions of the form $f:\mathbb{R}\to\mathbb{R},\ f(x)=e^{3x}-k$ where k is a real constant.

i. Investigate for which value(s) of k the graph of $y=f(x)$ has a positive y-intercept, a y-intercept of zero and a negative y-intercept respectively.

ii. Investigate for which value(s) of k the graph of $y=f(x)$ has a positive x-intercept, an x-intercept at $x=0$ and a negative x-intercept respectively.

b. Consider functions of the form $f : D \rightarrow \mathbb{R},\ f(x) = \log_e(3x - h)$, where h is a real constant.

i. Find the maximal domain D of f, giving your answer in terms of h.

ii. Investigate for which value(s) of h the graph of $y = f(x)$ has an x-intercept at point $(2, 0)$.

iii. Investigate for which value(s) of h the graph of $y = f(x)$ has a negative x-intercept.

Component II

a. Consider functions of the form $f:\mathbb{R}\to\mathbb{R},\ f(x)=e^{ax}-k$ where a and k are real constants and $a>0$.

i. Investigate for which value(s) of k the graph of $y=f(x)$ has a positive y-intercept, a y-intercept of zero and a negative y-intercept respectively.

ii. Investigate for which value(s) of k the graph of $y=f(x)$ has a positive x-intercept, an x-intercept at $x=0$ and a negative x-intercept respectively.

b. Consider functions of the form $f:\mathbb{R}\to\mathbb{R},\ f(x)=ax^3-4ax+6$, where $a>0$.

i. By considering the coordinates of stationary points, or otherwise, investigate the number of x-intercepts the graph of $y=f(x)$ may have, giving the range of possible a values in each case.

ii. Let $a=2$. Find the value(s) of k for which the graph of $y=f(k-x)$ has zero, one, two and three positive x-intercepts, respectively.

Component III

Consider functions of the form $f:\mathbb{R}\to\mathbb{R},\ f(x)=ax^3+bx^2+cx+5$, where a, b and c are real constants. The graph of $y=f(x)$ has a turning point at $(2,-2)$.

a. Generate two simultaneous equations and hence express a and b in terms of c.

b. Hence investigate for which value(s) of c all stationary points on the graph of $y=f(x)$ have positive x-coordinates.

END OF SAC 1

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, \ n \neq -1$
$\frac{d}{dx}\left((ax+b)^n\right) = an(ax+b)^{n-1}$	$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, n \neq -1$
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e(x) + c, \ x > 0$
$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$	$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$
$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$	$\int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$
$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$	

product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$	Newton's method	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
trapezium rule approximation	$Area \approx \frac{x_n - x_0}{2n}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + 2f(x_{n-2}) + 2f(x_{n-1}) + f(x_n)\right]$		