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Scotch College

Teacher's Name

MATHEMATICAL METHODS

Unit 3-SAC 1b – Application Task: Test

June 2023

| Reading Time | none | |
|--------------|------------|--|
| Writing Time | 45 minutes | |

| Task Sections | Marks | Your Marks |
|-----------------------------|-------|------------|
| Extended Response Questions | 30 | |
| Total Marks | 30 | |

Declaration

I declare that any work I have submitted for this VCE assessment is wholly my own, unless properly referenced or authorised for use by my teacher. I have had no assistance from any person in my home nor have I been assisted by, or given assistance to, a boy in my class or cohort unless specifically permitted to do so by my teacher. I have not used the internet or other sources to assist me in my responses unless specifically permitted by my teacher. I acknowledge my work may be reproduced, communicated, compared and archived for the purposes of detecting plagiarism and collusion.

Signature:

General Instructions

- Answer all questions in the spaces provided.
- In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working must be shown.
- Unless otherwise indicated, the diagrams in this task are not drawn to scale.

Allowed Materials

- Calculators are not allowed
- Notes and/or references are not allowed.

At the end of the task

• Ensure you cease writing upon request.

Electronic Devices

Students are **not** allowed to have a mobile phone, smart watch and/or any other unauthorised electronic device in the SAC, unless it is TURNED OFF and is placed on the front teacher desk.

Question 1 (4 marks)

a. If
$$y = \frac{\log_e(3x)}{2x}$$
, show that $\frac{dy}{dx} = \frac{1 - \log_e(3x)}{2x^2}$. 2 marks

b. Let $f(x) = x(2x+1)^3$. Find f'(1).

Question 2 (7 marks)

Consider the polynomial function *p* with rule $p(x) = x^3 - 3x^2 - 8x + 10$ over the domain [m, n) where $m, n \in \mathbb{R}$.

a. i. Show that (x-1) is a factor of p(x).

1 mark

ii. Hence, express p(x) in the form $p(x) = f(x) \times g(x)$, where f(x) = x - 1 and $g(x) = x^2 - ax - b$ where $a, b \in \mathbb{Z}$.

1 mark

b. The domain of f is (-10,10) and the domain of g is $[1,\infty)$. Using this information, find the domain of p, in the form [m,n).

1 mark

- c. The function f undergoes the following sequence of transformations to produce the function f_t .
 - Dilation of factor 2 from the *x*-axis
 - Reflection in the *x*-axis
 - Translation of 3 units in the positive direction of the *x*-axis.

Write down the rule for the transformed function f_t .

d. Find the domain and range for the transformed function f_t .

2 marks

Question 3 (4 marks)

b.

Let $f(x) = \sqrt{x-3}$ and $g(x) = x^2 - 13$, which are both defined over their maximal domains.

a. State the maximal domain and range of $g \circ f$.

 Find the maximal domain of g such that $f \circ g$ is defined.
 2 marks

Question 4 (4 marks)

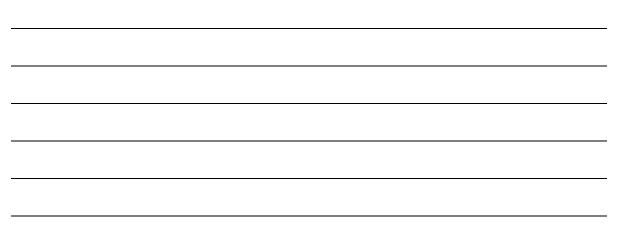
A hybrid function *h* is defined as follows:

$$h(x) = \begin{cases} \log_e(x) + a & \text{for } 1 \le x < e \\ e^{x - (e+b)} & \text{for } e \le x \le 2e \end{cases}$$

where $a, b \in \mathbb{Z}$.

a. Express a in terms of b if h is a continuous function.

b. Assuming *h* is a continuous function, find the value of *b* such that *h* is differentiable for $x \in (1, 2e)$.





Question 5 (2 marks) Let $T_1 : \mathbb{R}^2 \to \mathbb{R}^2$, $T_1(x, y) = (x - 2, 2y)$ and $T_2 : \mathbb{R}^2 \to \mathbb{R}^2$, $T_2(x, y) = (3x, -y)$. Find the image of the curve $y = \frac{1}{x}$, under the transformation $T_1 \circ T_2$

Question 6 (6 marks)

Consider the function $f:(a,\infty) \to \mathbb{R}$, $f(x) = -4\log_e(\sqrt{2x+3})$, where *a* is the smallest real number such that *f* is defined.

a. What is the value of *a*?

1 mark

b. The function $f(x) = -4\log_e(\sqrt{2x+3})$ can be written as $f(x) = k\log_e(2x+3)$. Show that k = -2.

1 mark

c. List the sequence of transformations which maps the graph y = f(x) to the graph $y = 6\log_e(x-5)$.

| d. | The gradient of the normal to the graph of f at the point $(b, f(b))$ is $\frac{2}{3}$. | |
|----|--|---------|
| | Find the value of <i>b</i> . | 2 marks |
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Question 7 (3 marks)

The function $g(x) = 25 - x^2$ has a tangent at the point (p, g(p)) which has the equation $y = -2px + p^2 + 25$, where p > 0.

Find the value of p for which the area enclosed by the tangent at the point (p, g(p)), the x-axis and the y-axis is a minimum and find this minimum area.

