	Scotch Student ID #			
Circle the relevant digits	0	0	0	0
	1	1	1	1
	2	2	2	2
	3	3	3	3
	4	4	4	4
	5	5	5	5
	6	6	6	6
	7	7	7	7
	8	8	8	8
	9	9	9	9

Scotch College

Teacher's Name

MATHEMATICAL METHODS

Unit 4-SAC 1b – Application Task: Test

August 2023

Reading Time	none
Writing Time	45 minutes

Task Sections	Marks	Your Marks
Extended Response Questions	30	
Total Marks	**30**	

Declaration

I declare that any work I have submitted for this VCE assessment is wholly my own, unless properly referenced or authorised for use by my teacher. I have had no assistance from any person in my home nor have I been assisted by, or given assistance to, a boy in my class or cohort unless specifically permitted to do so by my teacher. I have not used the internet or other sources to assist me in my responses unless specifically permitted by my teacher. I acknowledge my work may be reproduced, communicated, compared and archived for the purposes of detecting plagiarism and collusion.

Signature: _____

General Instructions

- Answer all questions in the spaces provided.
- In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working must be shown.
- Unless otherwise indicated, the diagrams in this task are not drawn to scale.

Allowed Materials

- Calculators are not allowed
- Notes and/or references are not allowed.

At the end of the task

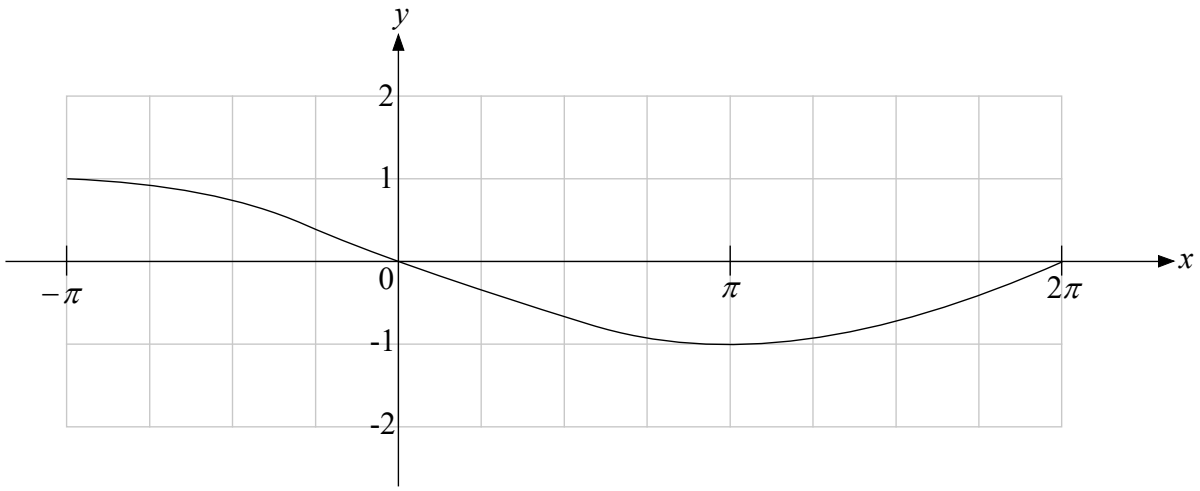
- Ensure you cease writing upon request.

Electronic Devices

Students are **not** allowed to have a mobile phone, smart watch and/or any other unauthorised electronic device in the SAC, unless it is TURNED OFF and is placed on the front teacher desk.

Question 1 (4 marks)

Part of the graph of $y = -\sin(nx)$ is shown on the axes below.



a. Find the value of n. 1 mark

The graph above is transformed under the following sequence of transformations.

1. Dilation of factor $\frac{1}{2}$ from the y-axis.
2. Reflection in the x-axis.
3. Translation of 1 unit in the negative direction of the y-axis.

b. On the axes above, sketch the transformed graph over the domain $x \in [-\pi, 2\pi]$.

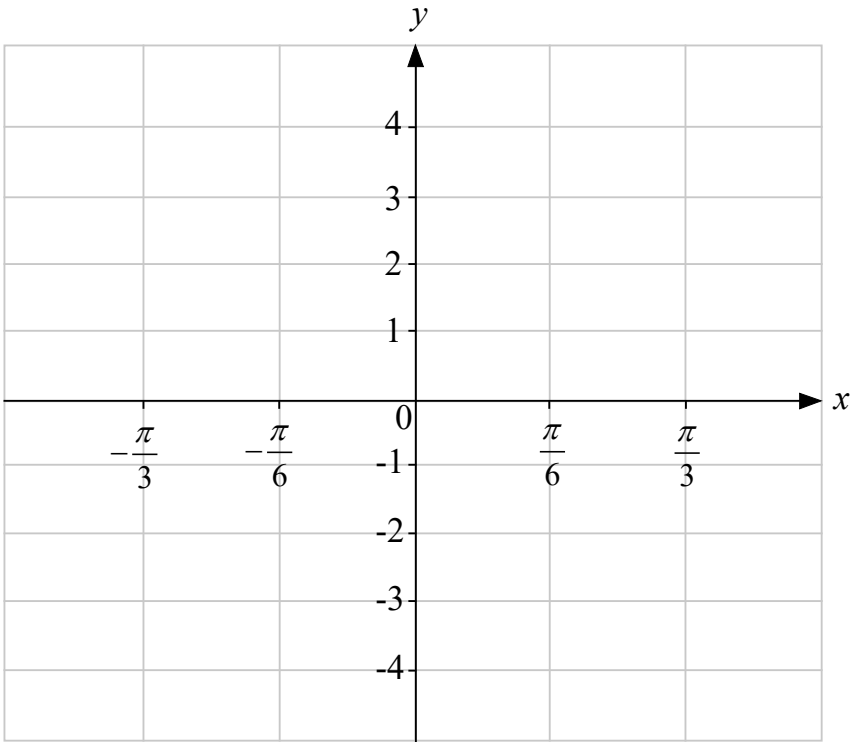
Label all end points, intercepts and stationary points with their coordinates. 3 marks

Question 2 (5 marks)

Let $f:\left[-\frac{\pi}{3},\frac{\pi}{3}\right]\to\mathbb{R}$, where $f(x)=\tan(3x)-1$.

a. Solve $f(x)=0$ for x. 2 marks

b. Sketch the graph of f on the axes below. Label any asymptotes with the appropriate equation and label the end points and axis intercepts with their coordinates. 3 marks



Question 3 (4 marks)

a. Evaluate $\int_{\frac{\pi}{2}}^{\pi}\left(x-\sin\left(x\right)\right)dx.$ 2 marks

b. Let $f'\left(x\right)=\left(2-x\right)^3$. Find $f\left(x\right)$ given that $f\left(1\right)=\frac{3}{2}.$ 2 marks

Question 4 (6 marks)

Let $f:[0,\pi]\to\mathbb{R},\ f(x)=\sin(2x)$

$g:[0,\pi]\to\mathbb{R},\ g(x)=k\cos(2x)$

a. If the graphs of $y=f(x)$ and $y=g(x)$ have a point of intersection at $x=\frac{2\pi}{3}$, show that the value of k is $\sqrt{3}$. 2 marks

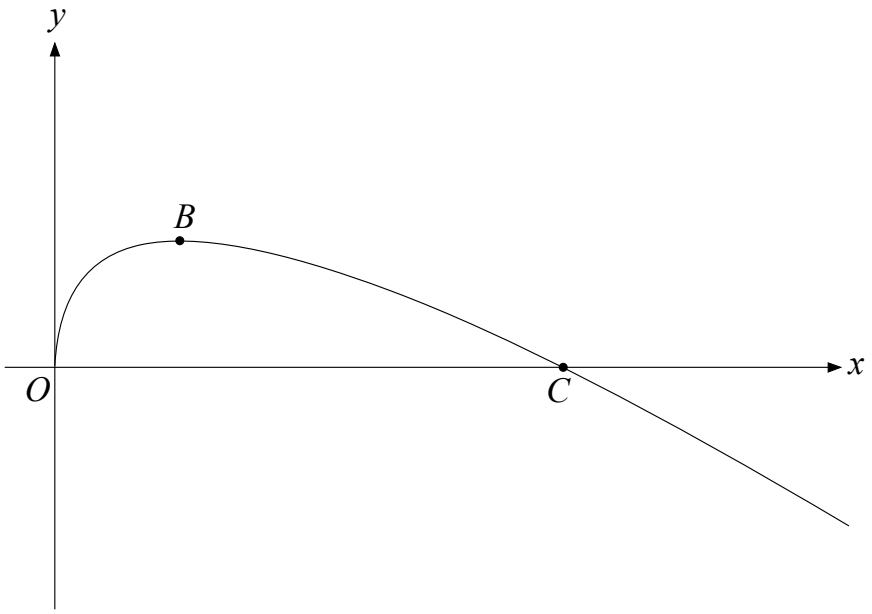
b. Find the x-coordinate of the other point of intersection of the two graphs. 1 mark

c. Hence, find the area enclosed between the two graphs. 3 marks

Question 5 (7 marks)

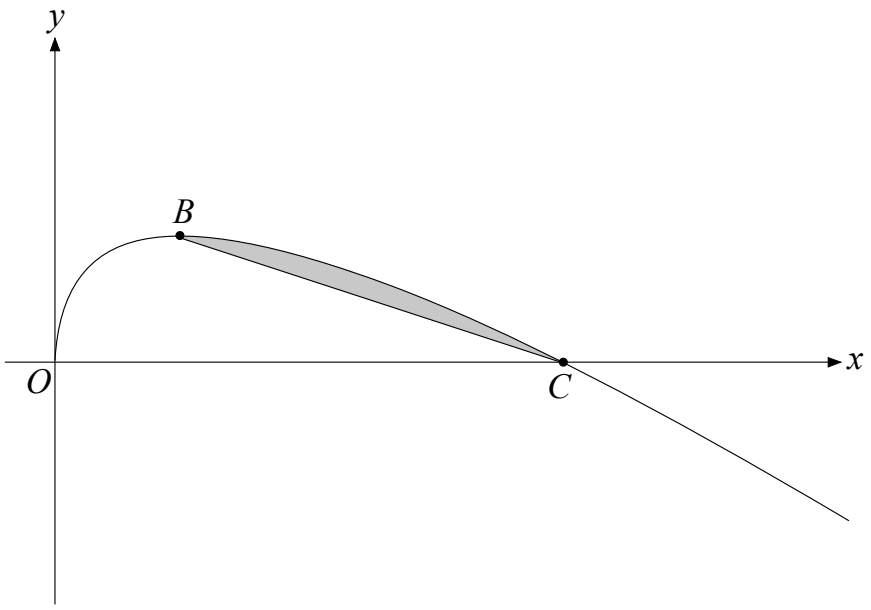
Consider the family of functions $f:[0,\infty)\to\mathbb{R},\ f(x)=2a\sqrt{x}-x$, where $a\in\mathbb{R}^+$.

Part of the graph of f is shown below.



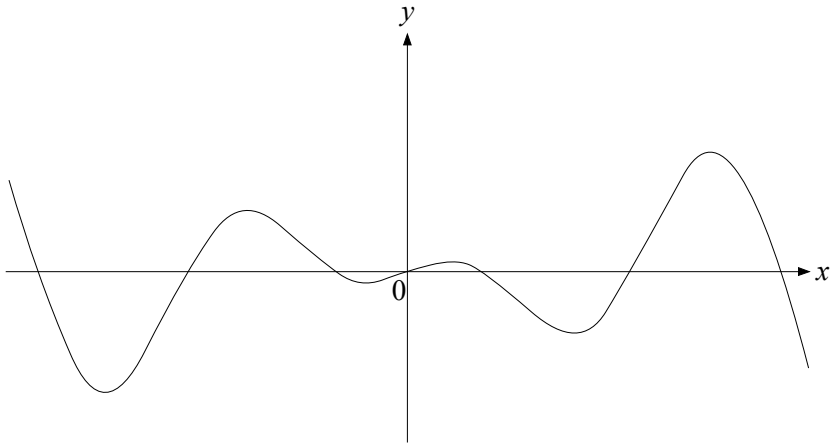
a. Point B is a stationary point and point C is an x-intercept. Show that the coordinates of points B and C, respectively, are (a^2,a^2) and $(4a^2,0)$. 4 marks

b. If the area of the region bounded by the graph of f and the line segment BC is 3 square units, find the value of a. 3 marks



Question 6 (4 marks)

Part of the graph of $f:\mathbb{R}\to\mathbb{R},\ f(x)=x\cos(2x)$ is shown below.



a. Find the derivative of $x\sin(2x)$. 1 mark

b. Hence, show that the average value of f on the interval $\left[\frac{\pi}{2},\pi\right]$ is $\frac{1}{\pi}$. 2 marks

c. Hence, or otherwise, evaluate $\int_{\pi+1}^{2\pi+1} 4f\left(\frac{x-1}{2}\right)+1\ dx$. 1 mark

END OF SAC 1b

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$		$\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$	
$\frac{d}{dx}\left((ax+b)^n\right) = an(ax+b)^{n-1}$		$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, n \neq -1$	
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$		$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$	
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$		$\int \frac{1}{x} dx = \log_e(x) + c,\ x > 0$	
$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$		$\int \sin(ax) dx = -\frac{1}{a}\cos(ax) + c$	
$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$		$\int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$	
$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$			
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$	Newton's method	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
trapezium rule approximation	$Area \approx \frac{x_n - x_0}{2n}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + 2f(x_{n-2}) + 2f(x_{n-1}) + f(x_n)\right]$		