

2021 Mathematical Methods (Unit 1-2)

Task 4

Paper 1 – Calculator not allowed

Number of marks: 10

Writing time: 15 minutes

Name: **SOLUTIONS**

Marks:

Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

Four-digit numbers are to be formed from the digits $\{2,3,4,5\}$.
Assume no repetition of digits in any number can occur.

- a. How many four-digit odd numbers can be formed?

1 mark

$$\begin{array}{cccc} \underline{3} & \underline{2} & \underline{1} & \underline{2} \\ & & & \{3,5\} \end{array} \rightarrow 12 \text{ numbers}$$

- b. One of the four-digit odd number is chosen at random. What is the probability the number is greater than 3000?

2 marks

$$\begin{array}{cccc} \underline{2} & \underline{2} & \underline{1} & \underline{2} \\ \{3,4,5\} & & & \{3,5\} \end{array} \rightarrow 8 \text{ numbers greater than } 3000 \quad (1 \text{ mark})$$

$$\Pr(>3000 | \text{odd}) = \frac{8}{12} = \frac{2}{3} \quad (1 \text{ mark for correct answer})$$

Question 2

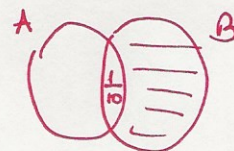
Two events, A and B, from a given event space, are such that $\Pr(A) = \frac{2}{5}$ and $\Pr(B) = \frac{1}{6}$. 2 marks

Calculate $\Pr(A' \cap B)$ when $\Pr(A \cap B) = \frac{1}{10}$.

$$\begin{aligned} \Pr(A' \cap B) &= \Pr(B) - \Pr(A \cap B) \quad (1 \text{ mark}) \\ &= \frac{1}{6} - \frac{1}{10} \end{aligned}$$

$$\neq \frac{1}{15} \quad (1 \text{ mark}) \quad \frac{2}{30}$$

OR



1 mark for identifying the correct region

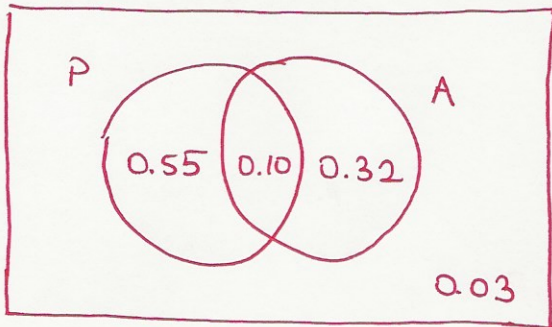
$$\frac{1}{6} - \frac{1}{10} = \left(\frac{1}{15}\right) + \left(\frac{2}{30}\right) \quad 1 \text{ mark for correct answer}$$

Question 3

According to a survey conducted in Australia, 65% of the residents consented to receiving Pfizer Covid-19 vaccine and 42% consented to receiving AstraZeneca, whereas 3% refused to both vaccine.

a. Draw a Venn diagram to illustrate the above information.

2 marks



any two correct numbers ~~answers~~ → 1 mark

all numbers are correct → 2 marks

b. Find the probability that a resident of Australia consented to Pfizer only.

1 mark

$$\Pr(\text{Pfizer only}) = \frac{55}{100} = 0.55$$

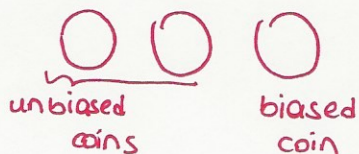
Question 4

2 marks

Adam has three coins in his pocket, two are unbiased and one is biased. When the biased coin is tossed, the probability of tossing a tail is $\frac{1}{5}$.

Adam selects a coin from his pocket and tosses it.

Find the probability that he tosses a tail.



$$\Pr(B^c \cap T) + \Pr(B \cap T) = \frac{2}{3} \times \frac{1}{2} + \frac{1}{3} \times \frac{1}{5}$$

1 mark for finding

any of these two prob.

$$= \frac{6}{15} = \frac{2}{5}$$

→ 1 mark for the correct answer



2021 Mathematical Methods (Unit 1-2)

Task 4

Paper 2 – Calculator allowed

Number of marks: 15

Writing time: 25 minutes

Name:

Marks – Section 1:

Section 2:

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Question 1

If A and B are independent events such that $\Pr(A) = 0.28$ and $\Pr(B) = 0.65$, then $\Pr(A \cup B)$ is equal to:

- A. 0.930
- B. 0.854
- C. 0.748
- D. 0.692
- E. 0.648

Question 2

A set of 4 white, 5 green and 6 blue mugs that are identical except for the colour are to be placed on a shelf. In how many ways can this be done if the same colour mugs are next to each other?

- A. 6
- B. 72
- C. 23450
- D. 630630
- E. 2073600

Question 3

Two dice are rolled. The probability of getting a greater number on the first die than the one on the second, given that the sum equals to 8 is:

A. $\frac{1}{2}$

B. $\frac{5}{9}$

C. $\frac{7}{8}$

D. $\frac{1}{9}$

E. $\frac{2}{5}$

Question 4

How many ways can the eleven-letters of the word 'COEFFICIENT' be arranged in a circle with the vowels together?

A. $\frac{6!5!}{2!2!2!2!}$

B. $\frac{6!}{2!2!2!2!}$

C. $\frac{10!}{2!2!2!2!}$

D. $\frac{7!}{3!2!}$

E. $10!2!2!$

Question 5

A card is drawn randomly from a standard pack of 52 cards.

The probability that the card is a 7 or a diamond is closest to:

A. 0.308

B. 0.390

C. 0.410

D. 0.480

E. 0.510

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.

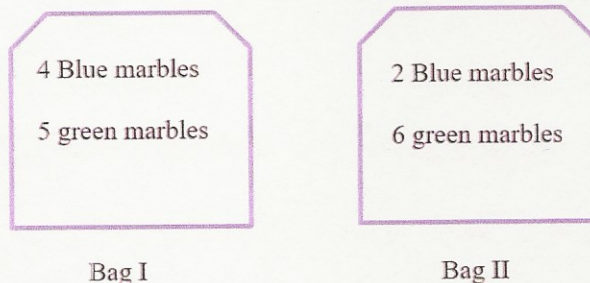
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Question 1

A bag is chosen randomly and one marble is drawn randomly from it. Each bag is equally likely to be chosen.



- a. What is the probability that the randomly drawn marble is blue?

2 marks

$$\begin{aligned} \Pr(\text{Blue}) &= \Pr(\text{Bag}_1 \text{ Blue}) + \Pr(\text{Bag}_2 \text{ Blue}) \\ &= \frac{1}{2} \frac{4}{9} + \frac{1}{2} \frac{2}{8} \end{aligned}$$

1 mark for finding just one of the prob.

$$\Pr(\text{Blue}) = \frac{25}{72}$$

2 marks for full correct answer

- b. Given that the marble is drawn is blue, what is the probability that it was drawn from Bag II?

2 marks

$$\Pr(\text{Bag}_2 / \text{Blue}) = \frac{\frac{1}{2} \frac{2}{8}}{\frac{25}{72}} = \frac{9}{25}$$

1 mark
1 mark

- c. A marble is drawn from Bag I and is put into Bag II then a marble is drawn from Bag II. What is the probability that the marble drawn from Bag II is green?

2 marks

$$\Pr(\overset{\text{Bag 1}}{\uparrow} \text{Blue } \overset{\text{Bag 2}}{\uparrow} \text{Green}) + \Pr(\overset{\text{Bag 1}}{\uparrow} \text{Green } \overset{\text{Bag 2}}{\uparrow} \text{Green}) = \frac{4}{9} \frac{6}{9} + \frac{5}{9} \frac{7}{9} = \frac{59}{81}$$

1 mark for finding either $\Pr(\text{Blue Green})$ or $\Pr(\text{Green Green})$

~~1 mark~~ 1 mark for the correct answer

Question 2

Among a group of 13 books on a shelf, 7 are hardbacks and 6 are paperbacks.

a. In how many ways can any set of 8 books be selected from this group of books?

1 mark

$${}^{13}C_8 = 1287$$

b. In how many ways can 4 hardbacks and 2 paperbacks be chosen from this group of books?

1 mark

$${}^7C_4 \times {}^6C_2 = 525$$

c. Five books will be chosen from this group. What is the probability of selecting two hardbacks and three paperbacks?

2 marks

$$\Pr(2H 3P) = \frac{{}^7C_2 \times {}^6C_3}{{}^{13}C_5}$$

1 mark for the equation

$$\Pr(2H 3P) = 0.326$$

1 mark for the correct answer