Supervision Instructions

Mathematics Methods (Unit 1-2)
Task #6
24th August 2021 – Period 4

Task will be completed remotely.

Students will be given 40 minutes writing time. There will not be any separation of **Paper 1** and **Paper 2**. However, students are expected not to use calculators while attempting Paper 1.

At the beginning of the session all students should turn on their videos and keep them on until the end of the session.

The whole task (Paper 1 and Paper 2) will be sent to all students as a pdf via SEQTA DM.

Students are required to complete their responses on a blank paper clearly indicating each question number.

At the end of writing time (40 minutes) students will have around 2-5 minutes to take a picture and upload their work via SEQTA DM. They can leave the session once you confirm receipt of the work.

2021 Mathematical Methods (Unit 1-2)
Task 6

Paper 1 – Calculator not allowed

Number of marks: 10
Writing time: 15 minutes

Name: Marks:

Instructions

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

2 marks

Solve by hand $\cos(x)\left(\cos(x)-\sin(x)\right)=0$ for x, where $0\le x\le 2\pi$.

$\cos x = 0$ or $\cos x - \sin x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

1 mark

$\cos x = \sin x$

$1 = \frac{\sin x}{\cos x} = \tan x$ 1 mark

$\tan^{-1}(1) = x = \frac{\pi}{4}, \frac{5\pi}{4}$

Question 2

3 marks

Solve by hand $\cos^2(2x)=\frac{3}{4}$ for x, where $0\le x\le 2\pi$.

1 mark

$\sqrt{\cos^2(2x)} = \sqrt{\frac{3}{4}}$

$\cos(2x) = \pm\frac{\sqrt{3}}{2}$

$2x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$2x = \frac{\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}, \frac{23\pi}{6}$

$x = \frac{\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{23\pi}{12}$

$\cos(2x) = -\frac{\sqrt{3}}{2}$

$2x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$2x = \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{17\pi}{6}, \frac{19\pi}{6}$

$x = \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}$

1 mark

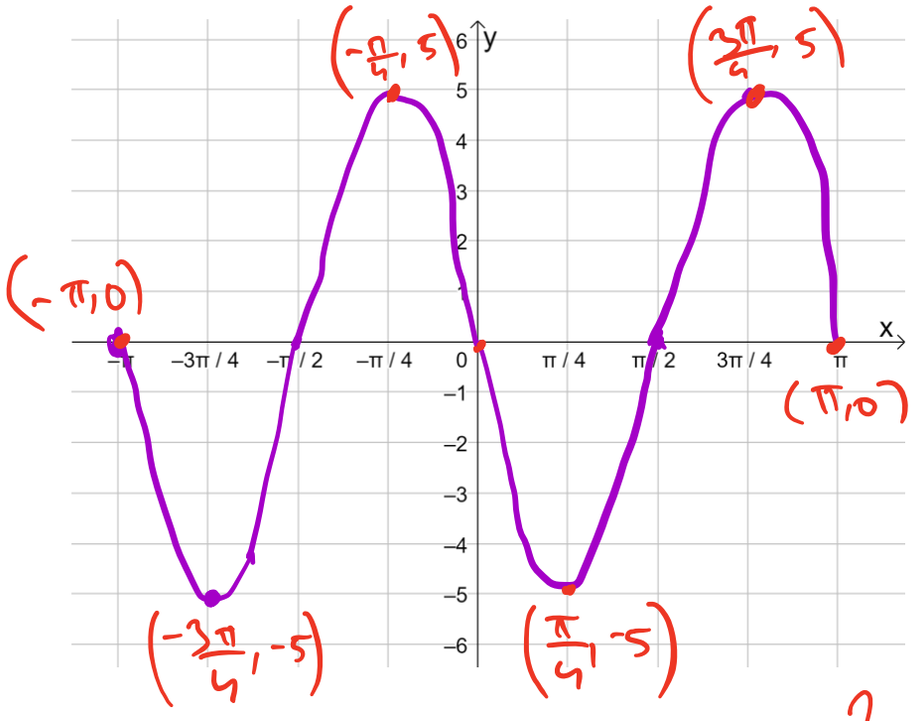
1 mark

Question 3

Consider the function $f:[-\pi,\pi]\to R,\ f(x)=5\cos\left(2x+\frac{\pi}{2}\right)$.

$5\cos\left(2\left(x+\frac{\pi}{4}\right)\right)$

a. Sketch the graph of $y=f(x)$ on the set of axes below. Label endpoints and axis intercepts with their coordinates. 2 marks



Period

$\frac{2\pi}{2}=\frac{2\pi}{2}=\pi$

amp = 5

$\frac{\pi}{4}$ to the left

1 mark – correct period and amplitude

2 mark – correct graph with endpoints

b. State a sequence of transformations applied to $y=10\cos(x)$ to obtain $y=f(x)$. 3 marks

— dilation of $\frac{1}{2}$ from x-axis

1 mark

$y=5\cos(x)$

— dilation of $\frac{1}{2}$ from y-axis

1 mark

$y=5\cos(2(x))$

— phase shift of $\frac{\pi}{4}$ to the left

1 mark

$y=5\cos\left(2x+\frac{\pi}{2}\right)$

2021 Mathematical Methods (Unit 1-2)
Task 6

Paper 2 – Calculator allowed

Number of marks: 15
Writing time: 25 minutes

Name: Marks – Section 1: Section 2:

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Question 1

An arc subtends an angle of $30°$ at the centre of a circle of radius 3 cm. The length of the arc, in cm, is:

A. $\frac{\pi}{2}$

B. 90

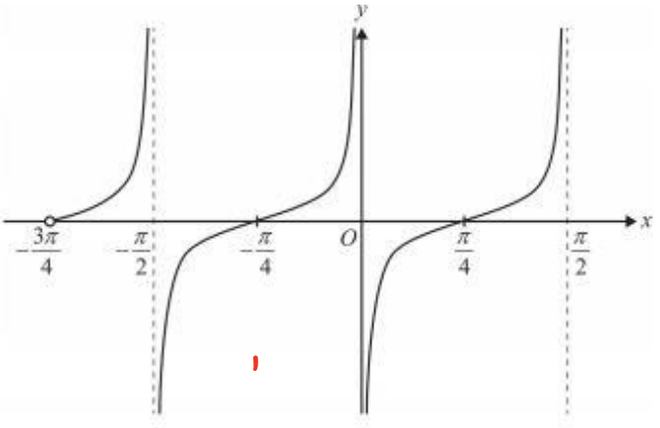
C. π

D. 180

E. $\frac{3\pi}{4}$

Question 2

A section of the graph of f is shown on the right.



The rule of f could be:

A. $f(x) = \tan x$

B. $f(x) = \tan(x - \frac{\pi}{4})$

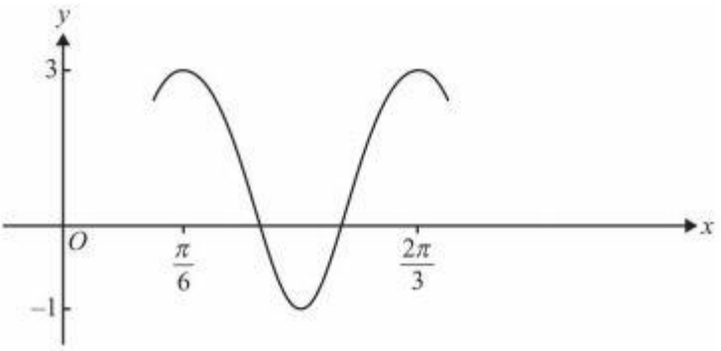
C. $f(x) = \tan(2(x - \frac{\pi}{4}))$

D. $f(x) = \tan(2(x - \frac{\pi}{2}))$

E. $f(x) = \tan(\frac{1}{2}(x - \frac{\pi}{2}))$

Question 3

The graph shown could have equation:



A. $y = 2\cos(x + \frac{\pi}{6}) + 1$

B. $y = 2\cos(4(x - \frac{\pi}{6})) + 1$

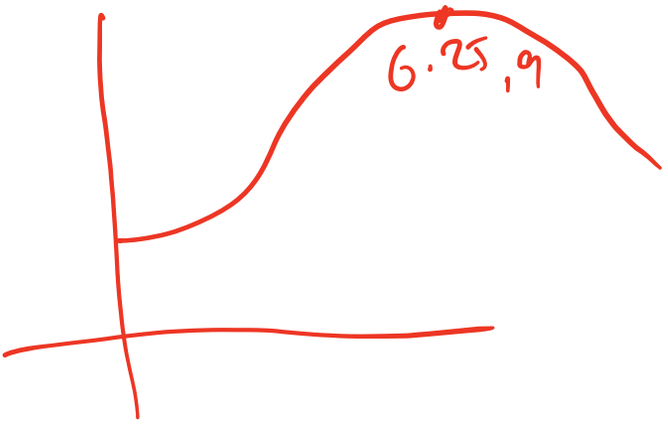
C. $y = -2\cos(x + \frac{\pi}{6}) - 1$

D. $y = 2\cos(3x + \frac{\pi}{6}) - 1$

E. $y = 2\cos(x + \frac{\pi}{6})$

Question 4

The depth of water near the Lorne Pier changes according to the rule $h(t) = 7 - 2\sin(\frac{4\pi t}{25} + \frac{\pi}{2})$, where t is the time in hours after low tide and h is the depth in metres. A low tide occurred at midnight. The time of the **next high** tide is:



A. 6.00 am

B. 6.15 am

C. 12.30 pm

D. 6.30 pm

E. 1.00 am on the next day

Question 5

A function f has the following two properties for all real values of ϕ:

$f(\pi - \phi) = -f(\varphi)$ and $f(\pi - \phi) = -f(-\varphi)$

The rule of f could be:

A. $f(x) = \sin(x)$

B. $f(x) = \cos(x)$

C. $f(x) = \tan(x)$

D. $f(x) = \sin(2x)$

E. $f(x) = \tan(2x)$

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
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Question 1

Triggy, the gardener, is working in a temperature-controlled greenhouse. During a particular 24-hour time interval, the temperature ($T\,°C$) is given by $T(t) = 25 + a\cos\left(\frac{\pi t}{8}\right)$, $0 \le t \le 24$,

where t is the time in hours from the beginning of the 24-hour time interval.

a. The temperature after 8 hours of the initial measurement is $23°C$. Show that $a = 2$. 2 marks

when $t = 8$

$25 + a\cos\left(\frac{8\pi}{8}\right) = 25 + a(-1)$

$25 - a = 23$

$25 - 23 = a = 2$

1 mark

b. State the period of the temperature function. 1 mark

$P = \frac{2\pi}{n} = \frac{2\pi}{\frac{\pi}{8}} = 2\pi \times \frac{8}{\pi} = 16$ 1 mark

c. Find the initial temperature. 1 mark

$t = 0$

$T = 25 + 2(\cos 0) = 27\,°C$ 1 mark

d. State the maximum temperature in the greenhouse and the values of t when this occurs. 2 marks

Maximum $27°C$ at $t = 0$

$t = 16$

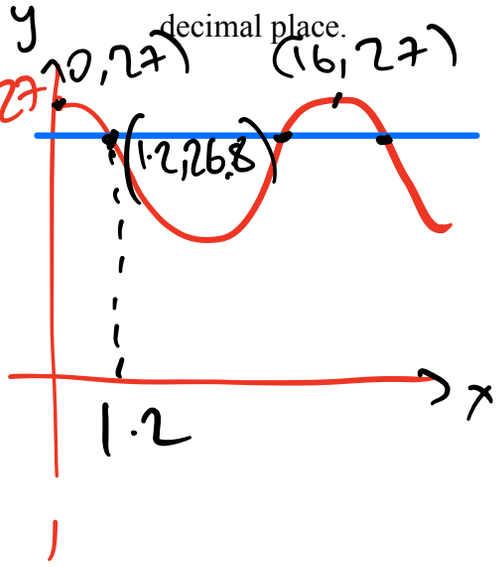
using CAS graph it

1 mark

00:00 and 16:00

1 mark

e. If the temperature was above k degrees for 3.6 hours, obtain the value of k correct to one decimal place. 2 marks



$\frac{3.6}{3} = 1.2$ → 1 mark

when $x = 1.2$ $y = 26.8$

1 mark

students lose a mark for wrong rounding or 2dp.

Question 2 2 marks

Consider the function $f:[-p,p] \to R, f(x) = 3\sin\left(5x - \frac{\pi}{7}\right)$.

Find the maximum value of p if $f(x)$ has an inverse function.

$y = 3\sin\left(5x - \frac{\pi}{7}\right)$

$5x - \frac{\pi}{7} = \frac{\pi}{2}$

$5x = \frac{\pi}{2} + \frac{\pi}{7}$

$5x = \frac{9\pi}{14}$

$x = \frac{9\pi}{70}$

$5x - \frac{\pi}{7} = -\frac{\pi}{2}$

$5x = -\frac{\pi}{2} + \frac{\pi}{7}$

$5x = \frac{-5\pi}{14}$

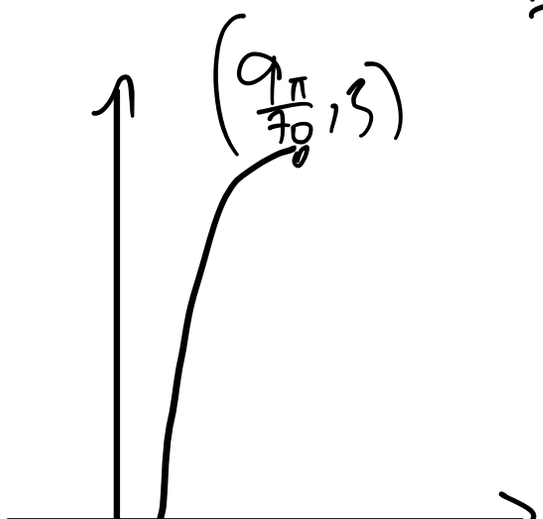
$x = -\frac{5\pi}{70}$

1 mark

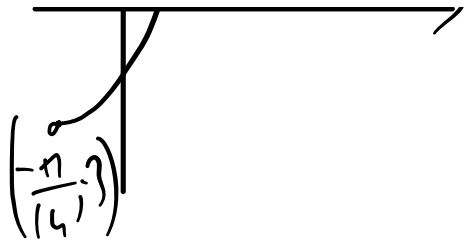
$x = -\frac{\pi}{14}$

maximum value

one-to-one



$[-p, p] = \left[-\frac{\pi}{14}, \frac{\pi}{14}\right]$



$$p = \frac{\pi}{14}$$ 1 mark