2021 Mathematical Methods (Unit 1-2)
Task 7

*Paper 1 – Calculator **not** allowed*

Number of marks: 10
Writing time: 15 minutes

Name: Marks:

Instructions

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

a. Solve by hand $24\times64^{x}+61\times2^{3x}-8=0$ for x. 3 marks

Let $8^x = a$ $\rightarrow$ $24a^2 + 61a - 8 = 0$ $\rightarrow$ 1m

$3a \quad 8$

$8a \quad -1$

$(3a+8)(8a-1)=0$

$a = -\frac{8}{3}$ $a = \frac{1}{8}$

a cannot be ≤ 0 1m

$8^x = \frac{1}{8}$

$8^x = 8^{-1}$

$x = -1$ // $\rightarrow$ 1m

b. Solve by hand the pair of simultaneous equations for a and b. 2 marks

$2a\times3^{2b-1}=486$

$a\times3^{b+1}=27$

$\frac{2a\times3^{2b-1}}{a\times3^{b+1}} = \frac{486}{27}$

$\frac{2\times3^{(2b-1)-(b+1)}}{2} = \frac{18}{2}$

$3^{2b-1-b-1} = 9$

$3^{b-2} = 3^2$

$b-2 = 2$

$b = 4$ //

$a\times3^{4+1} = 27$

$a\times3^5 = 3^3$

$a = \frac{1}{3^2}$ $a = \frac{1}{9}$

or

$2a\times3^{8-1} = 2\times3^5$

$a\times3^7 = 3^5$

$a = 3^{-2}$

$a = \frac{1}{9}$ //

1m

Question 2

Solve by hand the equation $\log_4(x+4)+\log_4(x-2)=2$ for x. 3 marks

$\log_4((x+4)(x-2)) = 2$

$(x+4)(x-2) = 4^2$

or $\log_4((x+4)(x-2)) = \log_4 4^2$

$(x+4)(x-2) = 4^2$

1m

$x^2+2x-8 = 16$

$x^2+2x-24 = 0$

$x \quad 6$

$x \quad -4$

1m

$(x+6)(x-4) = 0$

$x = -6, \ x = 4$

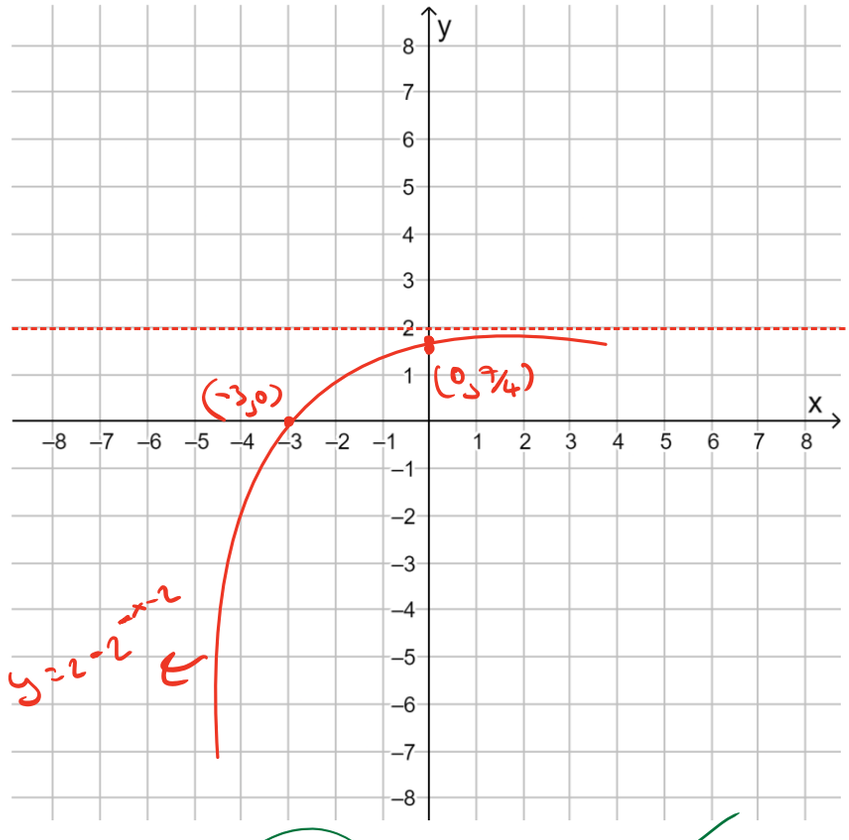
x cannot be ≤ 2

$x = 4 \rightarrow$ 1m

Question 3

2 marks

Sketch the graph of $y = 2 - 2^{-x-2}$ showing the intercepts and the asymptote.



1m

(sketching the graph)

$y = 2$ horizontal asymptote

$x=0$, $y = 2 - 2^{-2}$

$y = 2 - \frac{1}{4}$

$y = \frac{7}{4}$

$(0, \frac{7}{4})$ → y-intercept

$y=0$, $0 = 2 - 2^{-x-2}$

$2^{-x-2} = 2$

$-x-2 = 1$

$x = -3$

$(-3, 0)$ → x-intercept

1m

(finding x and y intercepts with working out)

2021 Mathematical Methods (Unit 1-2)
Task 7

Paper 2 – Calculator allowed

Number of marks: 15
Writing time: 25 minutes

Name: Marks – Section 1: Section 2:

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Question 1

The graph of the curve with rule $y = f(x)$, where f is one-to-one function, has exactly one asymptote whose equation is $x = 3$.

The graph of the curve with rule $y = f^{-1}(x)$, where f^{-1} is the inverse of f, will have

a) a horizontal asymptote at $y = -3$

b) a horizontal asymptote at $y = \frac{1}{3}$

c) a horizontal asymptote at $y = 3$

d) a vertical asymptote at $x = 3$

e) a vertical asymptote at $x = \frac{1}{3}$

Question 2

If $y = a^{b-4x} + 2$, where $a > 0$, then x is equal to

a) $\frac{1}{4}\left(b - \log_a(y-2)\right)$

b) $\frac{1}{4}\left(b - \log_a(y+2)\right)$

c) $b - \log_a\left(\frac{1}{4}(y+2)\right)$

d) $\frac{b}{4} - \log_a(y-2)$

e) $\frac{1}{4}\left(b + 2 - \log_a(y)\right)$

Question 3

Let $f:R^+ \to R, f(x) = k\log_2(x), k \in R$, Given that $f^{-1}(1) = 8$, the value of k is

a) 0

b) $\frac{1}{3}$

c) 3

d) 8

e) 12

Question 4

If $a^{\frac{1}{3}} + a^{-\frac{1}{3}} = x$ then $x^3 - 3x$ is

a) $\frac{1}{a}$

b) $\frac{3a^2+3}{a}$

c) $\frac{3a^2-3}{a}$

d) $a + \frac{1}{a}$

e) $a - \frac{1}{a}$

Question 5

The function f has rule $f(x) = 3\log_e(2x)$. If $f(5x) = \log_e(y)$ then y is equal to

a) $30x$

b) $6x$

c) $125x^3$

d) $50x^3$

e) $1000x^3$

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
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Question 1

Carbon 14 dating works by measuring the amount of Carbon 14 (a radioactive element) that is in a fossil. All living things have a constant level of Carbon 14 in them and once they die it starts to decay according to the formula,

$$Q = Q_0\, 3^{-0.000124\,t}$$

where t in years and Q_0 is the amount of Carbon 14, in milligrams present at death. It is known that an organism contains 100 milligrams of Carbon 14 initially after death.

a. How much Carbon 14 will there be after 1000 years? (correct to five significant figures) 2 marks

$1m \leftarrow Q = 100 \times 3^{-0.000124 \times 1000}$

$1m \leftarrow Q = 87.264$

b. How long will it take for half of the Carbon 14 to decay? (round to the nearest year) 2 marks

$50 = 100 \times 3^{-0.000124t}$

$\frac{1}{2} = 3^{-0.000124t}$

$\log_3\left(\frac{1}{2}\right) = -0.000124\,t$

$t = -\dfrac{\log_3\left(\frac{1}{2}\right)}{0.000124}$ $1m$

$t = 5088$ years

$1m$

Question 2

2 marks

Show that $\log_x(a)\log_a(b) - \log_x(c)\log_c(d) = \log_x\left(\dfrac{b}{d}\right)$.

$\log_x(a) \dfrac{\log_x(b)}{\log_x(a)} - \log_x(c) \dfrac{\log_x(d)}{\log_x(c)}$

$1m$

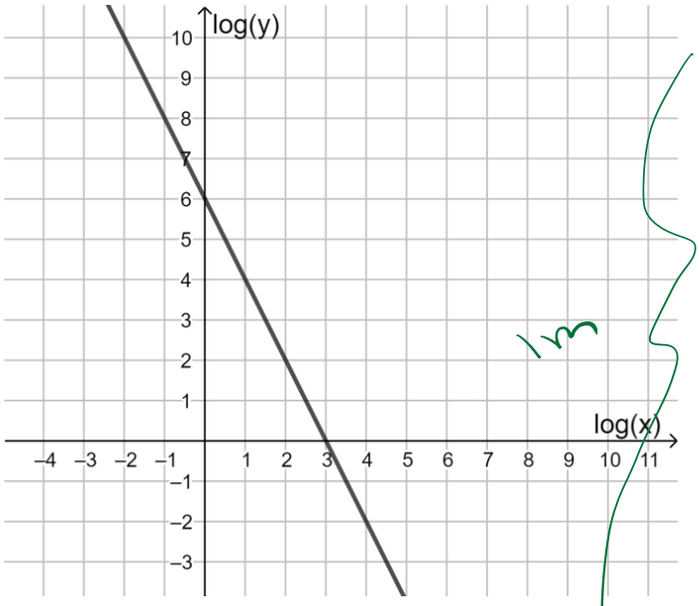
$\log_x(b) - \log_x(d) = \log_x\left(\frac{b}{d}\right)$

$1m$

Question 3 2 marks

For a set of data $\{(x,y)\}$, plotting $\log(y)$ versus $\log(x)$ gave straight line shown in the diagram.

Form the equation of the graph and hence determine the rule connecting y and x.



Let $\log(y) = Y$ and $\log(x) = X$

$m = -2 \quad c = 6$

$Y = -2X + 6$

$\log(y) = -2\log(x) + 6$

$\log(y) + 2\log(x) = 6$

$\log(yx^2) = 6$

$yx^2 = 10^6$

$y = \frac{10^6}{x^2}$ $\quad y = 10^6 x^{-2}$ or $y = 1000000\,x^{-2}$ 1m

Question 4

Consider the function $f: R \to R, f(x) = 4 - 7^{3x-6}$.

Form the rule for the inverse function and express the inverse function as a mapping. 2 marks

$x = 4 - 7^{3y-6}$

$7^{3y-6} = 4 - x$

$3y - 6 = \log_7(4-x)$

$3y = \log_7(4-x) + 6$

$y = \frac{\log_7(4-x)}{3} + 2$ 1m

Range of $f(x) = (-\infty, 4)$ (domain of inverse)

$f^{-1}(x) = (-\infty, 4) \to R$, $f^{-1}(x) = \frac{\log_7(4-x)}{3} + 2$

1m