# 2021 Mathematical Methods (Unit 1-2)
# Task 7

*Paper 1 – Calculator **not** allowed*

Number of marks: 10
Writing time: 15 minutes

Name: Marks:

**Instructions**

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

## Question 1

**a.** Solve by hand $24 \times 64^{x} + 61 \times 2^{3x} - 8 = 0$ for $x$. 3 marks

**b.** Solve by hand the pair of simultaneous equations for $a$ and $b$. 2 marks

$2a \times 3^{2b-1} = 486$

$a \times 3^{b+1} = 27$

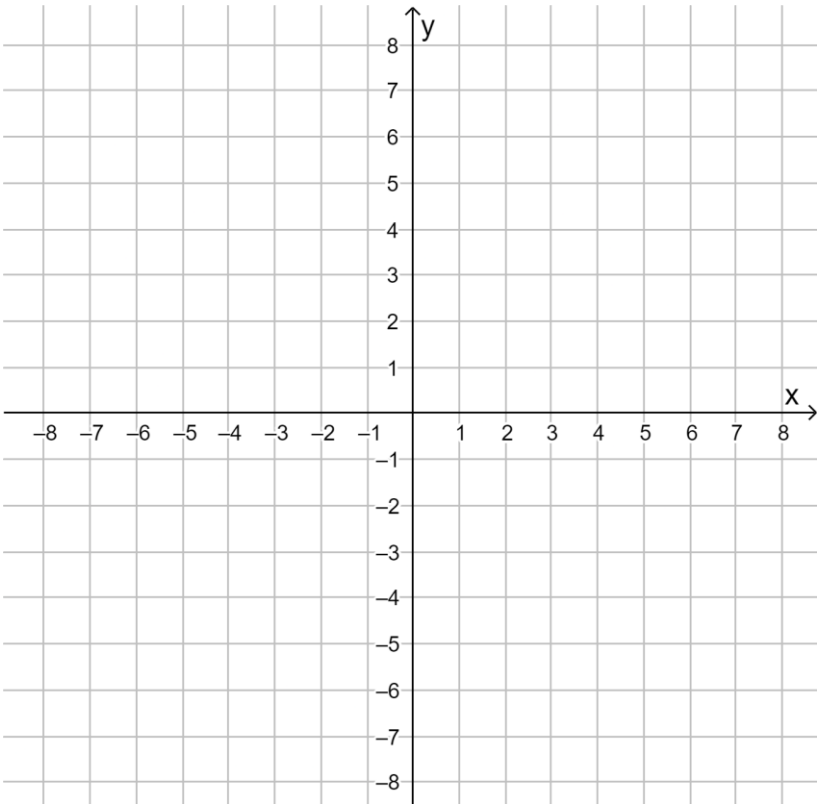
## Question 2

Solve by hand the equation $\log_4(x+4)+\log_4(x-2)=2$ for $x$. 3 marks

## Question 3

2 marks

Sketch the graph of $y=2-2^{-x-2}$ showing the intercepts and the asymptote.



# 2021 Mathematical Methods (Unit 1-2)
# Task 7
*Paper 2 – Calculator allowed*

Number of marks: 15
Writing time: 25 minutes

Name: Marks – Section 1: Section 2:

## SECTION 1

**Instructions for Section 1**

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

### Question 1

The graph of the curve with rule $y = f(x)$, where $f$ is one-to-one function, has exactly one asymptote whose equation is $x = 3$.

The graph of the curve with rule $y = f^{-1}(x)$, where $f^{-1}$ is the inverse of $f$, will have

a) a horizontal asymptote at $y = -3$

b) a horizontal asymptote at $y = \frac{1}{3}$

c) a horizontal asymptote at $y = 3$

d) a vertical asymptote at $x = 3$

e) a vertical asymptote at $x = \frac{1}{3}$

### Question 2

If $y = a^{b-4x} + 2$, where $a > 0$, then $x$ is equal to

a) $\frac{1}{4}\left(b - \log_a(y-2)\right)$

b) $\frac{1}{4}\left(b - \log_a(y+2)\right)$

c) $b - \log_a\left(\frac{1}{4}(y+2)\right)$

d) $\frac{b}{4} - \log_a(y-2)$

e) $\frac{1}{4}\left(b + 2 - \log_a(y)\right)$

**Question 3**

Let $f:R^+ \to R, f(x) = k\log_2(x), k \in R$, Given that $f^{-1}(1) = 8$, the value of $k$ is

a) $0$

b) $\frac{1}{3}$

c) $3$

d) $8$

e) $12$

**Question 4**

If $a^{\frac{1}{3}} + a^{-\frac{1}{3}} = x$ then $x^3 - 3x$ is

a) $\frac{1}{a}$

b) $\frac{3a^2+3}{a}$

c) $\frac{3a^2-3}{a}$

d) $a + \frac{1}{a}$

e) $a - \frac{1}{a}$

**Question 5**

The function $f$ has rule $f(x) = 3\log_e(2x)$. If $f(5x) = \log_e(y)$ then $y$ is equal to

a) $30x$

b) $6x$

c) $125x^3$

d) $50x^3$

e) $1000x^3$

## SECTION 2

**Instructions for Section 2**

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

### Question 1

Carbon 14 dating works by measuring the amount of Carbon 14 (a radioactive element) that is in a fossil. All living things have a constant level of Carbon 14 in them and once they die it starts to decay according to the formula,

$$Q = Q_0\, 3^{-0.000124\,t}$$

where $t$ in years and $Q_0$ is the amount of Carbon 14, in milligrams present at death. It is known that an organism contains 100 milligrams of Carbon 14 initially after death.

**a.** How much Carbon 14 will there be after 1000 years? (correct to five significant figures) 2 marks

**b.** How long will it take for half of the Carbon 14 to decay? (round to the nearest year) 2 marks

### Question 2

2 marks

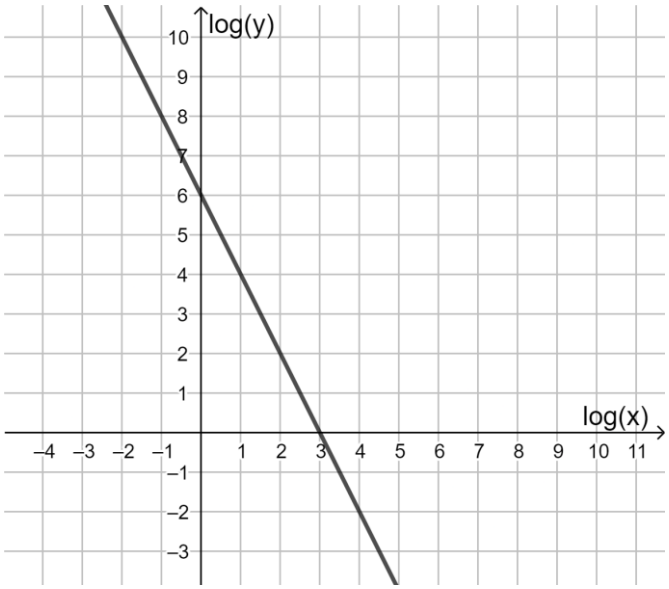
Show that $\log_x(a)\log_a(b) - \log_x(c)\log_c(d) = \log_x\left(\frac{b}{d}\right)$.

## Question 3

2 marks

For a set of data $\{(x,y)\}$, plotting $\log(y)$ versus $\log(x)$ gave straight line shown in the diagram.

Form the equation of the graph and hence determine the rule connecting $y$ and $x$.




## Question 4

Consider the function $f:R \to R, f(x) = 4 - 7^{3x-6}$.

Form the rule for the inverse function and express the inverse function as a mapping.

2 marks