2021 Mathematical Methods (Unit 1-2)
Task 8

*Paper 1 – Calculator **not** allowed*

Number of marks: 10
Writing time: 15 minutes

Name: Solutions / Marking guide Marks:

Instructions

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

Find $\frac{dy}{dx}$, If $y = \frac{5x^3 + 4x^2 - 2}{3x^2}$. 2 marks

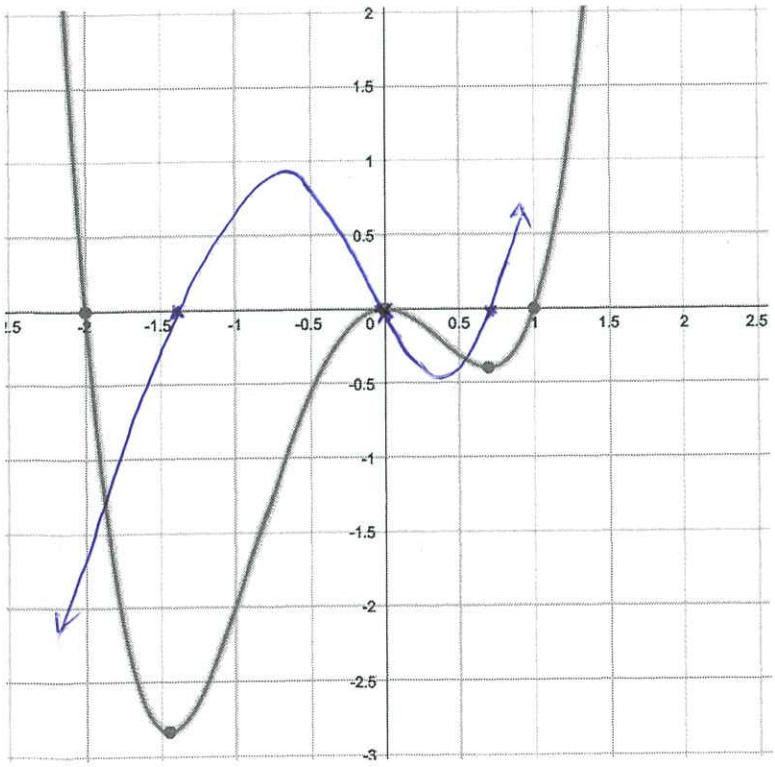
$= \frac{5}{3}x + \frac{4}{3} - \frac{2}{3}x^{-2}$

$\frac{dy}{dx} = \frac{5}{3} + 0 + \frac{4}{3x^3}$ — 2 m

deduct 1 mark for 1-2 mistakes.

Question 2 2 marks

Sketch a possible gradient graph of $f(x)$ on axis shown below.



1 mark for correct x-intercepts.

1 mark for sketching a positive cubic.

Question 3 2 marks

The line $y = ax + 3$ is tangent to the parabola, $y = x^2 + x + b$ when $x = 1$. Find the values of the constants a and b.

$\frac{dy}{dx} = 2x + 1 \Big|_{x=1} = a$

$\Rightarrow a = 3$ (1 mark)

$y = 3x + 3 \Big|_{x=1} = 6$

$6 = 1^2 + 1 + b$

$\Rightarrow b = 4$

1 mark.

Question 4

a. Find the x - coordinates of the stationary point of the following curve, 2 marks

$y = 2x^3 - 5x^2 - 4x + 13$.

1m.

$\frac{dy}{dx} = 6x^2 - 10x - 4 = 0$

$3x^2 - 5x - 2 = 0$

$(3x + 1)(x - 2) = 0$

$x = -1/3 \quad x = 2$

1m.

b. State the nature of the stationary points. 2 marks

Attempt to use Either double derivative or Sign diagram

1m.

$\frac{d^2y}{dx^2} = 6x - 5 \Big|_{x=2} > 0 \Rightarrow x = 2$ is a local min.

$\frac{d^2y}{dx^2} = 6x - 5 \Big|_{x=-1/3} < 0 \Rightarrow x = -1/3$ is a local max

1m

2021 Mathematical Methods (Unit 1-2)
Task 8

Paper 2 – Calculator allowed

Number of marks: 15
Writing time: 25 minutes

Name: Marks – Section 1: Section 2:

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Question 1

The gradient of the tangent to the curve $y = f(x)$ at the point $P(2, f(2))$ is given by

A. $f(2) - h$

B. $\dfrac{f(2+h)-f(2)}{2}$

C. $\dfrac{f(2+h)-f(2)}{h}$

D. $\lim\limits_{h\to\infty} \dfrac{f(2+h)-f(2)}{h}$

E. $\lim\limits_{h\to 0} \dfrac{f(2+h)-f(2)}{h}$

Question 2

If $y = x^4 - 4x^3 - 8x^2$, the points at which the tangent to the curve is parallel to the x-axis are

A. 1, 0 and 4

B. 0 and 1

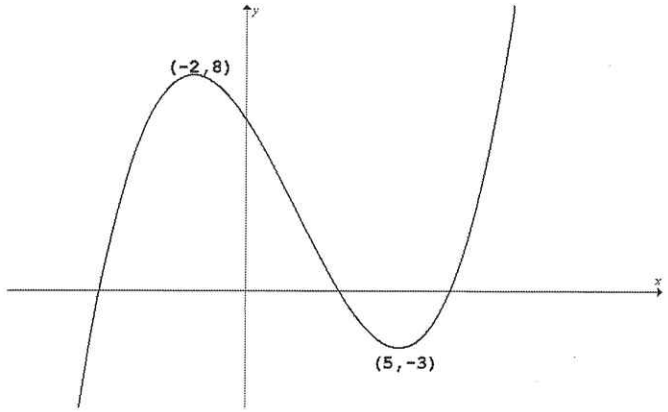
C. 1 and 4

D. 0 and 4

E. –1, 0 and 4

Question 3

The graph of $y = f(x)$ is shown.



$f'(x) < 0$ for

A. $-2 \leq x \leq -5$

B. $-3 < x < 8$

C. $x < -2$ or $x > 5$

D. $-2 < x < 5$

E. $x < -3$ or $x > 8$

Question 4

$\int (3x^2 - 1)\, dx$ is equal to

A. $6x$

B. $x^2 - x + c$

C. $\frac{1}{3}x^3 - x + c$

D. $x^3 - x + c$

E. $x^3 - x^2 + cx$

Question 5

If $f(x) = 3x^2 - x$, then $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ is

A. $6x$

B. $6x - 1$

C. $6x + 3h - 1$

D. $3x - 1$

E. $6x + h - 1$

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
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Question 1 2 marks

If $f(x) = 2\sqrt{x^3} - x^{-4}$, find $F(x)$, where $F'(x) = f(x)$.

$F(x) = \frac{2 \times 2x^{5/2}}{5} + \frac{x^{-3}}{3} + C$

1 mark for right answers
1 mark for C

Question 2

The curve for which $\frac{dy}{dx} = kx$, where k is a constant is such that the tangent at (1, 4) passes through the origin

a. Show that $k = 4$. 2 marks

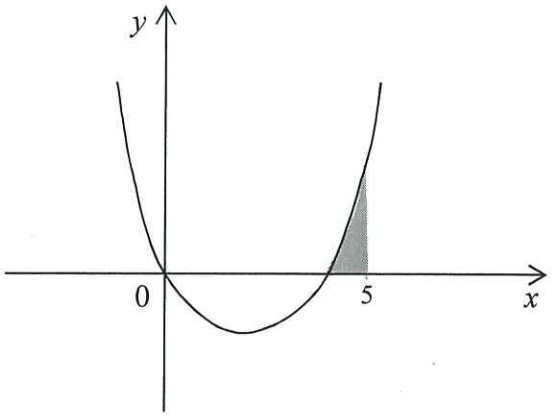
$y = \frac{kx^2}{2} + C$
Eqn of tangent at $x = 1$
$y - 4 = k(x-1)$ ——— 1m
Since tangent passes through (0,0) ——— 1m.
$-4 = k(-1) \Rightarrow k = 4$.
OR alternate correct method

b. Hence find the equation of the curve. 1 mark

$y = \frac{4x^2}{2} + c$
$4 = 2(1)^2 + c \Rightarrow c = 2$
$y = 2x^2 + 2$

Question 3

Part of the graph of $y = x^2 - 4x$ is shown.



a. Find the x –intercepts of $y = x^2 - 4x$. 1 mark

$x = 0, \quad x = 4$

b. Hence find the area enclosed by the shaded region using calculus. 2 marks

$$\int_4^5 (x^2 - 4x)\,dx = \frac{x^3}{3} - 2x^2 \Big]_4^5$$

$$= \left[\frac{125}{3} - 50\right] - \left[\frac{64}{3} - 32\right]$$

$$= 7/3 \text{ unit}^2$$

Question 4 2 marks

Tangents drawn to the curve, $f(x) = x^3 - x^2 + 5$ make an angle of 45 degrees with the positive direction of x-axis. Find equation of **ONE** of the tangents drawn to the curve $f(x)$.

$m_T = \tan(45) = 1$

$f'(x) = 1$

$3x^2 - 2x = 1$ ——— 1m.

$3x^2 - 2x - 1 = 0$

$(3x+1)(x-1) = 0$

$x = -1/3 \quad x = 1$

Eqn of tangent at $x = 1$ $\quad y = x + 4$

OR

Eqn of tangent at $x = -1/3$ $\quad y = x + \frac{140}{27}$

} 1 mark for any one correct eqnation.