Supervision Instructions

Mathematics Methods (Unit 1-2)
Task #5
23rd August 2022

Task consists of two papers: **Paper 1** and **Paper 2**. Students will have access to only one paper at a time.

Paper 1:
- 15 minutes
- Calculator is not allowed

After 15 minutes **Paper 1** is to be collected and **Paper 2** will be given.

Paper 2:
- 25 minutes
- Calculator is allowed

After 25 minutes **Paper 2** is to be collected.

Check that students put their names.

Sirius
College
" Reach for the stars "

Name: Marks:

Question 1   2 marks

Solve $2\sin(2\theta) = -\sqrt{3}$ for $0 \le \theta \le \pi$.

$$\sin(2\theta) = \frac{-\sqrt{3}}{2}, \quad 0 \le 2\theta \le 2\pi$$

$$2\theta = \pi + \frac{\pi}{3}, \; 2\pi - \frac{\pi}{3} \Big\} \text{— 1 m for solving for } 2\theta.$$

OR

$$2\theta = \frac{4\pi}{3}, \; \frac{5\pi}{3}$$

$$\theta = \frac{4\pi}{6}, \; \frac{5\pi}{6} \implies \theta = \frac{2\pi}{3}, \; \frac{5\pi}{6} \quad \text{— 1 m}$$

Question 2   2 marks

Write down a sequence of transformations that takes the graph of $y = \sin(x)$ to the graph of

$y = -4\sin(3x + \frac{\pi}{6})$. $= -4\sin\left(3\left(x + \frac{\pi}{18}\right)\right)$

The order of these 3 don't matter.

- Dilation factor of 4 from the x-axis
- Reflection in the x-axis
- Dilation factor of $\frac{1}{3}$ from the y-axis
- Translation $\frac{\pi}{18}$ units to the left

1 m for any 3 correct transformation with correct order.

OR
- Dilation factor of 4 from the x-axis
- Reflection in the x-axis
- Translation $\frac{\pi}{6}$ units to the left
- Dilation factor of $\frac{1}{3}$ from the y-axis

Question 3

2 marks

The graph of $y = a\tan(nx)$ has the domain $\left(-\dfrac{\pi}{3}, \dfrac{\pi}{3}\right)$ with vertical asymptotes at $x = -\dfrac{\pi}{3}$ and $x = \dfrac{\pi}{3}$ only. The graph passes through the origin and the point $\left(-\dfrac{\pi}{6}, -\dfrac{1}{2}\right)$ Determine its equation.

$$\frac{\pi}{n} = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3} \implies n = \frac{3}{2} \quad \text{—— 1m}$$

$$\left(-\frac{\pi}{6}, -\frac{1}{2}\right) \implies -\frac{1}{2} = a\tan\left(\frac{3}{2}x - \frac{\pi}{6}\right) \implies -\frac{1}{2} = a\tan\left(-\frac{\pi}{4}\right)$$

$$-\frac{1}{2} = -a$$

$$\implies a = \frac{1}{2} \quad \text{—— 1m}$$

Consequential marking applies for wrong value of n

Question 4

a.   2 marks

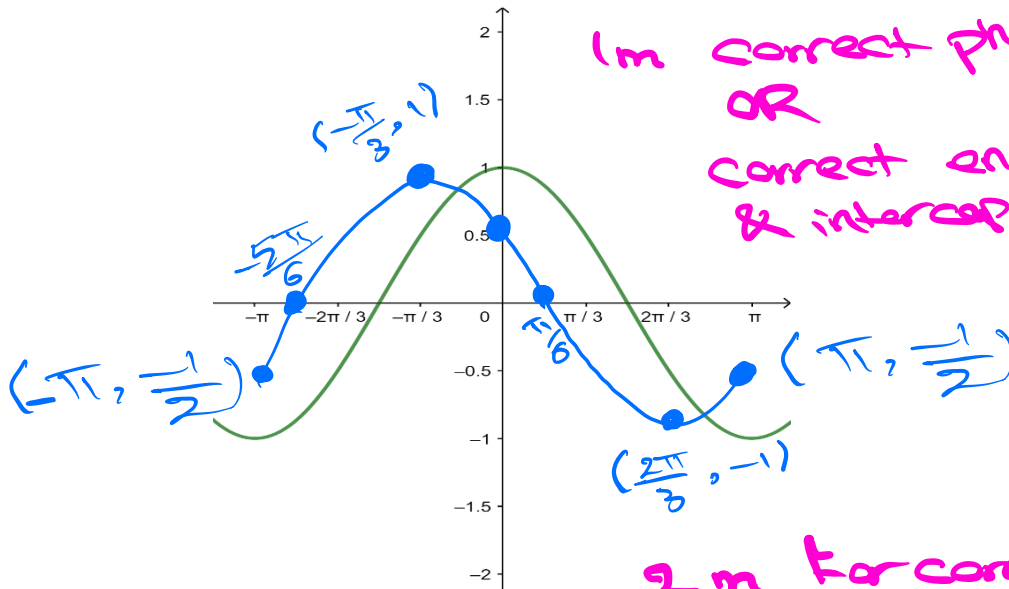Solve $\cos\left(x + \dfrac{\pi}{3}\right) = 0$ for $x \in [-\pi, \pi]$

$$x + \frac{\pi}{3} = \frac{\pi}{2}, \ -\frac{\pi}{2} \quad \text{—— 1m for solving for } x + \frac{\pi}{3}$$

$$x = \frac{\pi}{2} - \frac{\pi}{3}, \ -\frac{\pi}{2} - \frac{\pi}{3} \implies x = \frac{\pi}{6}, \frac{-5\pi}{6} \quad \text{—— 1m}$$

b.   The function $g(x) = \cos(x)$ is shown on the axes below.   2 marks

1m correct phase shift
OR
correct endpoints & intercepts

2m for correct shape including intercepts & end points

$\left(-\frac{\pi}{3}, 1\right)$, $\frac{-5\pi}{6}$, $(-\pi, -\frac{1}{2})$, $x = \frac{\pi}{6}$, $\left(\pi, -\frac{1}{2}\right)$, $\left(\frac{2\pi}{3}, -1\right)$

Let $f : [-\pi, \pi] \to R, \ f(x) = \cos\left(x + \dfrac{\pi}{3}\right)$.

Sketch the graph of the function f on the same set of axes above. Label y-intercept and turning points with their coordinates. Label endpoints of the graph with their coordinates.

$$f(-\pi) = -\cos\left(\frac{\pi}{3}\right) = -\frac{1}{2}$$

$$f(\pi) = -\cos\left(\frac{\pi}{3}\right) = -\frac{1}{2}$$

x-ints: $\left(\frac{\pi}{6}, 0\right), \left(\frac{-5\pi}{6}, 0\right)$

y-int: $\left(0, \frac{1}{2}\right)$

Name: Marks – Section 1:   Section 2:

SECTION 1

Question 1

Given that $\sin(\theta) = 0.465$ and θ is in the first quadrant, which of the following is **not** true?

A $\sin(5\pi + \theta) = -0.465$

B $\sin(4\pi - \theta) = 0.465$

C $\sin(\pi + \theta) = -0.465$

D $\sin(-\theta) = -0.465$

E $\sin(3\pi - \theta) = 0.465$

Question 2

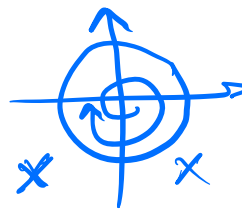The number of solutions of the equation $\sqrt{2}\sin(3x) = -1, \; -\pi \le x \le \pi$ is
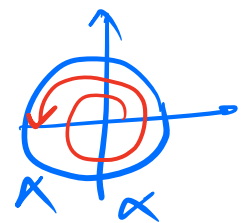
A 2
B 3
C 4
D 5
E 6

$-3\pi \le x \le 3\pi$

4   +   2

Question 3
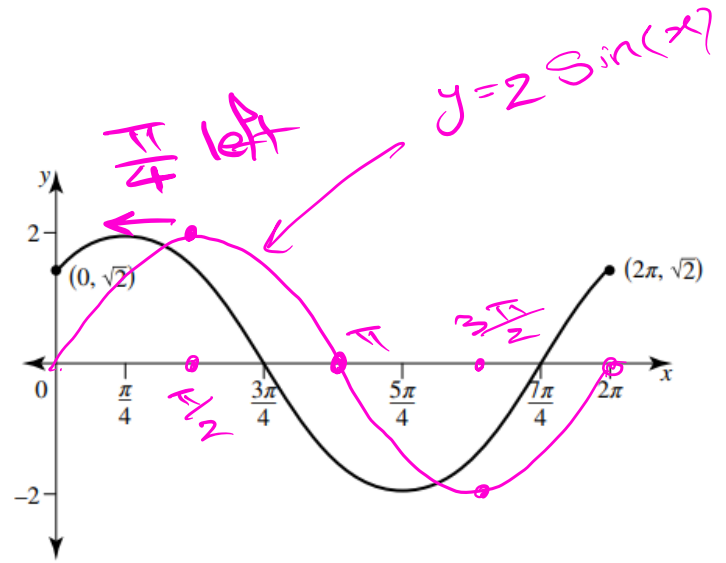
The graph is most likely to be the graph of

A $y = 2\sin(x + \dfrac{\pi}{3})$

B $y = 2\sin(x + \dfrac{\pi}{4})$

C $y = 2\sin(x + \dfrac{\pi}{4}) + \sqrt{2}$

D $y = 2\sin(x) + \sqrt{2}$

E $y = 2\cos(x + \dfrac{\pi}{4})$

(handwritten annotations: $\frac{\pi}{4}$ left; $y = 2\sin(x)$; $(0, \sqrt{2})$; $(2\pi, \sqrt{2})$)

Question 4

If $\cos(a) = -\dfrac{4}{5}$ and $\pi \le a \le \dfrac{3\pi}{2}$, the **exact** value for $\sin(a)$ is

A $\dfrac{4}{5}$

B $\dfrac{3}{4}$

C $-\dfrac{3}{4}$

D $\dfrac{3}{5}$

E $-\dfrac{3}{5}$

(handwritten: triangle with sides 4, 3, 5 and angle a)
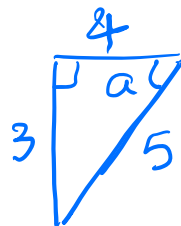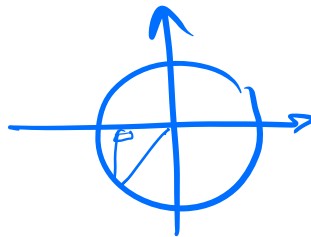
Question 5

Sum of the solutions to the equation $2\cos(2\theta) = -\sqrt{2}$, $0 \le x \le \pi$ is

A $\dfrac{5\pi}{8}$

B $\dfrac{3\pi}{8}$

C π

D $\dfrac{\pi}{2}$

E $\dfrac{\pi}{4}$

(handwritten: $\theta = \dfrac{3\pi}{8}, \dfrac{5\pi}{8}$)

SECTION 2

Question 1

Ms G is in a boat out at sea fishing. The weather makes a change for the worse and the water becomes very choppy. The depth of water above the sea bed can be modelled by the function with equation

$$d(t) = 1.5\sin\left(\frac{\pi t}{12}\right) + 12.5$$

Where d is the depth of water in metres and t is the time in hours since the change of weather began at **5 am.**

a. How far from the sea bed was the boat when the change of weather began?   1 mark

$$d(0) = 12.5 \text{ m}$$

b. What is the period of the function?   1 mark

$$\frac{2\pi}{\frac{\pi}{12}} = 24 \text{ hrs}$$

c. What are the maximum and minimum heights of the boat above the sea bed and the times when these occurs?   2 marks

$$Max = 12.5 + 1.5 = 14 \quad \text{at } t = 6 : \text{11am}$$
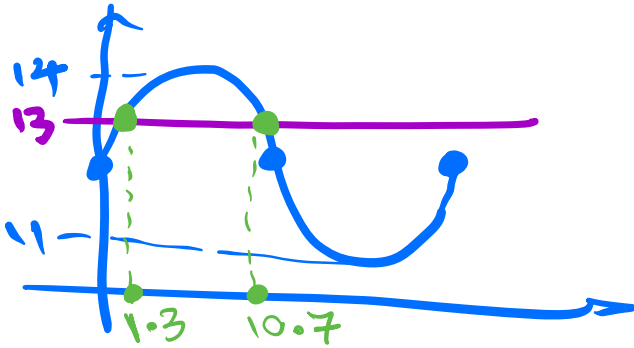$$Min = 12.5 - 1.5 = 11 \quad \text{at } t = 18 : \text{11pm}$$

1m for correct max & min values

2m for correct max & min values AND Correct times

d. What is the depth of water above the sea bed, correct to two decimal places, at 11:30am? 1 mark

$$t = 6.5 \qquad d(6.5) = 13.99 \text{ m}$$

e. Ms G is interested in when the depth of water above the sea bed is 13 meters at least. For how long, in minutes this occurs? 2 marks

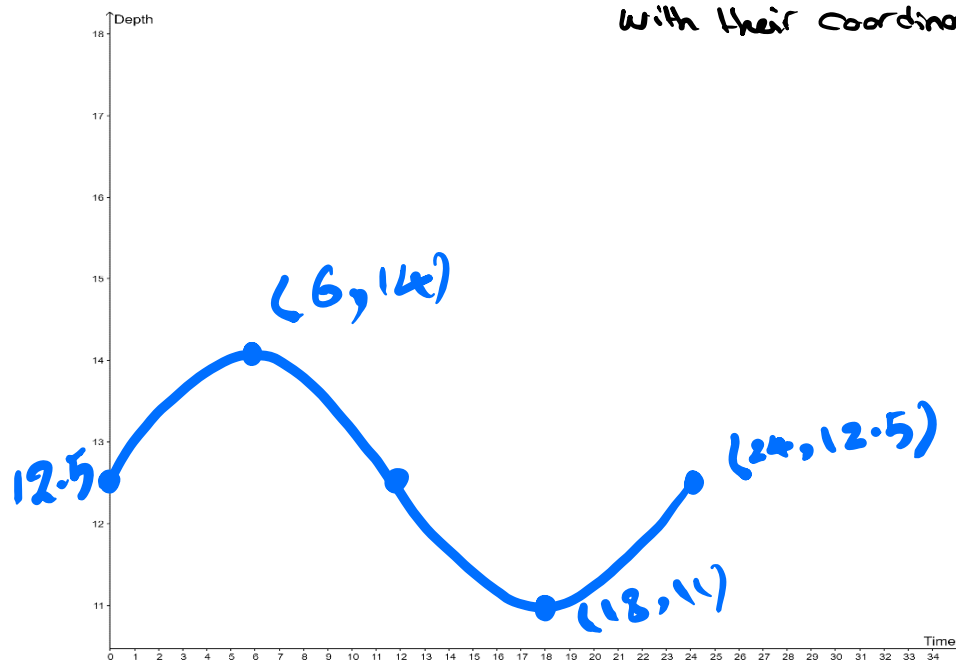
$$d(t) = 13$$
$$\Rightarrow t = 1.3 , 10.7$$

1m for correct times
2m for correct times & duration

$$10.7 - 1.3 = 9.4 \times 60 = 564 \text{ mins}$$

f. Sketch one cycle of the graph of the function $d(t)$. Label max, min & endpoints with their coordinates. 3 marks

$(6, 14)$

12.5

$(18, 11)$

$(24, 12.5)$

1m for correct shape
OR
correct endpoints
OR
correct min & max points

2m for
correct shape
AND
correct endpoints
OR
correct min & max points

3m for correct shape
including intercepts,
end points,
min & max
coordinates