2022 Mathematical Methods (Unit 1-2)
Task 6
Paper 1 – Calculator not allowed

Number of marks: 10
Writing time: 15 minutes

Name: Marks:

Instructions

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

Solve the following for x.

a. $4^x + 2^{3+x} - 9 = 0$ let $m = 2^x$ 2 marks

$m^2 + 2^3 \times 2^x - 9 = 0$

$m^2 + 8m - 9 = 0$

$(m+9)(m-1)$

$m = -9$ or $m = 1$ (1 mark)

$\cancel{2^x = -9}$ $2^x = 1$

$x = 0$ (1 mark)

b. $\log_3(2x-2) - \log_3(x-2) = 2$ 2 marks

$\log_3\left(\frac{2x-2}{x-2}\right) = 2$

$\frac{2x-2}{x-2} = 3^2$

$2x-2 = 9(x-2)$

$2x-2 = 9x-18$

$16 = 7x$

$x = 16/7$

Question 2

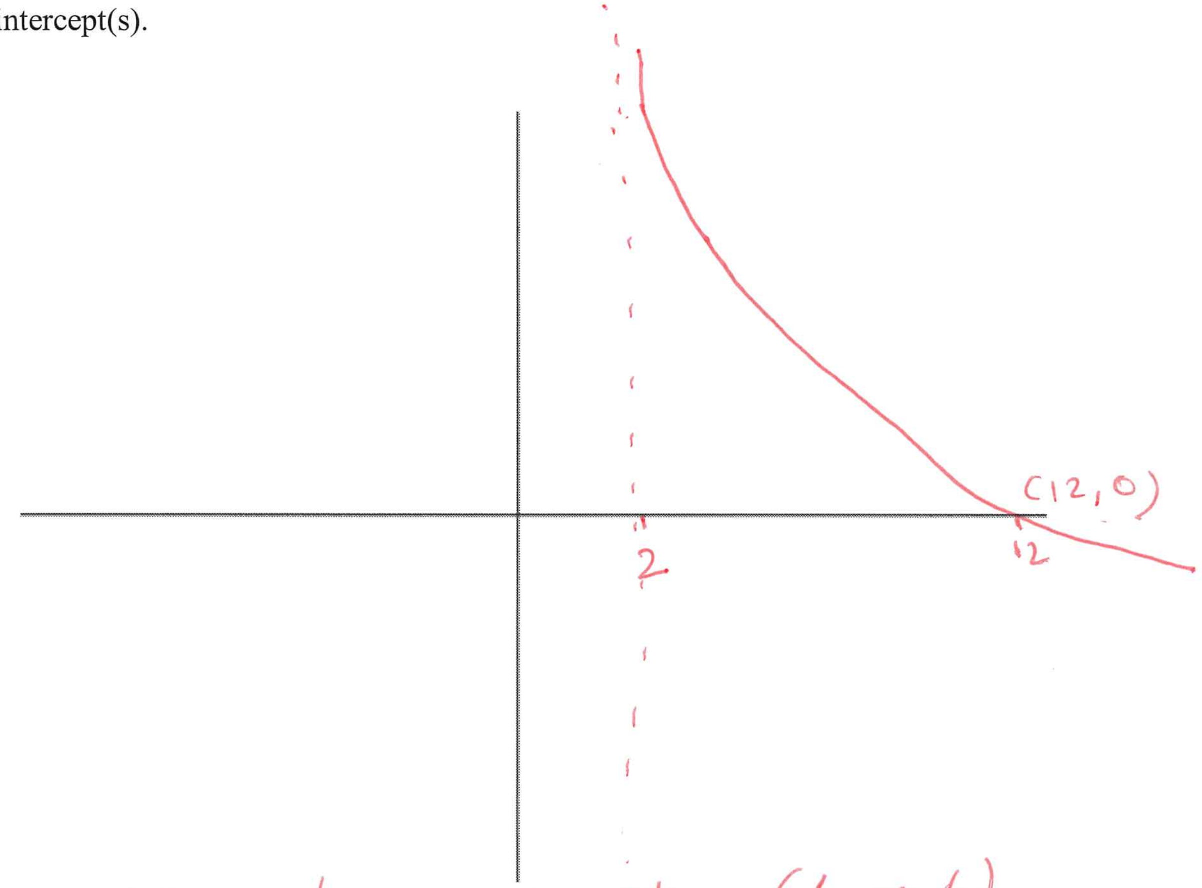
Express $\frac{8^{2-m} \times 16^{m+2}}{32^{m+3}}$ as a power of 2. 2 marks

$\frac{(2^3)^{2-m} \times (2^4)^{m+2}}{(2^5)^{m+3}} = \frac{2^{6-3m} \times 2^{4m+8}}{2^{5m+15}}$ (1 mark)

$= \frac{2^{14+m}}{2^{5m+15}} = 2^{-4m-1}$ (1 mark)

Question 3 2+2 marks

a. Sketch the graph of $y = -3\log_{10}(x-2)+3$ showing any asymptote(s) and axis intercept(s).



- Correct asymptote and x-intercept (1 mark)
- Correct shape and sketch (1 mark)

b. State the domain and range of $y = -3\log_{10}(x-2)+3$.

D: $(2, \infty)$ (1 mark)

R: R (1 mark)

2022 Mathematical Methods (Unit 1-2)
Task 6
Paper 2 – Calculator allowed

Number of marks: 15
Writing time: 25 minutes

Name: Marks – Section 1: Section 2:

SECTION 1

Instructions for Section 1
Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Question 1

The range of the function g with the rule $g(x) = 4 + 2^{3-2x}$ is:

A $y \in (2, \infty)$

B $y \in (-\infty, \infty)$

C $y \in [4, \infty)$

D $y \in [2, \infty)$

E $y \in (4, \infty)$

Question 2

The graph of $y = 3^x$ undergoes the following transformations in the order below:

- A reflection in the x-axis
- A translation of 1 unit parallel to the x-axis in the positive direction
- A translation of 3 units parallel to the y-axis in the negative direction

The rule for the graph of the transformed function is:

A $y = 3^{1-x} - 3$

B $y = -3^{x-1} - 3$

C $y = -3^{x+1} - 3$

D $y = 3^{1-x} + 3$

E $y = 3^{x+1} - 3$

Question 3

The function f has rule $f(x) = 2\log_e(3x)$. If $f(3x) = \log_e(m)$ then m is equal to:

A $81x^2$

B $18x^2$

C $6x$

D $18x$

E $8x^2$

Question 4

$\dfrac{2(p^2r^{-\frac{1}{2}})^3}{(4p^4r^2)^{\frac{1}{2}}}$ in simplest form is:

A $\dfrac{2}{p^4r^{\frac{3}{2}}}$

B $\dfrac{p^4}{r^{\frac{5}{2}}}$

C $\dfrac{2p^2}{r^{\frac{5}{2}}}$

D $\dfrac{p^4}{2r^{\frac{3}{2}}}$

E $\dfrac{p^6}{2r^{\frac{1}{2}}}$

Question 5

If $f(x) = 2^{4x+8} + 5$, then $f^{-1}(x) =$

A $\frac{1}{4}\log_2(x-5)+2$

B $\frac{1}{2}\log_4(x-5)-2$

C $\frac{1}{4}\log_2(x+5)+2$

D $\frac{1}{4}\log_2(x-5)-2$

E $\frac{1}{2}\log_2(x+5)-2$

SECTION 2

Instructions for Section 2
Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
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Question 1

The mass of a radioactive substance in grams remaining after t months is given by:

$M(t) = 500 \times 2^{-0.15t} + 10 \quad t \in [0, \infty)$

a. **i.** Find the initial mass of the substance. Round your answer to 4 significant figures. 1 mark

$M(0) = 500 \times 2^{-0.15(0)} + 10$

$= 510.0$ grams (1 mark)

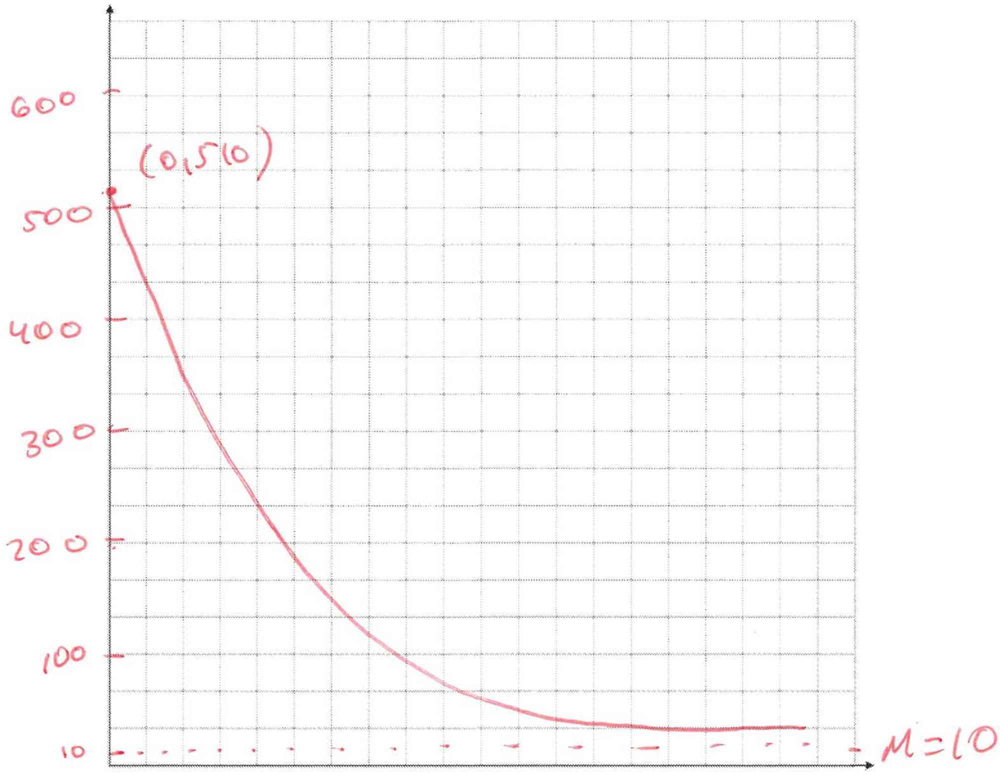
ii. Find the mass of the substance after 2 months. Round your answer to 4 significant figures. 1 mark

$M(2) = 416.1$ grams (1 mark)

b. What does the horizontal asymptote suggest about the decay of the substance? 1 mark

The remaining radioactive substance can't be less than or equal to 10 grams (1 mark)

c. Sketch the graph of $M(t) = 500 \times 2^{-0.15t} + 10 \quad t \in [0, \infty)$ on the set of axes below. Clearly indicate any axis intercepts, end points and asymptotes. 2 marks



- Correct asymptote and y-intercept/end point (1 mark)
- Correct shape and sketch. (1 mark)

d. The radioactive substance is safe to handle when it decays to 200 grams or less. Use a suitable method to find the time in months, correct to 3 significant figures, when it is first safe to handle the substance. 2 marks

Solve $500 \times 2^{-0.15t} + 10 = 200$ (1 mark)

$t = 9.31$ months (1 mark)

After investigation, scientists discover that a more accurate model to determine the mass of the decaying substance can be found using the function $M(t) = 515 \times 2^{kt} + 20 \quad t \in [0, \infty)$ where $M(t)$ is in grams and t is in months.

e. If after 10 months the mass was to be 150 grams, show that k is -0.20 correct to two significant figures for this new model. 2 marks

$M(10) = 150$

solve $515 \times 2^{10k} + 20 = 150$ (1 mark)

$k = -0.20$ (1 mark)

f. State the range of this new model. 1 mark

$R: (20, 535]$ (1 mark)