

Supervision Instructions

Mathematics Methods (Unit 1-2)

Task #6

5th of October 2022 – Period 4

Task consists of two papers: **Paper 1** and **Paper 2**. Students will have access to only one paper at a time.

Paper 1:

- 15 minutes
- Calculator is not allowed

After 15 minutes **Paper 1** is to be collected and **Paper 2** will be given.

Paper 2:

- 25 minutes
- Calculator is allowed

After 25 minutes **Paper 2** is to be collected.

Check that students put their names.



2022 Mathematical Methods (Unit 1-2)

Task 6

Paper 1 – Calculator not allowed

Number of marks: 10

Writing time: 15 minutes

Name:

Marks:

Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

Solve the following for x .

a. $4^x + 2^{3+x} - 9 = 0$

2 marks

b. $\log_3(2x-2) - \log_3(x-2) = 2$

2 marks

Question 2

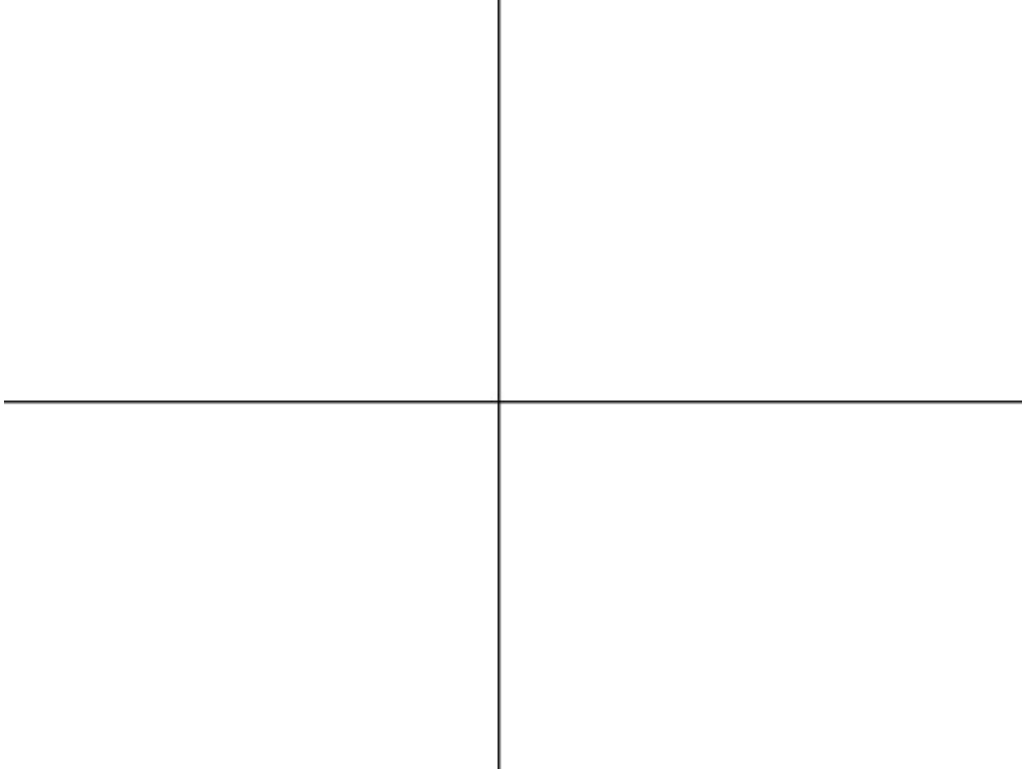
Express $\frac{8^{2-m} \times 16^{m+2}}{32^{m+3}}$ as a power of 2.

2 marks

Question 3

2+2 marks

- a. Sketch the graph of $y = -3 \log_{10}(x-2) + 3$ showing any asymptote(s) and axis intercept(s).



- b. State the domain and range of $y = -3 \log_{10}(x-2) + 3$.



2022 Mathematical Methods (Unit 1-2)

Task 6

Paper 2 – Calculator allowed

Number of marks: 15

Writing time: 25 minutes

Name:

Marks – Section 1:

Section 2:

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Question 1

The range of the function g with the rule $g(x) = 4 + 2^{3-2x}$ is:

- A $y \in (2, \infty)$
- B $y \in (-\infty, \infty)$
- C $y \in [4, \infty)$
- D $y \in [2, \infty)$
- E $y \in (4, \infty)$

Question 2

The graph of $y = 3^x$ undergoes the following transformations in the order below:

- A reflection in the x -axis
- A translation of 1 unit parallel to the x -axis in the positive direction
- A translation of 3 units parallel to the y -axis in the negative direction

The rule for the graph of the transformed function is:

- A $y = 3^{1-x} - 3$
- B $y = -3^{x-1} - 3$
- C $y = -3^{x+1} - 3$
- D $y = 3^{1-x} + 3$
- E $y = 3^{x+1} - 3$

Question 3

The function f has rule $f(x) = 2 \log_e(3x)$. If $f(3x) = \log_e(m)$ then m is equal to:

- A $81x^2$
- B $18x^2$
- C $6x$
- D $18x$
- E $8x^2$

Question 4

$\frac{2(p^2 r^{\frac{1}{2}})^3}{(4p^4 r^2)^{\frac{1}{2}}}$ in simplest form is:

- A $\frac{2}{p^4 r^{\frac{3}{2}}}$
- B $\frac{p^4}{r^{\frac{5}{2}}}$
- C $\frac{2p^2}{r^{\frac{5}{2}}}$
- D $\frac{p^4}{2r^{\frac{3}{2}}}$
- E $\frac{p^6}{2r^{\frac{1}{2}}}$

Question 5

If $f(x) = 2^{4x+8} + 5$, then $f^{-1}(x) =$

- A $\frac{1}{4} \log_2(x-5) + 2$
- B $\frac{1}{2} \log_4(x-5) - 2$
- C $\frac{1}{4} \log_2(x+5) + 2$
- D $\frac{1}{4} \log_2(x-5) - 2$
- E $\frac{1}{2} \log_2(x+5) - 2$

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

The mass of a radioactive substance in grams remaining after t months is given by:

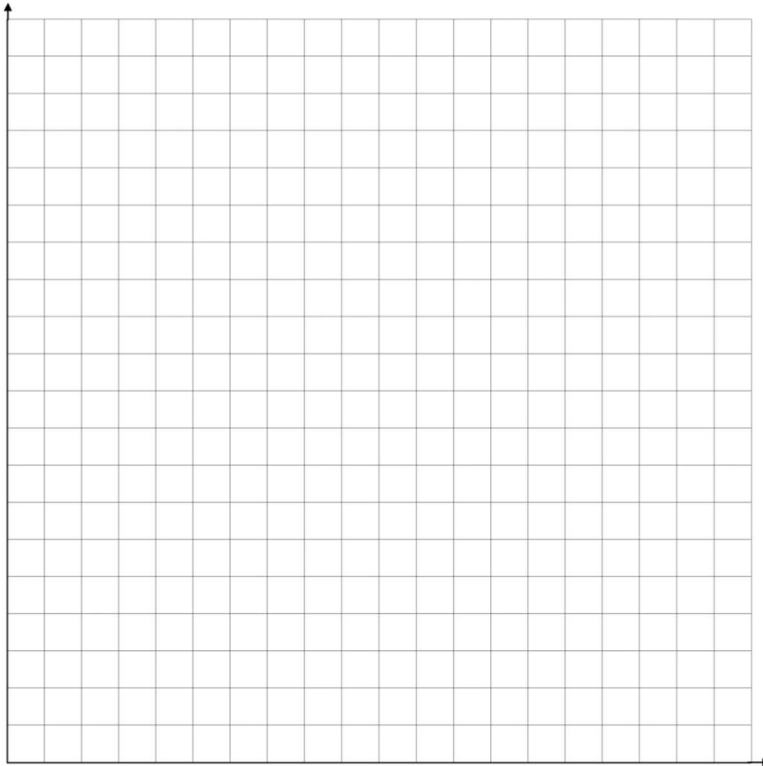
$$M(t) = 500 \times 2^{-0.15t} + 10 \quad t \in [0, \infty)$$

- a. i. Find the initial mass of the substance. Round your answer to 4 significant figures. 1 + 1 marks

ii. Find the mass of the substance after 2 months. Round your answer to 4 significant figures.

- b. What does the horizontal asymptote suggest about the decay of the substance? 1 mark

- c. Sketch the graph of $M(t) = 500 \times 2^{-0.15t} + 10$ $t \in [0, \infty)$ on the set of axes below. 2 marks
Clearly indicate any axis intercepts, end points and asymptotes.



- d. The radioactive substance is safe to handle when it decays to 200 grams or less. Use a suitable method to find the time in months, correct to 3 significant figures, when it is first safe to handle the substance. 2 marks

After investigation, scientists discover that a more accurate model to determine the mass of the decaying substance can be found using the function $M(t) = 515 \times 2^{kt} + 20$ $t \in [0, \infty)$ where $M(t)$ is in grams and t is in months.

e. If after 10 months the mass was to be 150 grams, show that k is -0.20 correct to two significant figures for this new model. **2 marks**

f. State the range of this new model.

1 mark