Marking Guide

Supervision Instructions

Mathematics Methods (Unit 1-2)
Task #7

30th of November 2022 – Period 4

Task consists of two papers: **Paper 1** and **Paper 2**. Students will have access to only one paper at a time.

Paper 1:
- 15 minutes
- Calculator is not allowed

After 15 minutes **Paper 1** is to be collected and **Paper 2** will be given.

Paper 2:
- 25 minutes
- Calculator is allowed

After 25 minutes **Paper 2** is to be collected.

Check that students put their names.

2022 Mathematical Methods (Unit 1-2)
Task 7

Paper 1 – Calculator not allowed

Sirius College
" Reach for the stars "

Number of marks: 10
Writing time: 15 minutes

Name: Marks:

Instructions
Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

2 marks

Let $y = \frac{3}{2}x^2(4x + \frac{1}{\sqrt{x}})$. Find $\frac{dy}{dx}$.

$y = 6x^3 + \frac{3}{2}x^{3/2}$ → 1 mark

$\frac{dy}{dx} = 18x^2 + \frac{9}{4}\sqrt{x}$ → 1 mark

Question 2

2 marks

A particle moves in a straight line so that its displacement, x metres, from a fixed origin at time t seconds is given by $x = t^3 - t^2 - 8t + 9$. At what position is the particle temporarily at rest?

$v = \frac{dx}{dt}$

$v = 3t^2 - 2t - 8$

$0 = 3t^2 - 2t - 8$

$0 = (3t+4)(t-2)$ → 1 mark

$t = -\frac{3}{4}$ or $t = 2$

reject

$x(2) = 2^3 - (2)^2 - 8(2) + 9$

$= -3m$

$\therefore$ 3 m to the left

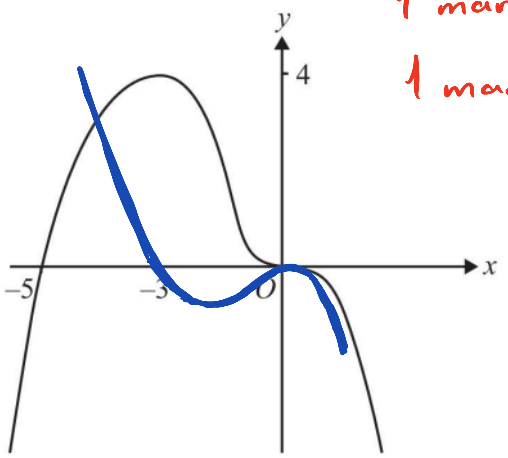
1 mark

Question 3 2 marks

The graph of $f(x)$ is given. Sketch the graph of its derivative function on the same set of the axes shown below.

1 mark for correct intercepts.

1 mark for the correct shape



Question 4 3+1 marks

The point (2, 4) is a stationary point of the curve $y = ax^2 + bx$

i) Calculate the values a and b.

$\frac{dy}{dx} = 0$ when $x = 2$

$\frac{dy}{dx} = 2ax + b$ sub. $x = 2$

1mark → $4a + b = 0$ ①

Sub. (2, 4) into $y = ax^2 + bx$

1mark → $4 = 4a + 2b$ ②

Solving sim. equations

$4a + b = 0$

$4a + 2b = 4$

$\boxed{\begin{matrix} a = -1 \\ b = 4 \end{matrix}}$ } 1 mark

ii) State the nature of the stationary point by showing your work.

	1	2	3
$f'(x)$	2	0	−2
slope	/	—	\

OR $f''(x) = -2 < 0$

Maximum Turning Point

2022 Mathematical Methods (Unit 1-2)
Task 7

Paper 2 – Calculator allowed

Number of marks: 15
Writing time: 25 minutes

Name: Marks – Section 1: Section 2:

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Question 1

The derivative of $f(x)=x^2-3x$ can be found from first principles by evaluating:

A. $\lim\limits_{h\to 0}\dfrac{(x+h)^2-3xh-x^2-3x}{h}$

B. $\lim\limits_{h\to 0}\dfrac{(x+h)^2-3(x+h)^2-x^2-3x}{h}$

C. $\lim\limits_{h\to 0}\dfrac{(x+h)^2-3(x+h)-x^2-3x}{h}$

D. $\lim\limits_{h\to 0}\dfrac{(x+h)^2-3(x+h)-x^2+3x}{h}$

E. $\lim\limits_{h\to 0}\dfrac{(x+h)^2-3(x+h)-x^2+3xh}{h}$

Question 2

The coordinates of the point on the graph of $y=-x^2-2x$ at which the tangent is perpendicular to $y=\dfrac{x}{8}+1$ is:

A. $(3,-8)$

B. $(3,-15)$

C. $(-3,\dfrac{11}{8})$

D. $(-3,\dfrac{1}{8})$

E. $(-3,3)$

Question 3

If $y = x^2 - 5x - 24$, the interval(s) for which $\dfrac{dy}{dx} > 0$ is:

A. $x < -3$

B. $-3 < x < 8$

C. $x < \dfrac{5}{2}$

D. $x < -3 \cup x > 8$

E. $x > \dfrac{5}{2}$

Question 4

$$h(x) = \begin{cases} 1 - x^3, & x < -2 \\ 3, & x = -2 \\ x^2 + 5, & x > -2 \end{cases}$$

$\lim\limits_{x \to -2} h(x)$ is:

A. -2

B. 3

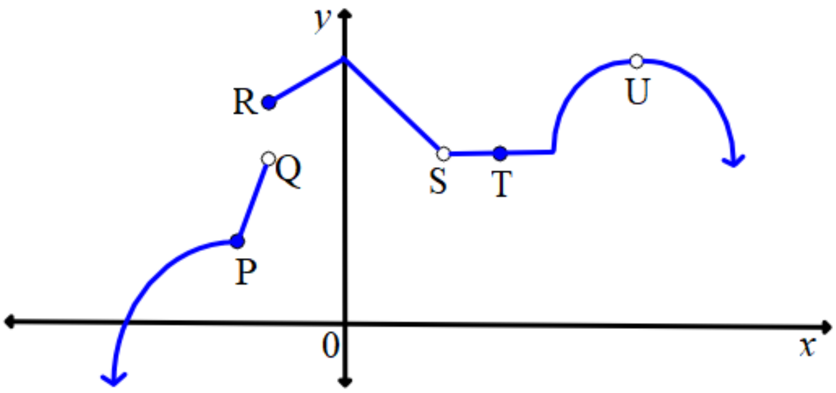
C. 8

D. 9

E. The limit does not exist.

Question 5

The graph below is differentiable at point(s):



A. P only

B. Q, S and U

C. T only

D. U only

E. Q and R

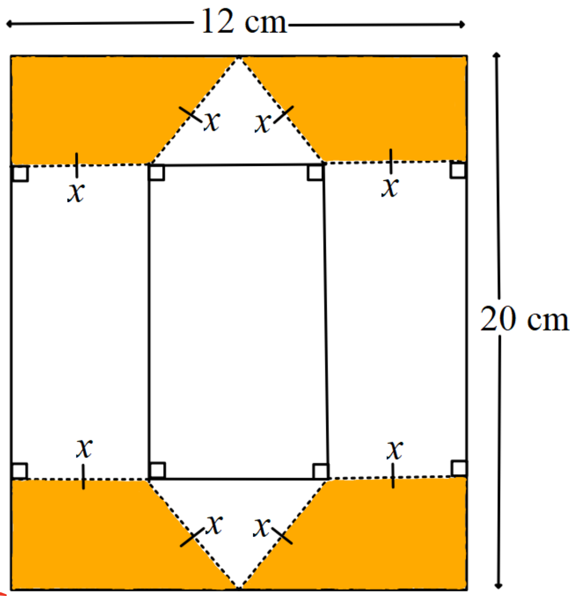
SECTION 2

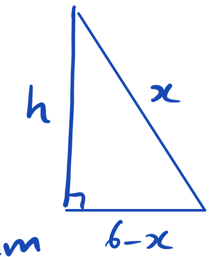
Instructions for Section 2
Answer **all** questions in the spaces provided. In all questions where a numerical answer is required, an exact value must be given unless otherwise specified. In questions where more than one mark is available, appropriate working **must** be shown. Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

A triangular prism shaped 'Toblerone' chocolate box is to be constructed from a rectangular sheet of cardboard measuring 20 cm by 12 cm as shown below. Equal lengths of x cm are cut along the dotted lines and remaining flaps are folded up.

a. Show that the height of the triangle is $h = \sqrt{12x - 36}$ — 2 marks





Base $= \frac{12-2x}{2} = 6-x$

Using Pythagoras' Theorem

$h^2 = x^2 - (6-x)^2 \longrightarrow$ 1 mark

$= x^2 - (36 - 12x + x^2)$

$= \cancel{x^2} - 36 + 12x - \cancel{x^2}$

$\sqrt{h^2} = \sqrt{12x - 36}$

$h = \sqrt{12x - 36}$

1 mark

b. Find the restriction(s) on x. Show your work — 2 marks

The prism exists when:

$x > 0$, $12 - 2x > 0$, $\sqrt{12x - 36} > 0$, $20 - 2\sqrt{12x - 36} > 0$

$\Rightarrow x < 6$ $\Rightarrow x > 3$ $\Rightarrow x < \frac{34}{3}$

1 mark for showing at least two restrictions.

The intersection of all this gives:

$\boxed{3 < x < 6} \longrightarrow$ 1 mark

c. Show that the volume of the box, V cm^3, is given by **2 marks**

$$V(x) = 2(6-x)(10\sqrt{12x-36} - (12x-36))$$

$$V = A \times H$$

$$V(x) = \left(\tfrac{1}{2}(12-2x)\sqrt{12x-36}\right)\left(20 - 2\sqrt{12x-36}\right) \rightarrow 1 \text{ mark}$$

$$= \left((6-x)\sqrt{12x-36}\right)\left(20-2\sqrt{12x-36}\right)$$

$$= 20(6-x)\sqrt{12x-36} - 2(6-x)\left(\sqrt{12x-36}\right)^2$$

$$= 2(6-x)\left(10\sqrt{12x-36} - (12x-36)\right)$$

1 mark

d. Use calculus to find the value of x for which the volume of the box is a maximum, correct to one decimal place. **2 marks**

Maximum volume at stationary point $\dfrac{dV}{dx} = 0$

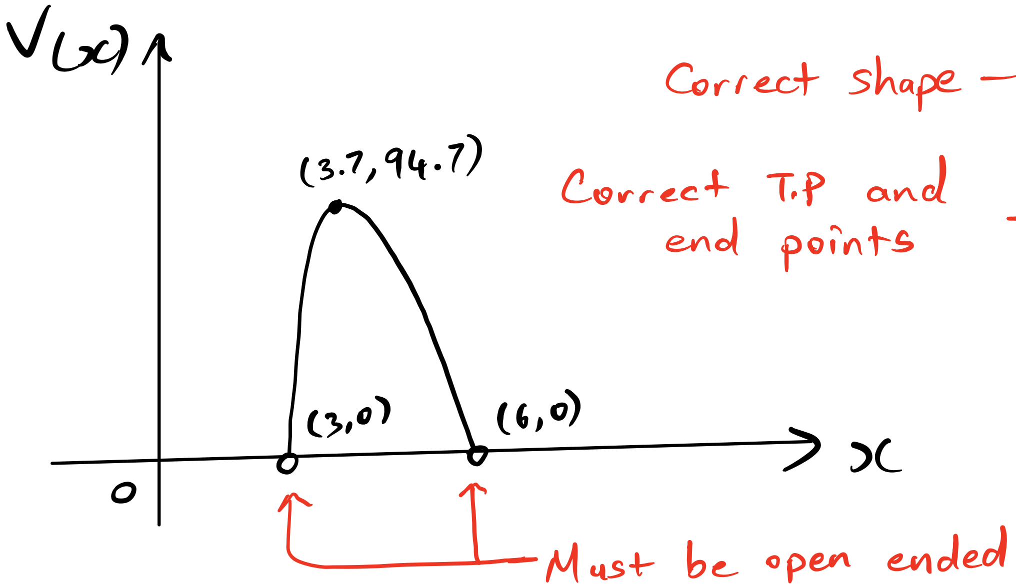
Solving $\dfrac{dV}{dx} = 0$ for x using CAS

$\boxed{x = 3.7 \text{ cm}}$ $\quad x = 8.8$ cm rejected since $3 < x < 6$

1 mark

1 mark

e. Sketch the graph of $V(x)$. Label any end points and stationary points with their coordinates correct to one decimal place. **2 marks**



Correct shape – 1 mark

Correct T.P and end points – 1 mark