

Supervision Instructions

Mathematics Methods (Unit 1-2) Task #7

30th of November 2022 – Period 4

Task consists of two papers: **Paper 1** and **Paper 2**. Students will have access to only one paper at a time.

Paper 1:

- 15 minutes
- Calculator is not allowed

After 15 minutes **Paper 1** is to be collected and **Paper 2** will be given.

Paper 2:

- 25 minutes
- Calculator is allowed

After 25 minutes **Paper 2** is to be collected.

Check that students put their names.



2022 Mathematical Methods (Unit 1-2)

Task 7

Paper 1 – Calculator not allowed

Number of marks: 10

Writing time: 15 minutes

Name:

Marks:

Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

2 marks

Let $y = \frac{3}{2}x^2(4x + \frac{1}{\sqrt{x}})$. Find $\frac{dy}{dx}$.

Question 2

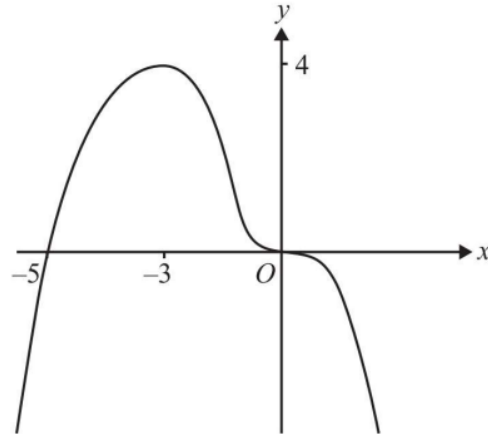
2 marks

A particle moves in a straight line so that its displacement, x metres, from a fixed origin at time t seconds is given by $x = t^3 - t^2 - 8t + 9$. At what position is the particle temporarily at rest?

Question 3

2 marks

The graph of $f(x)$ is given. Sketch the graph of its derivative function on the same set of the axes shown below.



Question 4

3+1
marks

The point $(2, 4)$ is a stationary point of the curve $y = ax^2 + bx$

i) Calculate the values a and b .

ii) State the nature of the stationary point by showing your work.



2022 Mathematical Methods (Unit 1-2)

Task 7

Paper 2 – Calculator allowed

Number of marks: 15

Writing time: 25 minutes

Name:

Marks – Section 1:

Section 2:

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Question 1

The derivative of $f(x) = x^2 - 3x$ can be found from first principles by evaluating:

- A. $\lim_{h \rightarrow 0} \frac{(x+h)^2 - 3xh - x^2 - 3x}{h}$
- B. $\lim_{h \rightarrow 0} \frac{(x+h)^2 - 3(x+h)^2 - x^2 - 3x}{h}$
- C. $\lim_{h \rightarrow 0} \frac{(x+h)^2 - 3(x+h) - x^2 - 3x}{h}$
- D. $\lim_{h \rightarrow 0} \frac{(x+h)^2 - 3(x+h) - x^2 + 3x}{h}$
- E. $\lim_{h \rightarrow 0} \frac{(x+h)^2 - 3(x+h) - x^2 + 3xh}{h}$

Question 2

The coordinates of the point on the graph of $y = -x^2 - 2x$ at which the tangent is perpendicular to

$y = \frac{x}{8} + 1$ is:

- A. (3, -8)
- B. (3, -15)
- C. $(-3, \frac{11}{8})$
- D. $(-3, \frac{1}{8})$
- E. (-3, 3)

Question 3

If $y = x^2 - 5x - 24$, the interval(s) for which $\frac{dy}{dx} > 0$ is:

- A. $x < -3$
- B. $-3 < x < 8$
- C. $x < \frac{5}{2}$
- D. $x < -3 \cup x > 8$
- E. $x > \frac{5}{2}$

Question 4

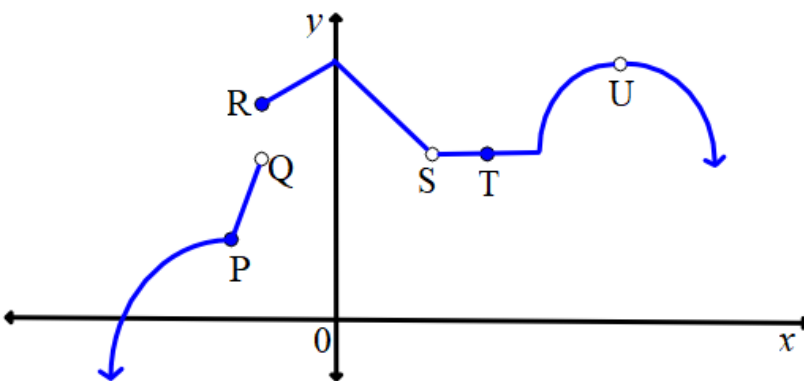
$$h(x) = \begin{cases} 1 - x^3, & x < -2 \\ 3, & x = -2 \\ x^2 + 5, & x > -2 \end{cases}$$

$\lim_{x \rightarrow -2} h(x)$ is:

- A. -2
- B. 3
- C. 8
- D. 9
- E. The limit does not exist.

Question 5

The graph below is differentiable at point(s):



- A. P only
- B. Q, S and U
- C. T only
- D. U only
- E. Q and R

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

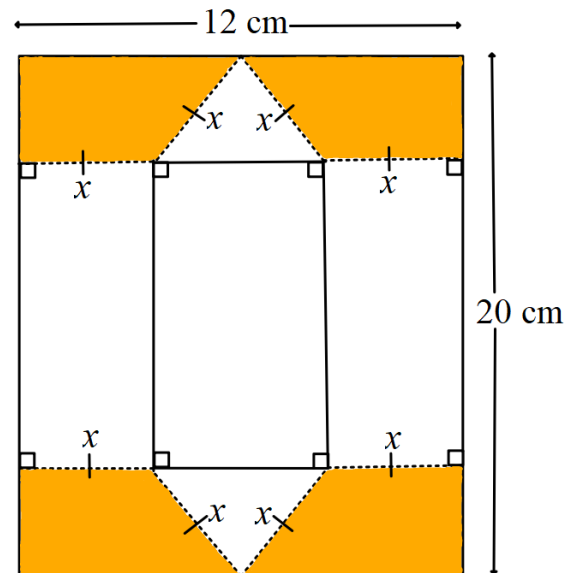
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Question 1

A triangular prism shaped ‘Toblerone’ chocolate box is to be constructed from a rectangular sheet of cardboard measuring 20 cm by 12 cm as shown below. Equal lengths of x cm are cut along the dotted lines and remaining flaps are folded up.

- a. Show that the height of the triangle is

$$h = \sqrt{12x - 36}$$



2 marks

- b. Find the restriction(s) on x . Show your work

2 marks

- c. Show that the volume of the box, $V \text{ cm}^3$, is given by

2 marks

$$V(x) = 2(6-x)(10\sqrt{12x-36} - (12x-36))$$

- d. Use calculus to find the value of x for which the volume of the box is a maximum, correct to one decimal place.

2 marks

- e. Sketch the graph of $V(x)$. Label any end points and stationary points with their coordinates correct to one decimal place.

2 marks