Sirius
College
" Reach for the stars "

Name: Marks:

| Instructions |
| --- |
| Answer **all** questions in the spaces provided. In all questions where a numerical answer is required, an exact value must be given unless otherwise specified. In questions where more than one mark is available, appropriate working **must** be shown. Unless otherwise indicated, the diagrams in this book are **not** drawn to scale. |

Question 1

a. Sketch $y = x^2 - 2x - 3, x \in [-2, 4)$, showing all key features and find its range.

3+1

marks
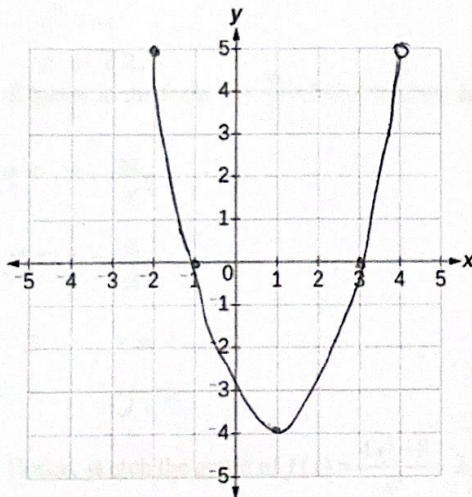
$b = -\frac{-2}{2(1)} = +1$

$T(+1, -4)$

y-int ; $(0, -3)$

x-int ; $0 = x^2 - 2x - 3$

$\begin{array}{r} -3 \\ +1 \end{array}$

$(3, 0) \ (-1, 0)$

$\begin{array}{l} [-2, 5] \\ (4, 5) \end{array}$ end points.

Range: $[-4, 5]$

b. Further restrict the domain of the function defined by $y = x^2 + 2x + 3$ so that it will be a one-to-one and decreasing function.

$[-2, 1] \longrightarrow$ this is restricted domain of $y = x^2 - 2x - 3$ (1.a) parabola in

$\longrightarrow$ for that parabola, restricted domain would be $(-\infty, -1]$

Question 2

State the range for the function $y = -\sqrt{16-x}$.

$$(-\infty, 0]$$

Question 3

Let $f : R/\{0\} \to R, f(x) = \dfrac{4x^2-8}{x^2} - 2$.

a. Evaluate $f(x) = 0$

$\dfrac{4x^2-8}{x^2} - 2 = 0$

$4x^2-8 = 2x^2$

$x^2 = 4$

$x = \mp 2$

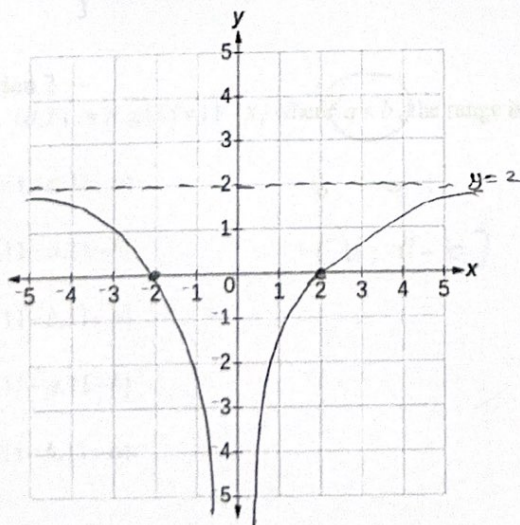b. Express in the form $y = \dfrac{a}{x^2} + b$ and state the equations of the asymptotes.

$y = 4 - \dfrac{8}{x^2} - 2$

$y = 2 - \dfrac{8}{x^2}$

h.a : $x = 0$

v.a : $y = 2$

c. Hence, sketch the graph of $f(x) = \dfrac{4x^2-8}{x^2} - 2$, showing all the key features.

x-int ; $(2,0)$ $(-2,0)$

no y-int.

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Name: Marks – Section 1:   Section 2:

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Question 1

If f is a function for which the rule is $f(x) = \frac{3}{4} - x$, where x is real, the rule for the inverse function f^{-1} is

A $\quad f^{-1}(x) = \frac{4}{3} + x$

B $\quad f^{-1}(x) = -\frac{4}{3}$

C $\quad f^{-1}(x) = \frac{3x+4}{4}$

D $\quad f^{-1}(x) = \frac{3}{4} - x$

E $\quad f^{-1}(x) = \frac{4}{3} - x$

Question 2

For $g : (a,b] \to R, g(x) = 11 - x$, where $a < b$, the range is

A $\quad (11-a, 11-b)$

B $\quad (11-a, 11-b]$

C $\quad (11-b, 11-a)$

D $\quad (11-a, 11-b]$

E $\quad [11-b, 11-a)$

Question 3

The maximal (implied) domain of the function with rule $f(x) = \dfrac{8}{(x-4)^2}$ is;

A $\quad (0, \infty)$

B $\quad (-4, \infty)$

C $\quad R \setminus \{4\}$

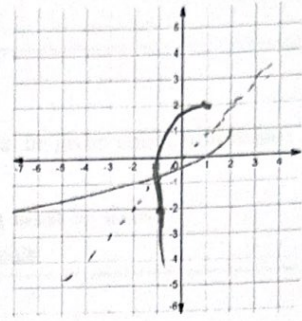D $\quad R \setminus \{-4\}$

E $\quad [0, \infty)$

Question 4

For the function with rule $f(x) = \begin{cases} x^2 + 3 & x \geq 4 \\ -x + 4 & x < 4 \end{cases}$, the value of $f(a+4)$ where a is a **negative** real number, is
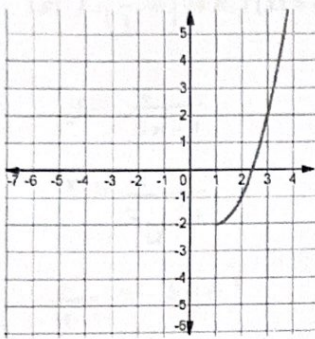
A $\quad a^2 + 8a + 16$ $\qquad -(a+4) + 4$

B $\quad -a$

C $\quad -a + 8$

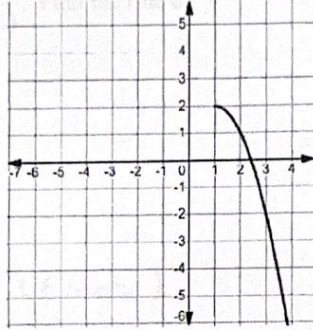D $\quad a^2 + 9$

E $\quad a^2 + 8a + 13$

Question 5

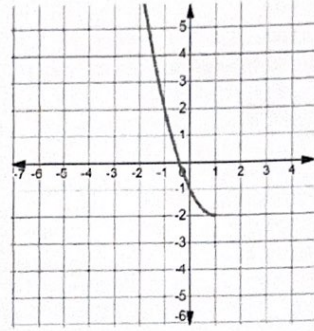The graph of the function f is shown. Which of the following is most likely to be the graph of the inverse function?
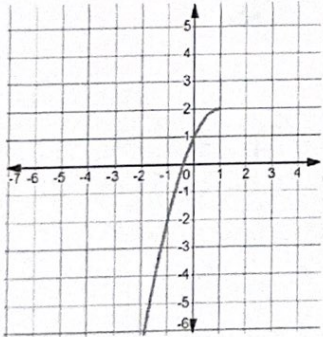
A

B

C

D
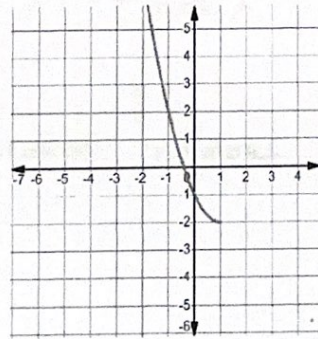
E

SECTION 2

Answer **all** questions in the spaces provided.

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Question 1

a.

Let $f:\left(\frac{1}{3},\infty\right) \to R, f(x) = \frac{1}{3x-1}$. Find the rule of f^{-1}

2 marks

$$y = \frac{1}{3x-1}$$

$$x = \frac{1}{3y-1}$$

$$\frac{1}{x} = 3y-1 \qquad (1 \text{ mark})$$

$$y = \frac{1}{3x} + \frac{1}{3}$$

$$f'(x) = \frac{1}{3x} + \frac{1}{3} \qquad (1 \text{ mark})$$

↑

if no f^{-1} notation, no marks

b.

State the domain of f^{-1}.

1 mark

$R - \{0\}$

Question 2

A colony of viruses is grown in a laboratory by putting a number of viruses in a dish of nutrient. The number of viruses in the dish, N million, at a time t days may be modelled by the formula $N(t) = \dfrac{100}{(t+1)^2} - 1, t \geq 0$.

Use this model to answer the following questions.

a. How many viruses were put in the dish initially?   1 mark
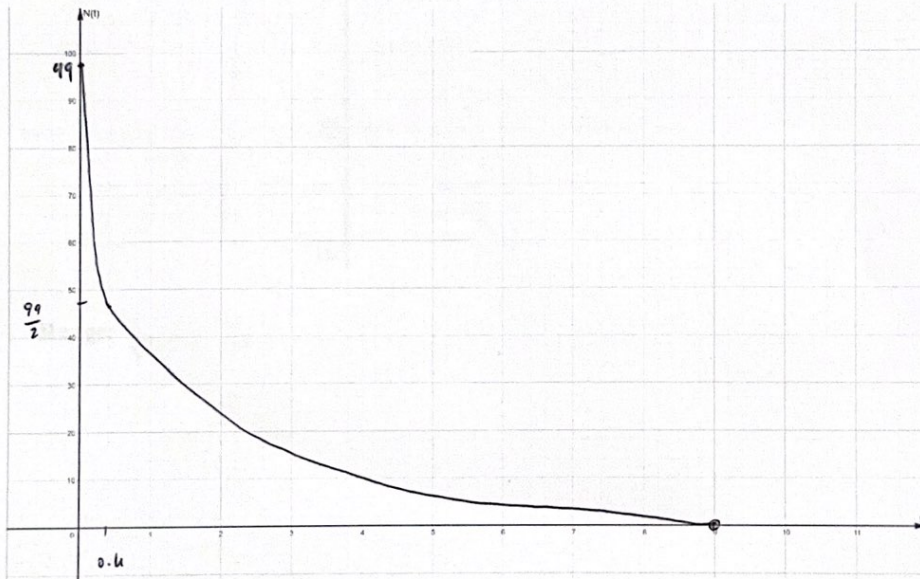
$$N(0) = \frac{100}{1} - 1 = 99 \text{ million}$$

b. How long does it take for the initial amount of viruses to halve? Round answer to the nearest **hour**.   1 mark

$$\frac{99}{2} = \frac{100}{(t+1)^2} - 1$$

$t = 0.4 \checkmark \rightarrow \quad 0.4 \times 24 = 9.6$

$t = -2.4 \times \qquad \hat{=} 10 \text{ hours}$

c. On the axes below, sketch the model of $N(t)$. Clearly label the results found in part a and b.   2 marks
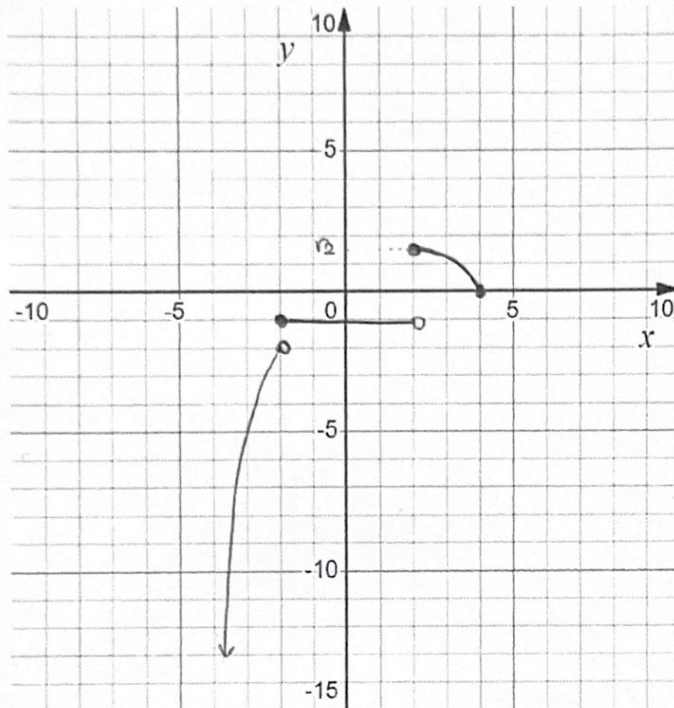
Question 3

Sketch the graph of the following function and state its range.   3 marks

$$f(x) = \begin{cases} x^3 + 6, & x < -2 \\ -1, & -2 \le x < 2 \\ \sqrt{4-x}, & x \ge 2 \end{cases}$$

Range: $\left(-\infty, \sqrt{2}\right]$