

Supervision Instructions

Mathematics Methods (Unit 1-2)

Task #3

24th May 2023 – Period 6

Task consists of two papers: **Paper 1** and **Paper 2**. Students will have access to only one paper at a time.

Paper 1:

- 15 minutes
- Calculator is not allowed

After 15 minutes **Paper 1** is to be collected and **Paper 2** will be given.

Paper 2:

- 25 minutes
- Calculator is allowed

After 25 minutes **Paper 2** is to be collected.

Check that students put their names.



2023 Mathematical Methods (Unit 1-2)

Task 3

Paper 1 – Calculator not allowed

Number of marks: 10

Writing time: 15 minutes

Name:

Marks:

Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

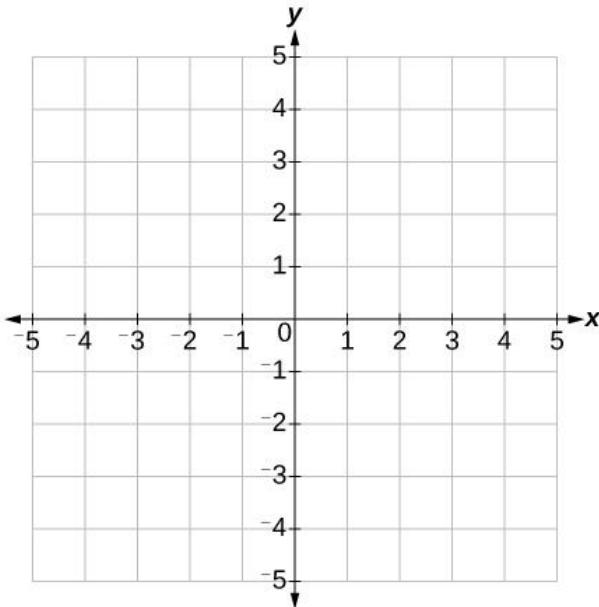
In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

- a. Sketch $y = x^2 - 2x - 3, x \in [-2, 4)$, showing all key features and find its range.

3+1
marks



Range:

- b. Further restrict the domain of the function defined by $y = x^2 + 2x + 3$ so that it will be a one-to-one and decreasing function.

Question 2

State the range for the function $y = -\sqrt{16-x}$.

1 mark

Question 3

Let $f : \mathbb{R} \setminus \{0\} \rightarrow \mathbb{R}$, $f(x) = \frac{4x^2 - 8}{x^2} - 2$.

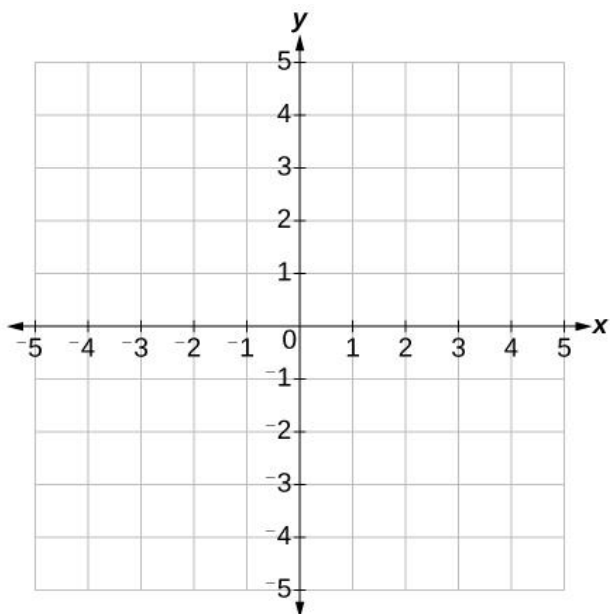
1+2+2

marks

a. Evaluate $f(x) = 0$

b. Express in the form $y = \frac{a}{x^2} + b$ and state the equations of the asymptotes.

c. Hence, sketch the graph of $f(x) = \frac{4x^2 - 8}{x^2} - 2$, showing all the key features.





2023 Mathematical Methods (Unit 1-2)

Task 3

Paper 2 – Calculator allowed

Number of marks: 15

Writing time: 25 minutes

Name:

Marks – Section 1:

Section 2:

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Question 1

If f is a function for which the rule is $f(x) = \frac{3}{4} - x$, where x is real, the rule for the inverse function f^{-1} is

A $f^{-1}(x) = \frac{4}{3} + x$

B $f^{-1}(x) = -\frac{4}{3}$

C $f^{-1}(x) = \frac{3x+4}{4}$

D $f^{-1}(x) = \frac{3}{4} - x$

E $f^{-1}(x) = \frac{4}{3} - x$

Question 2

For $g : (a, b] \rightarrow \mathbb{R}$, $g(x) = 11 - x$, where $a < b$, the range is

A $(11 - a, 11 - b)$

B $(11 - a, 11 - b]$

C $(11 - b, 11 - a)$

D $(11 - a, 11 - b]$

E $[11 - b, 11 - a)$

Question 3

The maximal (implied) domain of the function with rule $f(x) = \frac{8}{(x-4)^2}$ is;

- A $(0, \infty)$
- B $(-4, \infty)$
- C $\mathbb{R} \setminus \{4\}$
- D $\mathbb{R} \setminus \{-4\}$
- E $[0, \infty)$

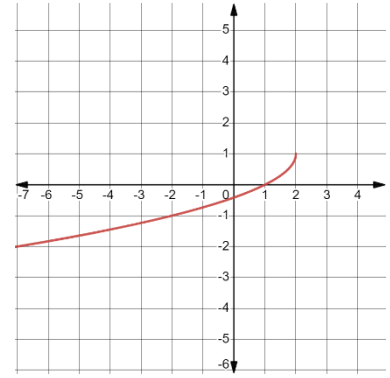
Question 4

For the function with rule $f(x) = \begin{cases} x^2+3 & x \geq 4 \\ -x+4 & x < 4 \end{cases}$, the value of $f(a+4)$ where a is a **negative** real number, is

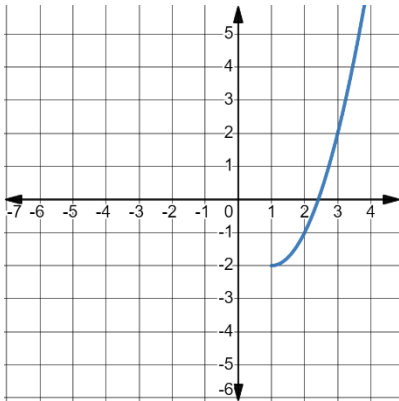
- A $a^2 + 8a + 16$
- B $-a$
- C $-a + 8$
- D $a^2 + 9$
- E $a^2 + 8a + 13$

Question 5

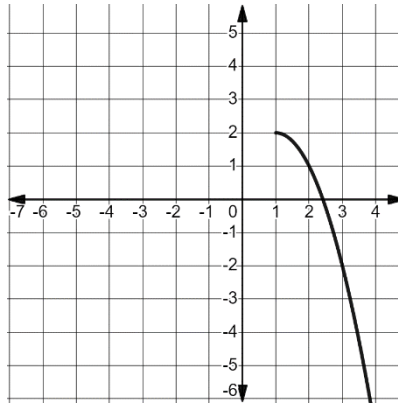
The graph of the function f is shown. Which of the following is most likely to be the graph of the inverse function?



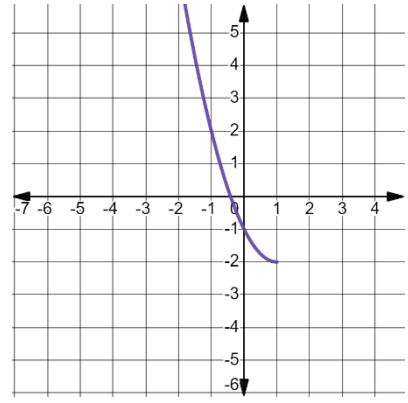
A



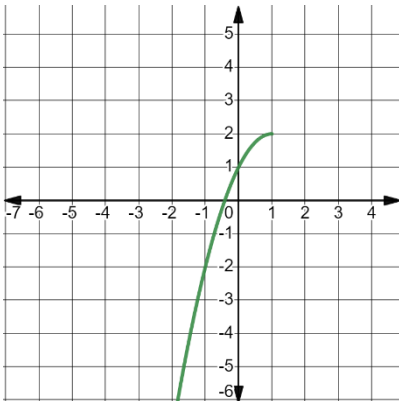
B



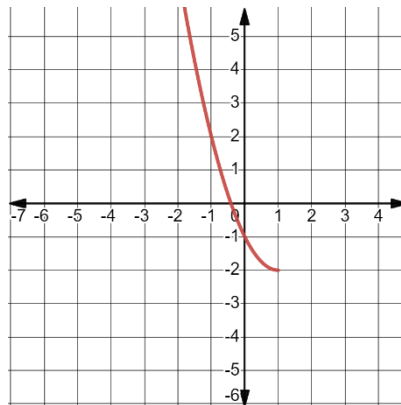
C



D



E



SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

a. Let $f : \left(\frac{1}{3}, \infty\right) \rightarrow \mathbb{R}$, $f(x) = \frac{1}{3x-1}$. Find the rule of f^{-1}

2 marks

b. State the domain of f^{-1} .

1 mark

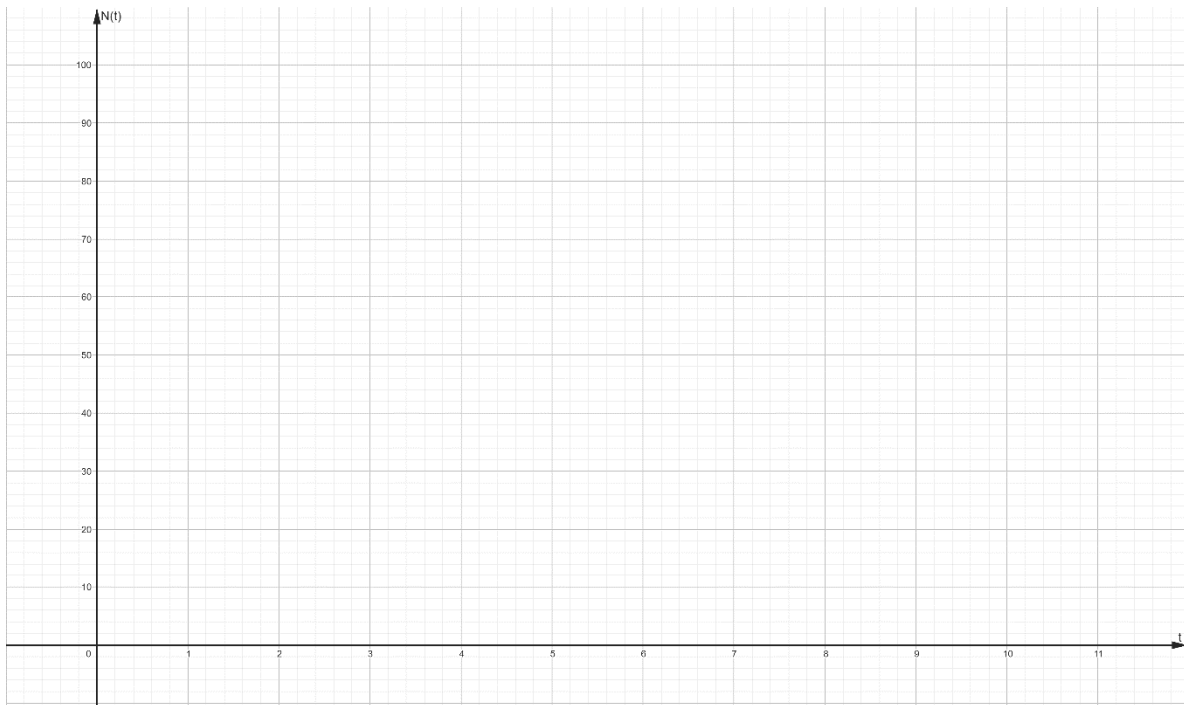
Question 2

A colony of viruses is grown in a laboratory by putting a number of viruses in a dish of nutrient. The number of viruses in the dish, N million, at a time t days may be modelled by

$$\text{the formula } N(t) = \frac{100}{(t+1)^2} - 1, t \geq 0.$$

Use this model to answer the following questions.

- a. How many viruses were put in the dish initially? 1 mark
- b. How long does it take for the initial amount of viruses to halve? Round answer to the nearest hour. 1 mark
- c. On the axes below, sketch the model of $N(t)$. Clearly label the results found in part a and b. 2 marks

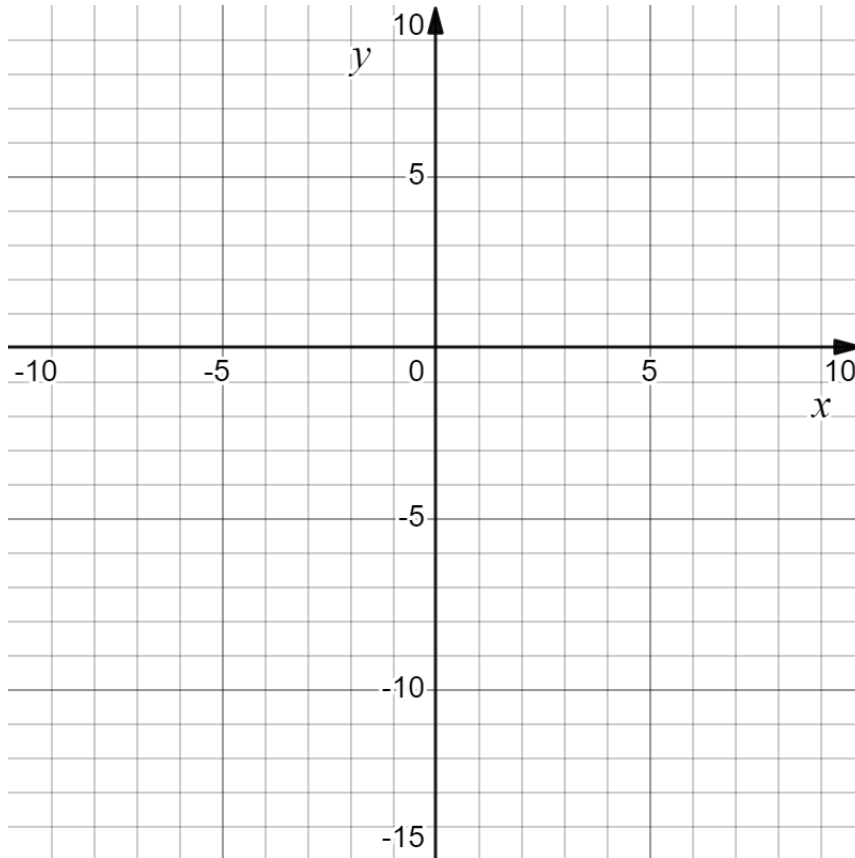


Question 3

Sketch the graph of the following function and state its range.

3 marks

$$f(x) = \begin{cases} x^3 + 6, & x < -2 \\ -1, & -2 \leq x < 2 \\ \sqrt{4-x}, & x \geq 2 \end{cases}$$



Range: