

MARKING GUIDE



2023 Mathematical Methods (Unit 1-2)

Task 4

Paper 1 – Calculator not allowed

Number of marks: 10

Writing time: 15 minutes

Name:

Marks:

Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

1+1+1 marks

In a class of 34 students, it was found that TikTok only users were twice as many as Instagram only users. 4 students used both TikTok and Instagram while 6 students used neither of them.

Let T be the set of TikTok users and let I be the set of Instagram users.

- a) Calculate the number of students that use Instagram only.

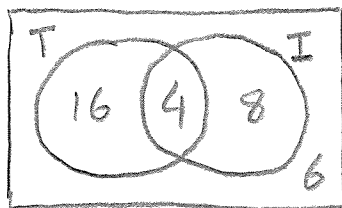
$$34 - 6 = 28$$

$$I + 2I + 4 = 28$$

$$3I = 24$$

$$\Rightarrow \boxed{I = 8} \quad (1 \text{ mark})$$

- b) Draw a Venn diagram to illustrate this situation.



(1 mark)

- c) Find $\Pr(T' \cup I)$

$$\Pr(T' \cup I) = \Pr(T') + \Pr(I) - \Pr(T \cap I)$$

$$= \frac{14}{34} + \frac{12}{34} - \frac{8}{34}$$

$$\Rightarrow \frac{18}{34} = \boxed{\frac{9}{17}} \quad (1 \text{ mark})$$

Question 2

2 marks

Given $\Pr(A) = k$, $\Pr(B) = \frac{1}{7}$ and $\Pr(A \cup B) = \frac{5}{7}$, determine the value of k if the events are independent.

$$\Pr(A \cap B) = k \times \frac{1}{7} = \frac{k}{7}$$

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

$$\frac{5}{7} = k + \frac{1}{7} - \frac{k}{7} \quad \leftarrow (1 \text{ mark for the equation})$$

$$\frac{4}{7} = \frac{6k}{7} \quad \Rightarrow \boxed{k = \frac{2}{3}} \quad (1 \text{ mark})$$

Question 3

1 mark

In how many ways the letters of the word CHEESE can be arranged in a circle?

$$\frac{(6-1)!}{3!} = \frac{5!}{3!} = \frac{5 \times 4 \times \cancel{3!}}{3!} = 20 \quad (1 \text{ mark})$$

Question 4

2 marks

If $\Pr(A \cap B) = \frac{1}{5}$ and $\Pr(B) = \frac{7}{10}$, calculate $\Pr(A' | B)$

$$\Pr(B) = \Pr(A \cap B) + \Pr(A' \cap B)$$

$$\frac{7}{10} = \frac{1}{5} + \Pr(A' \cap B)$$

$$\Pr(A' \cap B) = \frac{1}{2} \quad (1 \text{ mark})$$

$$\Pr(A' | B) = \frac{\Pr(A' \cap B)}{\Pr(B)}$$

$$= \frac{\frac{1}{2}}{\frac{7}{10}}$$

$$= \frac{5}{7} \quad (1 \text{ mark})$$

Question 5

2 marks

A box contains 4 green and 2 red marbles. A sample of two marbles is drawn from the box, without replacement.

- a) Find the probability that both marbles are green.

$$\Pr(GG) = \frac{4}{6} \times \frac{3}{5} = \frac{12}{30} = \frac{2}{5} \quad (1 \text{ mark})$$

- b) Find the probability that the marbles are different in colour.

$$\begin{aligned} & \Pr(GR) + \Pr(RG) \\ &= \frac{4}{6} \times \frac{2}{5} + \frac{2}{6} \times \frac{4}{5} \\ &= \frac{4}{15} + \frac{4}{15} \\ &= \frac{8}{15} \quad (1 \text{ mark}) \end{aligned}$$



2023 Mathematical Methods (Unit 1-2)

Task 4

Paper 2 – Calculator allowed

Number of marks: 15

Writing time: 25 minutes

Name:

Marks – Section 1:

Section 2:

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Question 1

A bag contains 4 red pencils and 3 blue pencils. Three pencils are drawn without replacement. The probability that at least one pencil is red:

- A $\frac{4}{35}$
- B $\frac{34}{35}$
- C $\frac{24}{35}$
- D $\frac{17}{210}$
- E $\frac{21}{35}$

$$\begin{aligned} \Pr(\text{at least one red}) &= 1 - \Pr(\text{BBB}) \\ &= 1 - \left(\frac{3}{7} \times \frac{2}{6} \times \frac{1}{5}\right) \\ &= \frac{34}{35} \end{aligned}$$

Question 2

In a soccer competition, the Australian team's chances of defeating the teams from France and Korea are 0.3 and 0.6 respectively. The probability that Australia loses both matches is:

- A 0.18
- B 0.28
- C 0.45
- D 0.72
- E 0.9

$$0.7 \times 0.4 = 0.28$$

Question 3

How many 4-digit even numbers can be formed using the digits 2,3,4,5 and 6, if the repetitions are not allowed?

- A 24
- B 60
- C 72
- D 120
- E 140

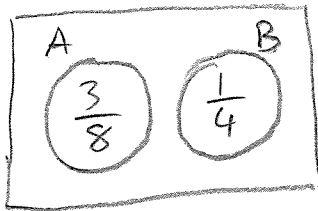
$$\begin{array}{|c|c|c|c|} \hline 4 & 3 & 2 & 3 \\ \hline \end{array} = 72$$

↑
2, 4, 6

Question 4

If two events A and B are mutually exclusive and $\Pr(A \cup B) = \frac{5}{8}$ and $\Pr(B) = \frac{1}{4}$ then $\Pr(A \cap B')$ is equal to:

- A $\frac{1}{8}$
- B $\frac{2}{5}$
- C $\frac{3}{5}$
- D $\frac{3}{8}$
- E $\frac{7}{8}$



$$\begin{aligned} \Pr(A \cap B') &= \Pr(A) \\ \Pr(A) &= \Pr(A \cup B) - \Pr(B) \\ &= \frac{5}{8} - \frac{1}{4} \\ &= \frac{3}{8} \end{aligned}$$

Question 5

In how many ways can a hand of 4 cards be dealt from a deck of 52 cards?

- A 208
- B 1248
- C 54236
- D 270725
- E 6497400

$${}^{52}C_4 = 270725$$

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.

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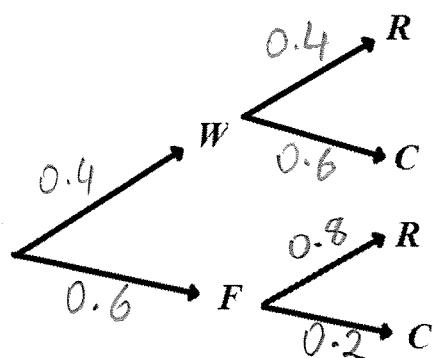
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Question 1

Jessica has two main hobbies; she either likes to run or play chess. If the weather is forecast as wet, there is a 60% chance that she will play chess. However, if the weather is forecast as fine, there is an 80% chance that Jessica will go for a run.

The weather bureau forecasts the chance of Sunday being wet as 0.4.

- a. Complete the tree diagram below by writing the probability values on each branch. 1 mark
Let W = wet, F = fine, R = run and C = chess.



- b. Calculate the probability that Sunday is fine and Jessica plays chess. 1 mark

$$\Pr(F \cap C) = 0.6 \times 0.2 = 0.12 \quad (1 \text{ mark})$$

- c. What is the probability that Jessica goes for a run on Sunday? 2 marks

$$\begin{aligned} \Pr(R) &= \Pr(W \cap R) + \Pr(F \cap R) \\ &= 0.4 \times 0.4 + 0.6 \times 0.8 \rightarrow 1 \text{ mark} \\ &= 0.64 \rightarrow 1 \text{ mark} \end{aligned}$$

- d. Given that Jessica went for a run, what is the probability that Sunday was wet? 2 marks

$$\begin{aligned} \Pr(W|R) &= \frac{\Pr(W \cap R)}{\Pr(R)} \quad (1 \text{ mark for the equation}) \\ &= \frac{0.4 \times 0.4}{0.64} = 0.25 \quad (1 \text{ mark}) \end{aligned}$$

Question 2

A panel of 6 is to be selected from a group of 5 girls and 8 boys.

a. How many panels can be formed if there are no restrictions?

1 mark

$${}^{13}C_6 = 1716 \quad (1 \text{ mark})$$

b. How many panels can be formed if there are at least 4 girls on the panel?

2 marks

$${}^5C_4 \times {}^8C_2 + {}^5C_5 \times {}^8C_1 = 148$$

(1 mark for the correct equation) ↓
1 mark

c. What is the probability that the oldest person being on the panel?

1 mark

$$Pr = \frac{{}^{12}C_5}{{}^{13}C_6}$$

$$= \frac{792}{1716} = \frac{6}{13} \quad (1 \text{ mark})$$