Supervision Instructions

Mathematics Methods (Unit 1-2)
Task #5
22 August 2023

Task consists of two papers: **Paper 1** and **Paper 2**. Students will have access to only one paper at a time.

Paper 1:

- 15 minutes
- Calculator is not allowed

After 15 minutes **Paper 1** is to be collected and **Paper 2** will be given.

Paper 2:

- 25 minutes
- Calculator is allowed

After 25 minutes **Paper 2** is to be collected.

Check that students put their names.

2023 Mathematical Methods (Unit 1-2)
Task 5

Paper 1 – Calculator not allowed

Number of marks: 10
Writing time: 15 minutes

Name: Marks:

Instructions

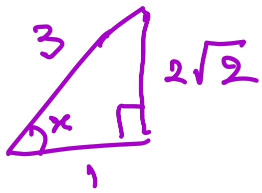
Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

2 marks

Given that $\cos(x)=\frac{1}{3}$ for $x\in[0,\frac{\pi}{2}]$, evaluate

a. $\sin(x) = \frac{2\sqrt{2}}{3}$



b. $\cos(x+\frac{\pi}{2}) = -\sin(x)$

$= -\frac{2\sqrt{2}}{3}$

Question 2

2 marks

Show that $\frac{\cos\theta}{1-\sin\theta}=\frac{1+\sin\theta}{\cos\theta}$

1 mark

$$\frac{\cos\theta}{1-\sin\theta}\times\frac{1+\sin\theta}{1+\sin\theta} = \frac{\cos\theta(1+\sin\theta)}{1-\sin^2\theta}$$

1 mark

$$= \frac{\cos\theta(1+\sin\theta)}{\cos^2\theta}$$

$$= \frac{1+\sin\theta}{\cos\theta}$$

OR any equivalent method

Question 3 1+2 marks

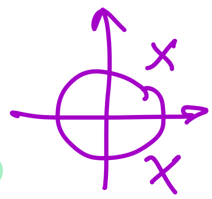
Let $h: R \to R,\ h(x) = 3\cos(4x)$

a. State the range of h.

$[-3, 3]$

b. Solve $3\cos(4x) = \frac{3}{2}$ for $x \in [-\frac{\pi}{2}, \pi]$.

$-\frac{\pi}{2} \le x \le \pi$



1 mark $\cos(4x) = \frac{1}{2}$, $-2\pi \le 4x \le 4\pi$

$4x = -2\pi + \frac{\pi}{3},\ -\frac{\pi}{3},\ \frac{\pi}{3},\ 2\pi - \frac{\pi}{3},\ 2\pi + \frac{\pi}{3},\ 4\pi - \frac{\pi}{3}$

$x = \frac{-5\pi}{12},\ -\frac{\pi}{12},\ \frac{\pi}{12},\ \frac{5\pi}{12},\ \frac{7\pi}{12},\ \frac{11\pi}{12}$

1 mark

Question 4

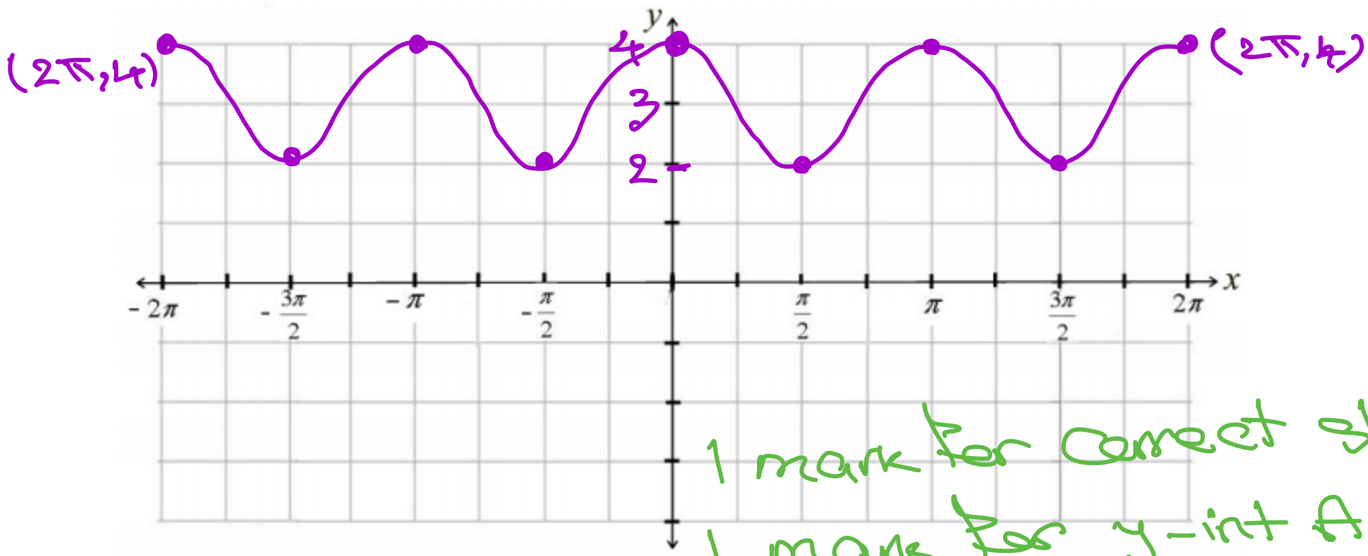
The graph of $y = \cos(x)$ undergoes the following transformations:

- a dilation of factor $\frac{1}{2}$ from the y-axis
- a translation of 3 units up

a. Write down the eqation of the image. 1 mark

$y = \cos(2x) + 3$

b. Sketch the transformed graph over $[-2\pi, 2\pi]$ on the axes below. Label all intercepts and endpoints. 2 marks



1 mark for correct shape

1 mark for y-int AND Endpoints

y-int: $(0, 4)$

endpoints: $(-2\pi, 4)$ & $(2\pi, 4)$

2023 *Math*ematical Methods (Unit 1-2)
Task 5
Paper 2 – Calculator allowed

Number of marks: 15
Writing time: 25 minutes

Name: Marks – Section 1: Section 2:

SECTION 1

Instructions for Section 1
Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Question 1

Given that $\sin(x) = \frac{4}{5}$ and $\cos(y) = -\frac{15}{17}$ where $x \in [\frac{\pi}{2}, \pi]$ and $y \in [\frac{\pi}{2}, \pi]$, then the expression $\sin(y) - \cos(x)$ is equal to

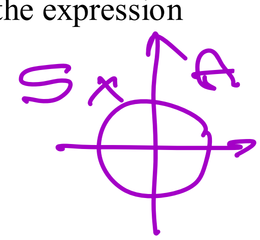
A. $\frac{-91}{85}$

B. $\frac{-11}{85}$

C. $\frac{11}{85}$

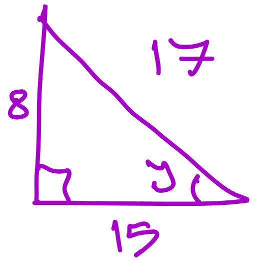
D. $\frac{91}{85}$

E. $\frac{126}{85}$



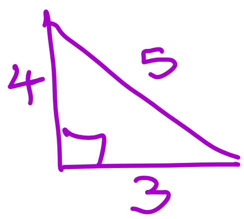
$\cos(y) = \frac{-15}{17}$

$\Rightarrow \sin(y) = \frac{8}{17}$



$\sin(x) = \frac{4}{5}$

$\Rightarrow \cos(x) = \frac{-3}{5}$



$\sin(y) - \cos(x)$

$= \frac{8}{17} - -\frac{3}{5} = \frac{91}{85}$

Question 2

Let $f : R \to R,\ f(x) = 2\sin(\frac{\pi x}{3}) - 1$.

The period and range of this function are given respectively by

A. 6 and $[-3,1]$

B. 6 and $[-2,2]$

C. 6π and $[-3,1]$

D. 6π and $[-2,2]$

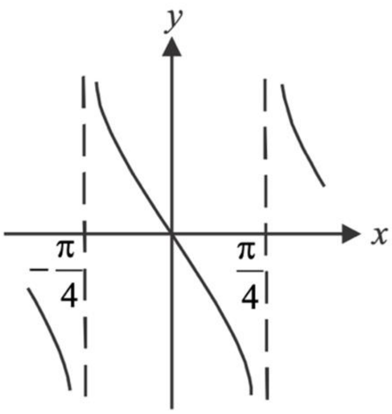
E. $\frac{2}{3}$ and $[-2,1]$

$P = \frac{2\pi}{\frac{\pi}{3}} = 6$

$[-1-2, -1+2]$
$= [-3, 1]$

Question 3

Part of the graph of f is shown below.



The rule could be

A. $f(x) = -\tan(\frac{x}{2})$

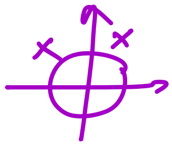
B. $f(x) = \tan(\frac{x}{2})$

C. $f(x) = -\tan(x)$

D. $f(x) = \tan(2x)$

E. $f(x) = -\tan(2x)$

Question 4



The sum of the solutions of the equation $\sin(2x) = \frac{1}{2}$ for $-\pi \le x \le \pi$ is

$\sin(2x) = \frac{1}{2}, \; -2\pi \le 2x \le 2\pi$

A. $-\frac{\pi}{12}$

B. $-\pi$

C. $\frac{\pi}{12}$

D. π

E. $\frac{3\pi}{2}$

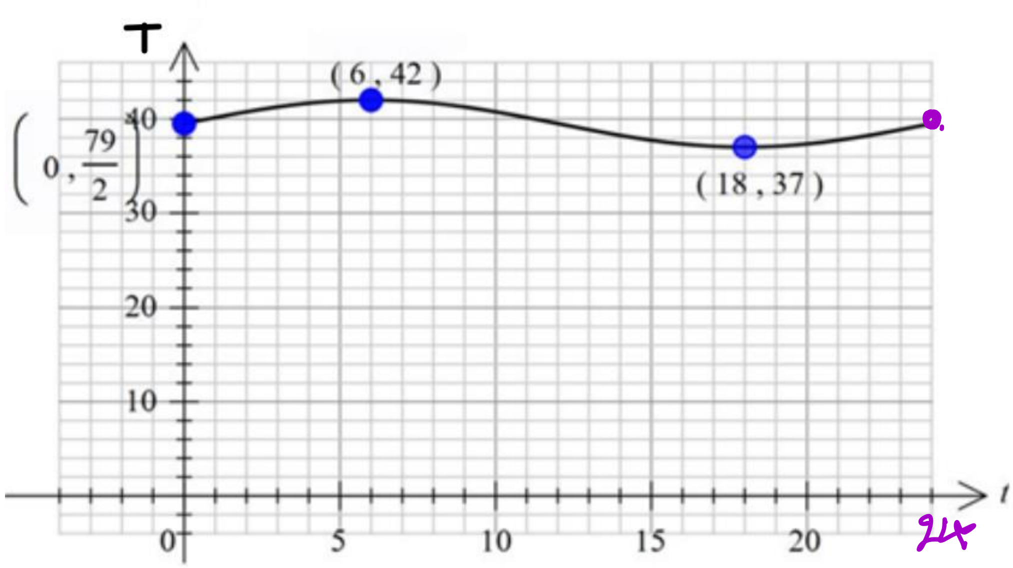
$2x = -2\pi + \frac{\pi}{6}, -\pi - \frac{\pi}{6}, \frac{\pi}{6}, \pi - \frac{\pi}{6}$

$x = \frac{-11\pi}{12}, \frac{-7\pi}{12}, \frac{\pi}{12}, \frac{5\pi}{2}$

$\frac{-11\pi}{12} + \frac{-7\pi}{12} + \frac{\pi}{12} + \frac{5\pi}{2} = -\pi$

Question 5

The temperature, $T^{\circ}C$ of a sick child in a hospital in Melbourne is illustrated in the graph below, where t is the number of hours after 8 am.



The graph is most likely to have the equation

A. $y = 38.5\sin(12t)$

B. $y = 2.5\sin(\frac{\pi t}{12}) + 42$

C. $y = 2.5\sin(\pi t) + 39.5$

D. $y = -2.5\sin(\frac{\pi t}{12}) + 39.5$

E. $y = 2.5\sin(\frac{\pi t}{12}) + 39.5$

$P = \frac{2\pi}{n} = 24$

$n = \frac{2\pi}{24} = \frac{\pi}{12}$

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

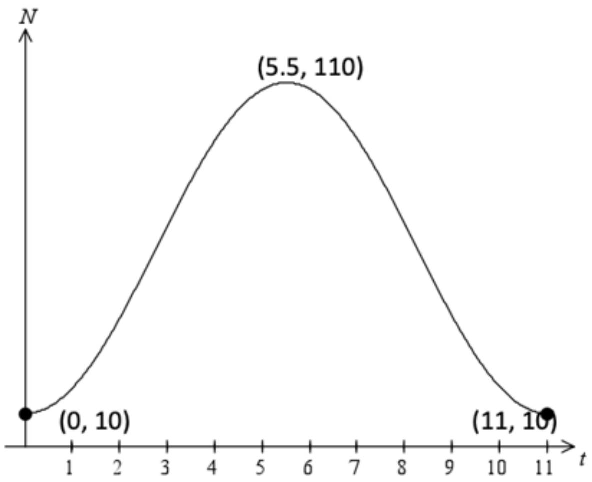
Question 1

Solar flares or "sunspots" are caused by the Sun's magnetic field. The average number of sunspots in any given year follows a periodic cycle, called a solar cycle.

Using historical data, Bryan, a solar astronomer, modelled the number of sunspots during a solar cycle with the function

$N:[0,11]\rightarrow R, N(t)=b-a\cos(nt)$

Where N is the number of sunspots t years after the start of a solar cycle and a, b and n are **positive** real constants. The graph of the function is shown.



According to the model:

a. What is the range of N? 1 mark

$[10, 110]$

Assume that a new solar cycle began 1 January 1982. $t=0$

b. How many complete solar cycles have occurred between 1 January 1982 and 1 January 2037? 1 mark

$$\frac{2037-1982}{11}=\frac{55}{11}=5$$

c. Show that $n=\frac{2\pi}{11}$. 1 mark

$$P=\frac{2\pi}{n}=11 \Rightarrow n=\frac{2\pi}{11}$$

d. Explain why $a=50$ and $b=60$. 1 mark for correct explanation of each. 2 mark

The amplitude is 50, therefore $a=50$.
The Cos graph has been translated 60 units up OR the average value for a complete cycle is 60, therefore $b=60$

OR any equivalent method

e. What is the predicted number of sunspots on 1 January 1992, correct to the nearest integer? 2 marks

1 mark for substituting $t=10$ into $N(t)$

$$N(t)=60-50\sin\left(\frac{2\pi t}{11}\right)$$

1 mark

$$N(10)=60-50\sin\left(\frac{20\pi}{11}\right)\approx 18 \text{ sunspots}$$

f. The level of UV radiation increases with the number of sunspots. Bryan proposes to monitor UV radiation levels during the period when $N \geq 80$. For what length of time is $N \geq 80$? Round your answer to the nearest month. 3 marks

1 mark $\rightarrow N(t)=80$ $\quad 0\leq t\leq 11$

1 mark $\rightarrow$ $t=3.47$ & $t=7.53$

$(7.53-3.47)\times 12=48.7$

$\simeq 49$ months

1 mark