Supervision Instructions

Mathematics Methods (Unit 1-2)
Task #5
22 August 2023

Task consists of two papers: **Paper 1** and **Paper 2**. Students will have access to only one paper at a time.

Paper 1:

- 15 minutes
- Calculator is not allowed

After 15 minutes **Paper 1** is to be collected and **Paper 2** will be given.

Paper 2:

- 25 minutes
- Calculator is allowed

After 25 minutes **Paper 2** is to be collected.

Check that students put their names.

2023 Mathematical Methods (Unit 1-2)
Task 5
Paper 1 – Calculator not allowed

Number of marks: 10
Writing time: 15 minutes

Name: Marks:

Instructions
Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

2 marks

Given that $\cos(x) = \frac{1}{3}$ for $x \in [0, \frac{\pi}{2}]$, evaluate

a. $\sin(x)$

b. $\cos(x + \frac{\pi}{2})$

Question 2

2 marks

Show that $\frac{\cos\theta}{1-\sin\theta} = \frac{1+\sin\theta}{\cos\theta}$

Question 3

1+2 marks

Let $h: R \to R,\ h(x) = 3\cos(4x)$

a. State the range of h.

b. Solve $3\cos(4x) = \frac{3}{2}$ for $x \in [-\frac{\pi}{2}, \pi]$.

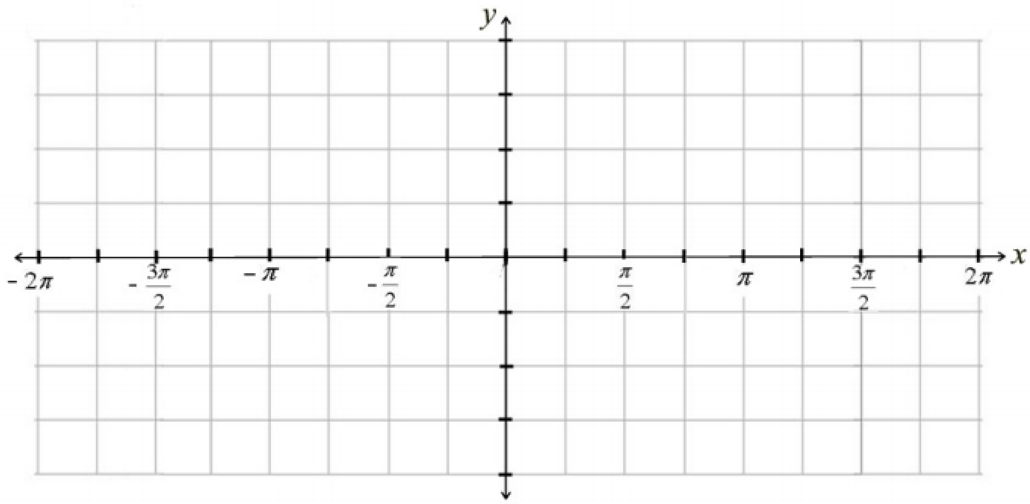
Question 4

The graph of $y = \cos(x)$ undergoes the following transformations:

- a dilation of factor $\frac{1}{2}$ from the y-axis
- a translation of 3 units up

a. Write down the eqation of the image. 1 mark

b. Sketch the transformed graph over $[-2\pi, 2\pi]$ on the axes below. Label all intercepts and endpoints. 2 marks



2023 $Math$ematical Methods (Unit 1-2)
Task 5

Paper 2 – Calculator allowed

Number of marks: 15
Writing time: 25 minutes

Name: Marks – Section 1: Section 2:

SECTION 1

Instructions for Section 1
Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Question 1

Given that $\sin(x) = \frac{4}{5}$ and $\cos(y) = -\frac{15}{17}$,where $x \in [\frac{\pi}{2}, \pi]$ and $y \in [\frac{\pi}{2}, \pi]$, then the expression $\sin(y) - \cos(x)$ is equal to

A. $\frac{-91}{85}$

B. $\frac{-11}{85}$

C. $\frac{11}{85}$

D. $\frac{91}{85}$

E. $\frac{126}{85}$

Question 2

Let $f: R \to R,\ f(x) = 2\sin(\frac{\pi x}{3}) - 1$.

The period and range of this function are given respectively by

A. 6 and $[-3,1]$

B. 6 and $[-2,2]$

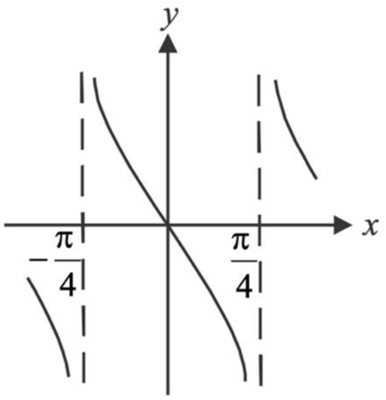
C. 6π and $[-3,1]$

D. 6π and $[-2,2]$

E. $\frac{2}{3}$ and $[-2,1]$

Question 3

Part of the graph of f is shown below.



The rule could be

A. $f(x) = -\tan(\frac{x}{2})$

B. $f(x) = \tan(\frac{x}{2})$

C. $f(x) = -\tan(x)$

D. $f(x) = \tan(2x)$

E. $f(x) = -\tan(2x)$

Question 4

The sum of the solutions of the equation $\sin(2x)=\frac{1}{2}$ for $-\pi \le x \le \pi$ is

A. $-\frac{\pi}{12}$

B. $-\pi$

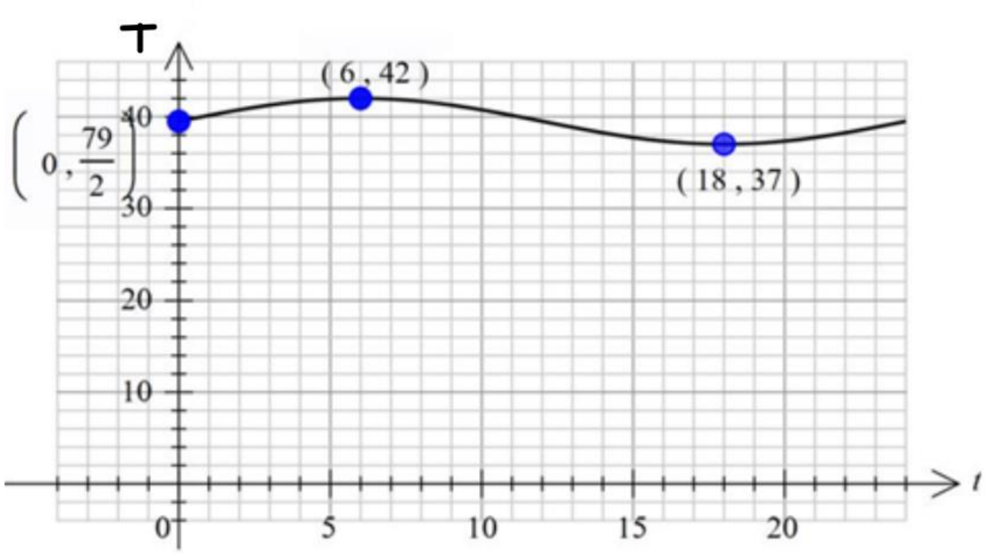
C. $\frac{\pi}{12}$

D. π

E. $\frac{3\pi}{2}$

Question 5

The temperature, $T^{\circ}C$ of a sick child in a hospital in Melbourne is illustrated in the graph below, where t is the number of hours after 8 am.



The graph is most likely to have the equation

A. $y=38.5\sin(12t)$

B. $y=2.5\sin(\frac{\pi t}{12})+42$

C. $y=2.5\sin(\pi t)+39.5$

D. $y=-2.5\sin(\frac{\pi t}{12})+39.5$

E. $y=2.5\sin(\frac{\pi t}{12})+39.5$

SECTION 2

Instructions for Section 2
Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
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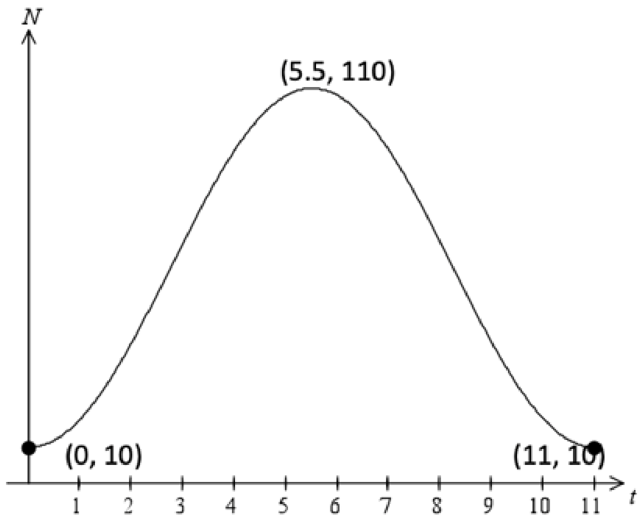
Question 1

Solar flares or "sunspots" are caused by the Sun's magnetic field. The average number of sunspots in any given year follows a periodic cycle, called a solar cycle.

Using historical data, Bryan, a solar astronomer, modelled the number of sunspots during a solar cycle with the function

$N:[0,11]\to R, N(t)=b-a\cos(nt)$

Where N is the number of sunspots t years after the start of a solar cycle and a, b and n are **positive** real constants. The graph of the function is shown.



According to the model:

a. What is the range of N? 1 mark

Assume that a new solar cycle began 1 January 1982.

b. How many complete solar cycles have occurred between 1 January 1982 and 1 January 2037? 1 mark

c. Show that $n=\frac{2\pi}{11}$. 1 mark

d. Explain why $a=50$ and $b=60$. 2 mark

e. What is the predicted number of sunspots on 1 January 1992, correct to the nearest integer? 2 marks

f. The level of UV radiation increases with the number of sunspots. Bryan proposes to monitor UV radiation levels during the period when $N \geq 80$. For what length of time is $N \geq 80$? Round your answer to the nearest **month**. 3 marks