Supervision Instructions

Mathematics Methods (Unit 1-2)
Task #6
5th of October 2023 – Period 4

Task consists of two papers: **Paper 1** and **Paper 2**. Students will have access to only one paper at a time.

Paper 1:

- 15 minutes
- Calculator is not allowed

After 15 minutes **Paper 1** is to be collected and **Paper 2** will be given.

Paper 2:

- 25 minutes
- Calculator is allowed

After 25 minutes **Paper 2** is to be collected.

Check that students put their names.

2023 Mathematical Methods (Unit 1-2)
Task 6
Paper 1 – Calculator not allowed

Number of marks: 10
Writing time: 15 minutes

Name: Marks:

Instructions

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

Solve the following for x.

a. $3^{2x} - 3^{x+2} + 18 = 0$ 2 marks

Let $3^x = a$

$a^2 - 9a + 18 = 0$

$a \quad -6$

$a \quad -3$

$(a-6)(a-3) = 0$

$a = 6$ or $a = 3$

$3^x = 6$ $\quad$ $3^x = 3$

$x = \log_3 6$ $\quad$ $x = 1$

b. $\log_e(2x) + \log_e(x+2) = \log_e(6)$ 2 marks

$\log_e(2x)(x+2) = \log_e 6$

$2x \cdot (x+2) = 6$

$2x^2 + 4x = 6$

$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$x = -3$ or $x = 1$

Question 2

Express $\dfrac{9^{1-2m} \times 3^{m-1}}{27^{-m+2}}$ as a power of 3. 2 marks

$= \dfrac{3^{2-4m} \times 3^{m-1}}{3^{-3m+6}} = \dfrac{3^{-3m+1}}{3^{-3m+6}} = 3^{-5} = \dfrac{1}{3^5}$

Question 3 2+1 marks

a. Sketch the graph of $y = 2 - 2\log_{10}(x-1)$ showing any asymptote(s) and axis intercept(s).

x int when $y = 0$

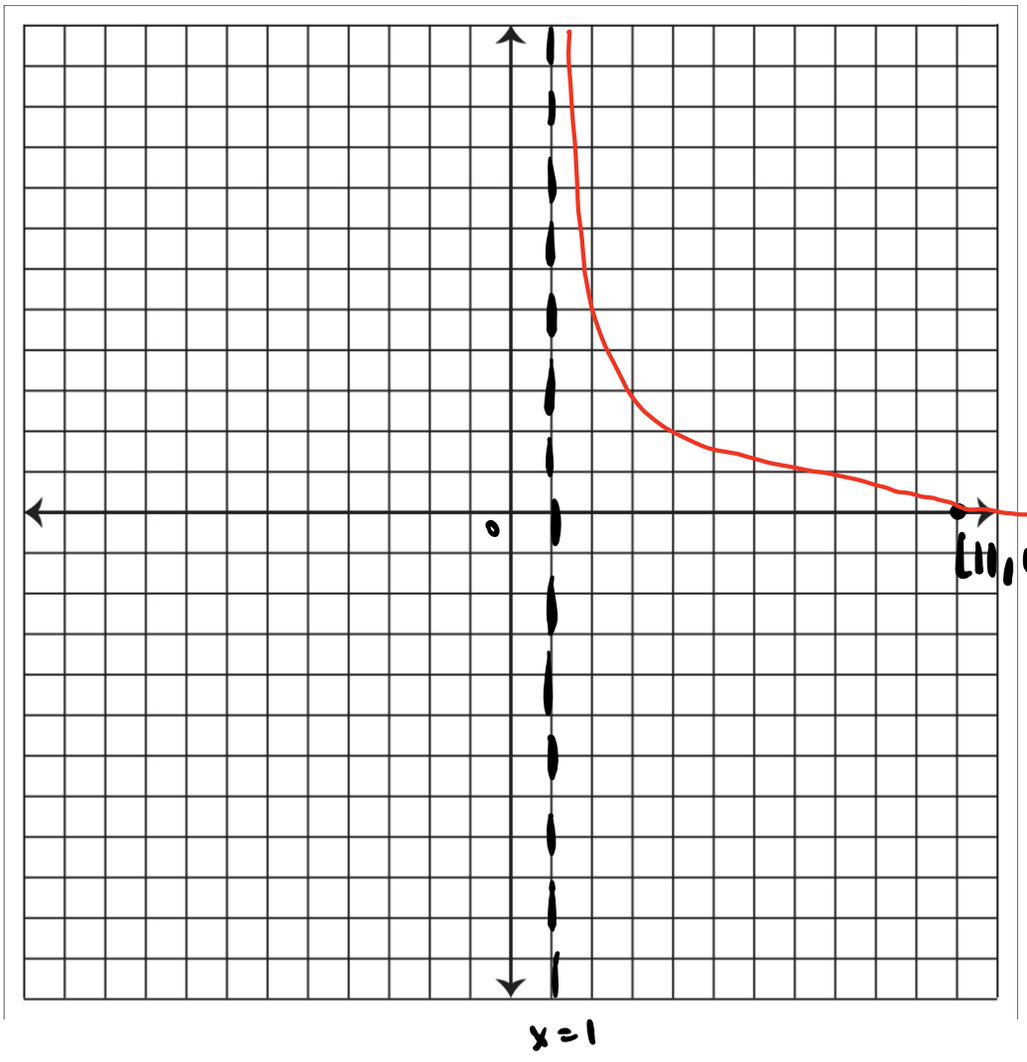
$2\log_{10}(x-1) = 2$

$\log_{10}(x-1) = 1$

$x - 1 = 10$

$x = 11$

$(11, 0)$



b. State the domain of $y = 2 - 2\log_{10}(x-1)$.

$(1, \infty)$

2023 Mathematical Methods (Unit 1-2)
Task 6
Paper 2 – Calculator allowed

Number of marks: 15
Writing time: 25 minutes

Name: Marks – Section 1: Section 2:

SECTION 1

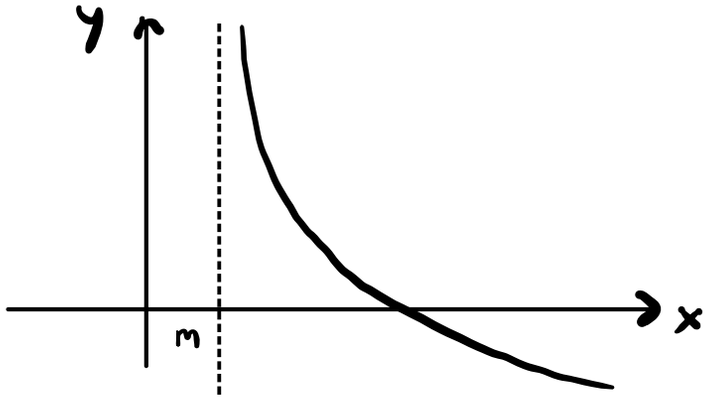
Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Question 1

The function has a rule $h(x) = r\log_e(x-m)+k$ where m and k are positive constants and r is a negative constant. What is the maximal domain for $h(x)$?

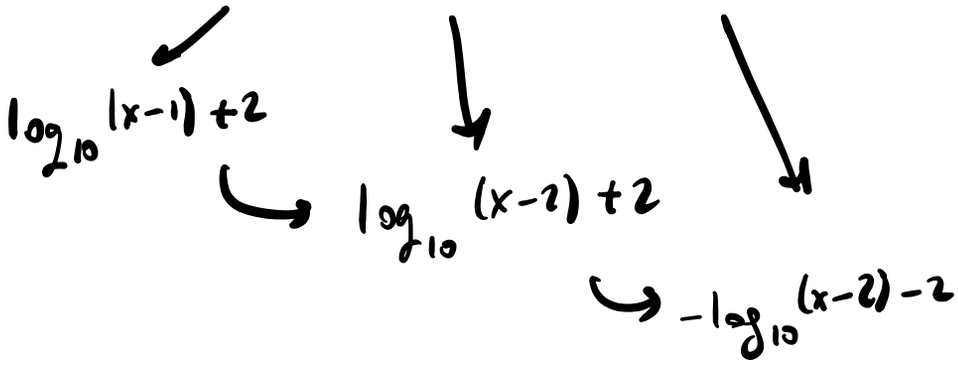
A R^+
B $R\setminus\{m\}$
C $R\setminus\{n\}$
D (m,n)
E (m,∞)



Question 2

When the function $f(x) = \log_{10}(x-1)$ is translated 2 units up, 1 unit right and then reflected in the x axis, the function becomes;

A $y = -\log_{10}(x-2)+2$
B $y = \log_{10}(x-2)+2$
C $y = -\log_{10}(x-2)-2$
D $y = \log_{10}(x-1)+2$
E $y = -\log_{10}(x-1)+2$



Question 3

If $7e^{ax} = 3$ then x equals

A $\frac{3}{7}\log_e a$

B $a\log_e\left(\frac{3}{7}\right)$

C $\frac{\log_e\left(\frac{3}{7}\right)}{a}$

D $\frac{\log_e(3)}{a\log_e(7)}$

E $\frac{a\log_e(3)}{\log_e(7)}$

$e^{ax} = \frac{3}{7}$

$ax = \log_e\left(\frac{3}{7}\right)$

$x = \frac{\log_e\left(\frac{3}{7}\right)}{a}$

Question 4

What is the inverse function of $f(x) = 4e^{2x-6} + 2$?

A $\log_e\frac{x-2}{4} + 6$

B $\log_e\frac{\sqrt{x-2}}{2} + 6$

C $\log_e\frac{\sqrt{x-2}}{2} + 3$

D $\frac{1}{2}\log_e\frac{\sqrt{x+2}}{2} - 3$

E $2\log_e\frac{x+2}{4} + 6$

$x = 4e^{2y-6} + 2$

$x - 2 = 4e^{2y-6}$

$\frac{x-2}{4} = e^{2y-6}$

$2y - 6 = \log_e\left(\frac{x-2}{4}\right)$

$2y = \log_e\left(\frac{x-2}{4}\right) + 6$

$y = \frac{1}{2}\log_e\left(\frac{x-2}{4}\right) + 3$

$y = \log_e\left(\frac{\sqrt{x-2}}{2}\right) + 3$

Question 5

What is $e^{(3\log_e x - \log_e(3x))}$ equal to?

A $3\log_e\left(\frac{1}{3}\right)$

B 0

C 1

D $x^3 - 3x$

E $\frac{x^2}{3}$

$e^{\log_e x^3 - \log_e(3x)}$

$e^{\log_e \frac{x^3}{3x}} = \frac{x^3}{3x} = \frac{x^2}{3}$

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

In her chemistry class, Sarah is preparing a special solution for an experiment that she has to complete. The concentration of the solution can be modelled by the rule.

$$C = A\log_e(kt)$$

Where C is the concentration in moles per litre (M) and t represents the time of the mixing in seconds. The concentration of the solution after 30 seconds as of mixing is 4 M, and the concentration of the solution after 2 seconds of mixing was 0.1 M.

a. Find the values of A and k, correct to 3 decimal places. 2 marks

$\text{Solve}\left(\begin{matrix} 4 = A.\log_e 30k \\ 0.1 = A.\log_e 2k \end{matrix}, A, k\right)$

$A = 1.440$

$k = 0.536$

b. Find the concentration of the solution after 15 seconds of mixing, correct to the nearest whole number. 1 mark

$C = 1.440\log_e 0.536 \times 15 = 3\ M$

c. How long does it take for the concentration of the solution to reach 10 M, correct to five significant figures? 1 mark

$\text{Solve}\left(10 = 1.44\log_e 0.536 \times t, t\right)$

$t = 1935.4$ seconds

Question 2

The population of the species of wallaby found on a reserve is increasing according to the equation $W = 150 \times 1.08^t$, W is the number of wallabies t years after records were first kept.

a. **i.** Find the initial size of the population. 1+1 marks

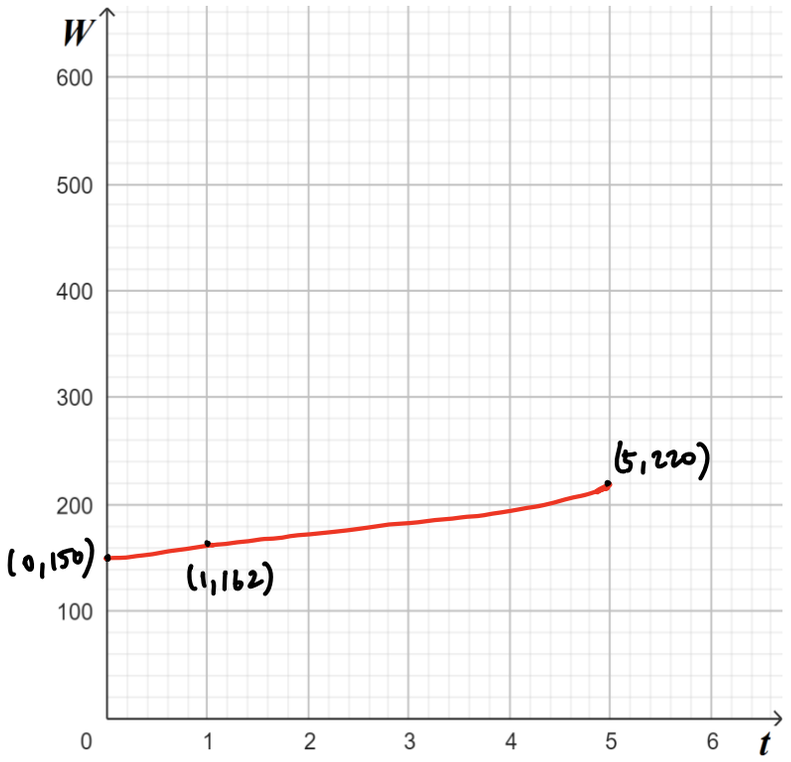
when $t = 0$, $W = 150$

ii. Find the population, to the nearest whole number, 1 year and 5 years after records were first kept.

when $t = 1$, $W = 150 \times 1.08 = 162$

$t = 5$, $W = 150 \times 1.08 = 220$

b. Plot a graph of W versus t, showing the coordinates found in part (a). 2 marks



c. Find how long, correct to the nearest whole number, would it take for the population to double. 2 marks

Solve $(300 = 150 \times 1.08^t, t)$

$t = 9$ years