2023 Mathematical Methods (Unit 1-2)
Task 8
Paper 1 – Calculator not allowed

Sirius College
"Reach for the stars"

Number of marks: 10
Writing time: 15 minutes

Name: Solutions / Marking guide. Marks:

Instructions

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 2 marks

Let $y = \frac{x^5 + 2\sqrt{x}}{x^2}$. Find $\frac{dy}{dx}$.

$y = x^3 + 2x^{-3/2}$ 1m

$\frac{dy}{dx} = 3x^2 - 3x^{-5/2}$ 1m

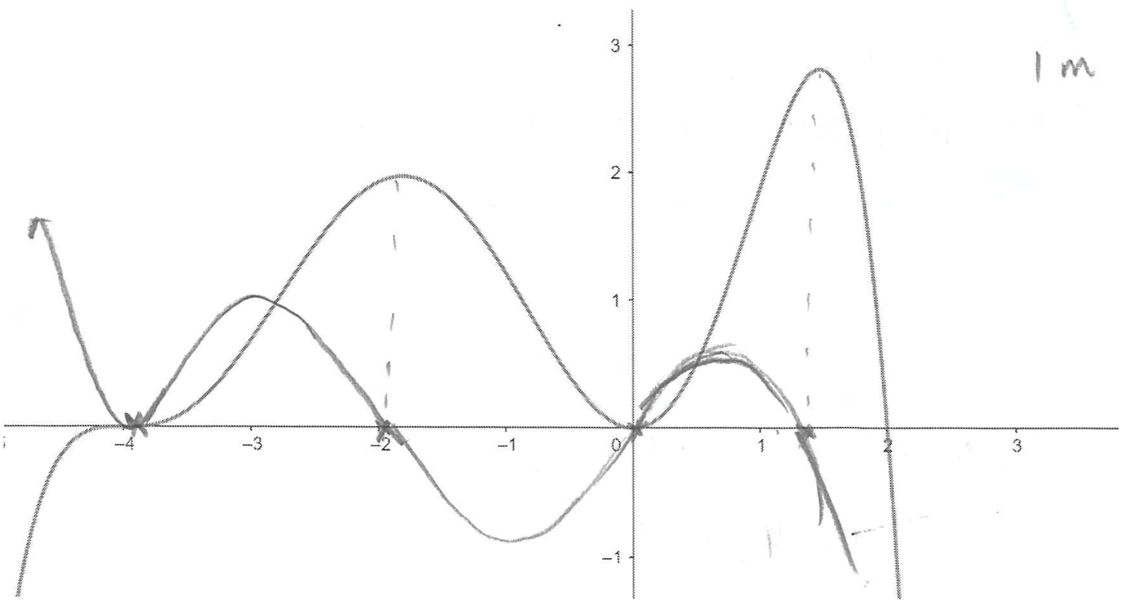
or

$3x^2 - \frac{3}{x^{5/2}}$

Question 2 2 marks

The graph of $f(x)$ is given. Sketch the graph of its derivative function on the same set of the axes shown below.

1m } 2 correct intercepts or right shape



Question 3

A particle moves in a straight line so that its displacement, x metres, from a fixed origin at time t seconds is given by $x=\frac{1}{3}t^3-2t^2+at-9$. The particle temporarily at rests at $t=1$.

a. Find a. 2 marks

$\frac{dx}{dt}=t^2-4t+a\Big|_{t=1}=0 \rightarrow 1m$

$1-4+a=0 \Rightarrow a=3 \longrightarrow 1m$

b. When is the second time the particle is at rest? 1 mark

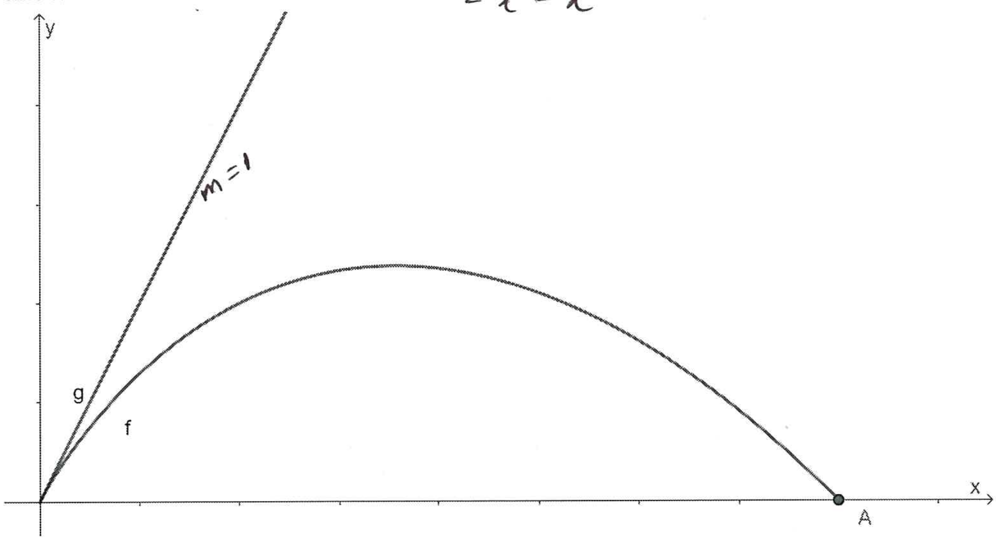
$t^2-4t+3=0 \quad (t-1)(t-3)=0$

$t=3$

Question 4

The graphs of $f:[0,4]\rightarrow R, f(x)=x(2-\sqrt{x})$ and part of the graph of $g:[0,\infty)\rightarrow R, g(x)=x$,are shown below.

$=2x-x^{3/2}$



a. A tangent is drawn to $f(x)$ at $x=a$, such that it is perpendicular to $g(x)$. Find a. **2 marks**

$f'(x)=$ $2-\frac{3}{2}x^{1/2}=-1$ — $1m$

$\frac{3}{2}x^{1/2}=3$

$x^{1/2}=2$

$x=4=a$ — $1m$

b. Hence find the equation of the tangent drawn to $f(x)$ at $x=a$. **1 mark**

$(4,0)\quad m=-1$

$y=-1(x-4)$

$y=-x+4$

2023 Mathematical Methods (Unit 1-2)
Task 8
Paper 2 – Calculator allowed

Number of marks: 15
Writing time: 25 minutes

Name: Marks – Section 1: Section 2:

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Question 1

The derivative of $f(x)=3x^2+5x$ can be found using central difference theorem by evaluating:

A. $\lim\limits_{h\to 0}\dfrac{3(x+h)^2-5xh-x^2-5x}{h}$

B. $\lim\limits_{h\to 0}\dfrac{3(x+h)^2-3(x-h)^2-x^2-3x}{2h}$

C. $\lim\limits_{h\to 0}\dfrac{3(x+h)^2+5(x+h)-3x^2-5x}{2h}$

D. $\lim\limits_{h\to 0}\dfrac{3(x+h)^2-5(x+h)-3(x-h)^2+5(x-h)}{2h}$

E. $\lim\limits_{h\to 0}\dfrac{3(x+h)^2+5(x+h)-3(x-h)^2-5(x-h)}{2h}$

Question 2

For the curve $y=x^2-5$, the tangent to the curve will be parallel to the line connecting the positive $x-$intercept and the $y-$intercept when x is equal to;

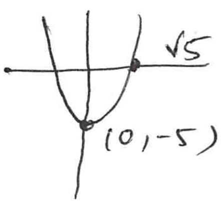
A. $\sqrt{5}$

B. $-\sqrt{5}$

C. $\dfrac{1}{\sqrt{5}}$

D. $\dfrac{-1}{\sqrt{5}}$

E. $\dfrac{\sqrt{5}}{2}$



$(0,-5)$ & $(\sqrt{5},0)$ $\quad m=\dfrac{5}{\sqrt5}$

$2x=\dfrac{5}{\sqrt5}$

$x=\dfrac{5}{2\sqrt5}=\dfrac{5\sqrt5}{\cancel{10}\,2}=\dfrac{\sqrt5}{2}$

Question 3

The average rate of change of the function with the rule $f(x) = x^2 - 2x$ over the interval $[1, a]$, where $a > 1$, is 8. The value of a is:

A. 9

B. 8

C. 7

D. 4

E. $1 + \sqrt{2}$

$$\frac{f(1) - f(a)}{1 - a} = 8$$

$$\frac{-1 - a^2 + 2a}{1 - a} = 8$$

$$-1 - a^2 + 2a = 8 - 8a$$

$$a^2 - 10a + 9 = 0$$

Question 4

Which one of the following functions is **not** continuous?

A. $f(x) = \begin{Bmatrix} x+1 & x<0 \\ x^2+1 & x \geq 0 \end{Bmatrix}$

B. $f(x) = \begin{Bmatrix} x & x<0 \\ x(x-1) & x \geq 0 \end{Bmatrix}$

C. $f(x) = \begin{Bmatrix} 2^x & x \leq 0 \\ 1-x & x>0 \end{Bmatrix}$

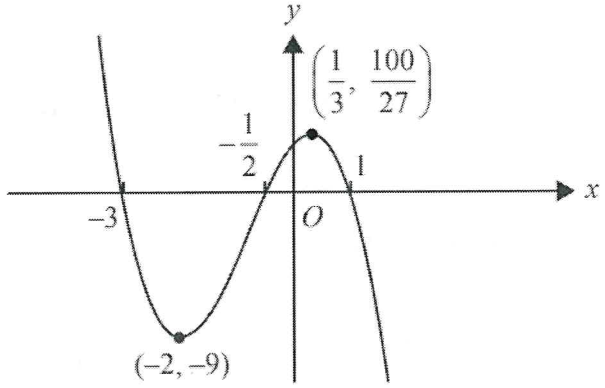
D. $f(x) = \begin{Bmatrix} x^2 & x<0 \\ x+1 & x \geq 0 \end{Bmatrix}$

E. $f(x) = \begin{Bmatrix} 0 & x \leq 0 \\ \sqrt{x} & x>0 \end{Bmatrix}$

Question 5

Part of the graph $y = f(x)$ is shown below;

$f'(x) < 0$ for



A. $x \in (-\infty, -2) \cup (\frac{1}{3}, \infty)$

B. $x \in (-\infty, -2] \cup [1, \infty)$

C. $x \in (-2, \frac{1}{3})$

D. $x \in (-9, \frac{100}{27})$

E. $x \in (-2, 0) \cup (\frac{1}{3}, \infty)$

Alisha wants the volume of the hats to be as large as possible.

c. Find the value of h that gives the maximum volume for the cone, and hence find the corresponding value of r. 3 marks

$$V'(h) = \frac{\pi}{3}(100 - 3h^2) = 0 \quad \Rightarrow \quad h = \sqrt{\frac{100}{3}} \quad \text{— 1m}$$

1m

$$\therefore r = \sqrt{100 - h^2} = \sqrt{100 - \frac{100}{3}} = \sqrt{\frac{200}{3}} = r$$

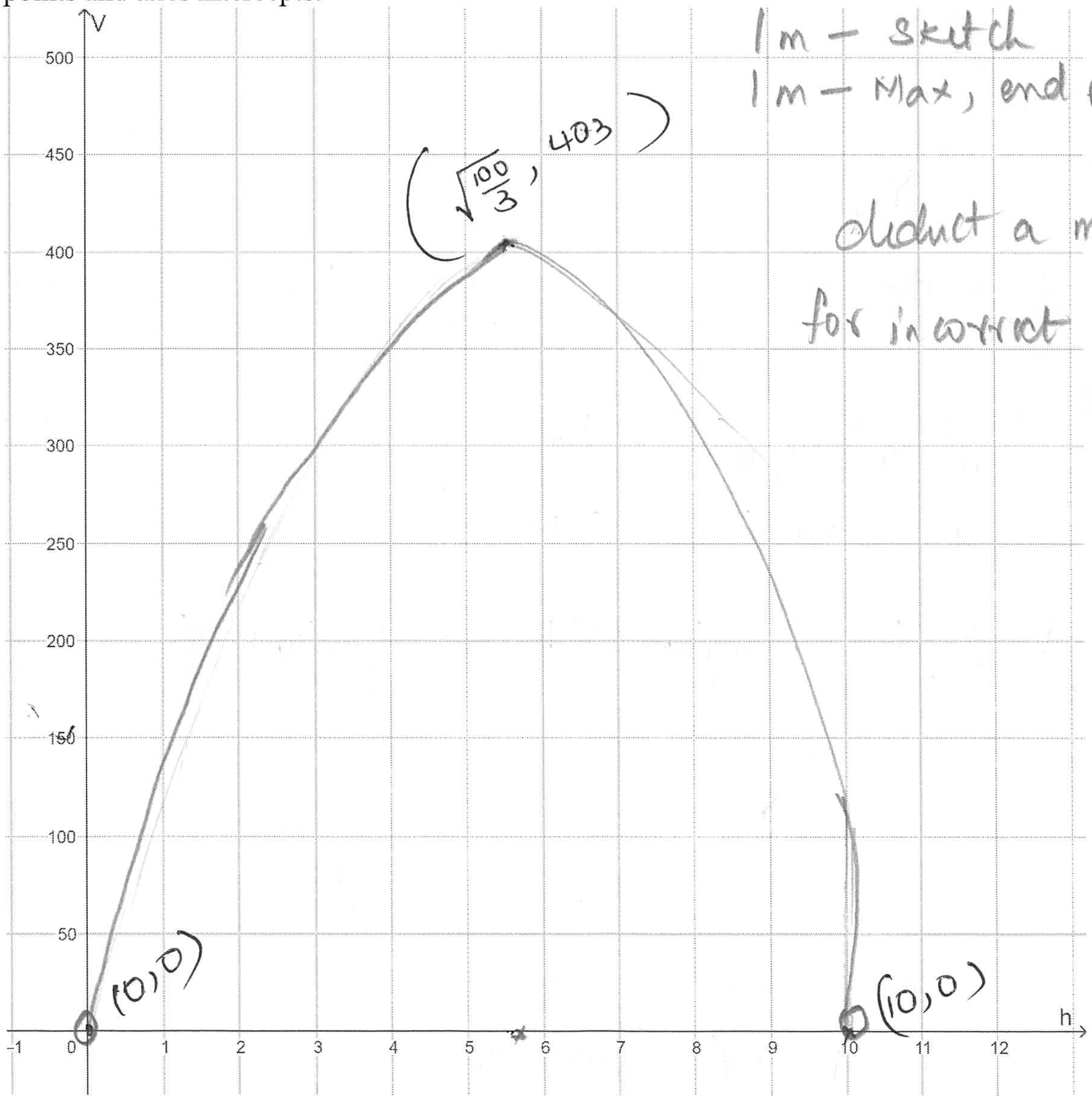
$r = 8.16$ → 1m.

d. Find the maximum volume of the cone (to the nearest cubic centimetre). 1 mark

$$V = \frac{\pi}{3}(100h - h^3)\Big|_{h=\sqrt{\frac{100}{3}}} = 403.066$$

$$\approx 403 \text{ cm}^3.$$

e. Hence sketch the graph of V against h over an appropriate domain. Label the turning points and axes intercepts. 2 marks



1m – sketch

1m – Max, end pts.

deduct a mark for incorrect end pts

SECTION 2

Instructions for Section 2

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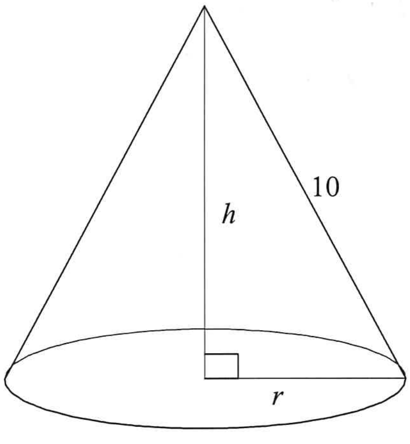
Question 1

Alisha is planning to make conical party hats. The conical hats are as shown in the diagram below, where h equals the height of the cone and r is equal to the radius of the base of the cone. All measurements are in centimetres.



a. Express r in terms of h and hence show that the volume of the cone in terms of its height h, is $V = \frac{1}{3}\pi(100h - h^3)$. (Volume of a cone is $\frac{\pi r^2 h}{3}$) 3 marks

$h^2 + r^2 = 100$ — 1m

$r = \sqrt{100 - h^2}$ — 2m

$V = \frac{\pi r^2 h}{3} = \frac{\pi(100-h^2)h}{3} = \frac{1}{3}\pi(100h - h^3) \rightarrow$ 3m

b. Find the volume of the cone if the height is 7.5 cm. 1 mark

$V = \frac{\pi(100 - 7.5^2)7.5}{3} = 343.61 \text{ cm}^3$