2023 Mathematical Methods (Unit 1-2)
Task 8
Paper 1 – Calculator not allowed

Number of marks: 10
Writing time: 15 minutes

Name: Marks:

Instructions
Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

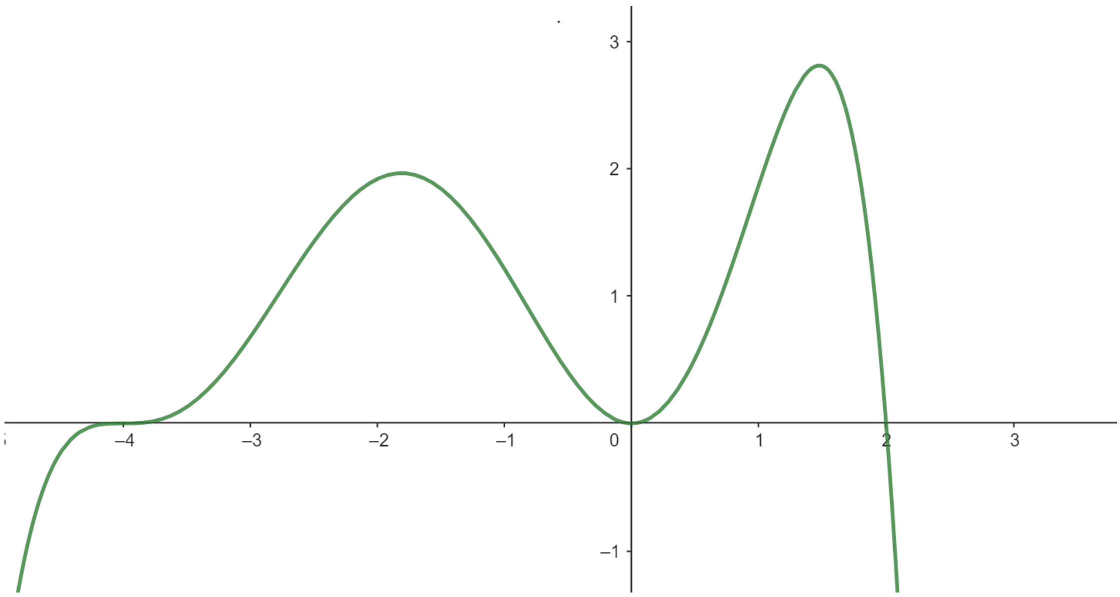
2 marks

Let $y = \dfrac{x^5 + 2\sqrt{x}}{x^2}$. Find $\dfrac{dy}{dx}$.

Question 2

2 marks

The graph of $f(x)$ is given. Sketch the graph of its derivative function on the same set of the axes shown below.



Question 3

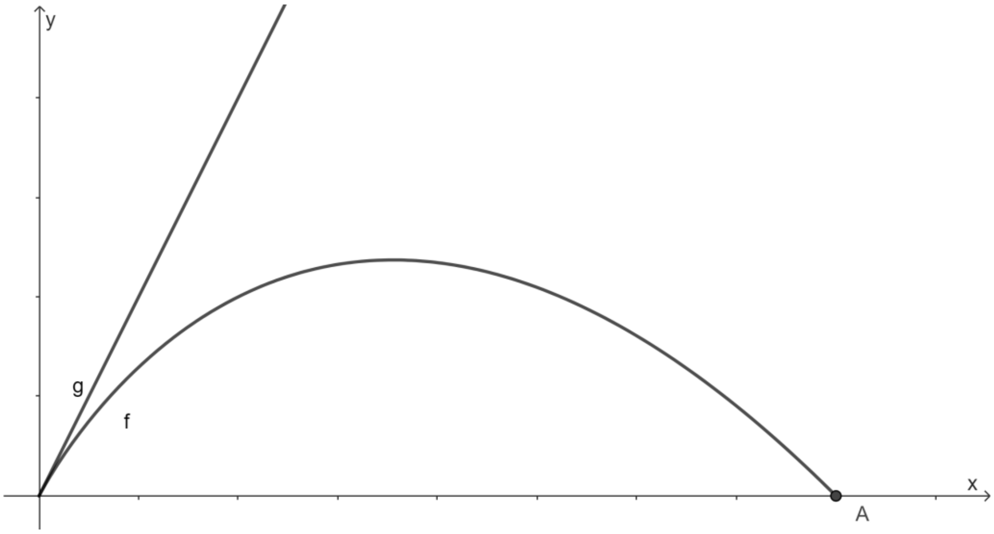
A particle moves in a straight line so that its displacement, x metres, from a fixed origin at time t seconds is given by $x = \frac{1}{3}t^3 - 2t^2 + at - 9$. The particle temporarily at rests at $t = 1$.

a. Find a. 2 marks

b. When is the second time the particle is at rest? 1 mark

Question 4

The graphs of $f : [0, 4] \to R,\ f(x) = x(2 - \sqrt{x})$ and part of the graph of $g : [0,\ \infty) \to R, g(x) = x$,are shown below.



a. A tangent is drawn to $f(x)$ at $x = a$, such that it is perpendicular to $g(x)$. Find a. 2 marks

b. Hence find the equation of the tangent drawn to $f(x)$ at $x = a$. 1 mark

2023 Mathematical Methods (Unit 1-2)
Task 8
Paper 2 – Calculator allowed

Number of marks: 15
Writing time: 25 minutes

Name: Marks – Section 1: Section 2:

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Question 1

The derivative of $f(x) = 3x^2 + 5x$ can be found using central difference theorem by evaluating:

A. $\lim\limits_{h \to 0} \dfrac{3(x+h)^2 - 5xh - x^2 - 5x}{h}$

B. $\lim\limits_{h \to 0} \dfrac{3(x+h)^2 - 3(x-h)^2 - x^2 - 3x}{2h}$

C. $\lim\limits_{h \to 0} \dfrac{3(x+h)^2 + 5(x+h) - 3x^2 - 5x}{2h}$

D. $\lim\limits_{h \to 0} \dfrac{3(x+h)^2 - 5(x+h) - 3(x-h)^2 + 5(x-h)}{2h}$

E. $\lim\limits_{h \to 0} \dfrac{3(x+h)^2 + 5(x+h) - 3(x-h)^2 - 5(x-h)}{2h}$

Question 2

The tangent to the curve $y = x^2 - 5$ is parallel to the line connecting the positive $x-$intercept and the $y-$intercept when x is equal to;

A. $\sqrt{5}$

B. $-\sqrt{5}$

C. $\dfrac{1}{\sqrt{5}}$

D. $\dfrac{-1}{\sqrt{5}}$

E. $\dfrac{\sqrt{5}}{2}$

Question 3

The average rate of change of the function with the rule $f(x)=x^2-2x$ over the interval $[1,a]$, where $a>1$, is 8. The value of a is:

A. 9

B. 8

C. 7

D. 4

E. $1+\sqrt{2}$

Question 4

Which one of the following functions is **not** continuous over R?

A. $f(x)=\begin{cases} x+1 & x<0 \\ x^2+1 & x\geq 0 \end{cases}$

B. $f(x)=\begin{cases} x & x<0 \\ x(x-1) & x\geq 0 \end{cases}$

C. $f(x)=\begin{cases} 2^x & x\leq 0 \\ 1-x & x>0 \end{cases}$

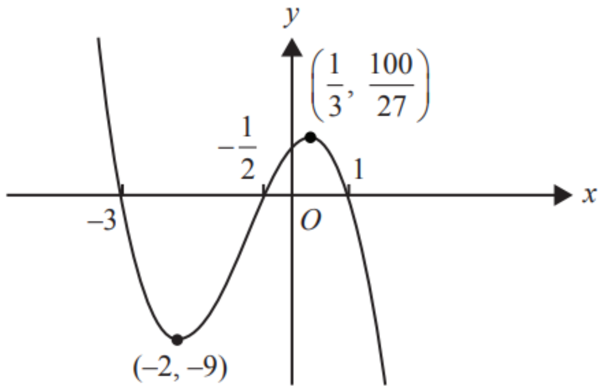
D. $f(x)=\begin{cases} x^2 & x<0 \\ x+1 & x\geq 0 \end{cases}$

E. $f(x)=\begin{cases} 0 & x\leq 0 \\ \sqrt{x} & x>0 \end{cases}$

Question 5

Part of the graph $y=f(x)$ is shown below;

$f'(x)<0$ for



A. $x\in(-\infty,-2)\cup(\frac{1}{3},\infty)$

B. $x\in(-\infty,-2]\cup[1,\infty)$

C. $x\in(-2,\frac{1}{3})$

D. $x\in(-9,\frac{100}{27})$

E. $x\in(-2,0)\cup(\frac{1}{3},\infty)$

SECTION 2

Instructions for Section 2
Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
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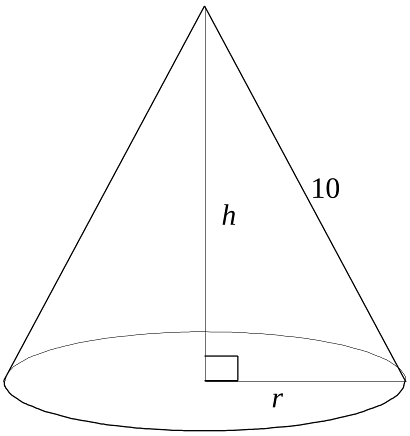
Question 1

Alisha is planning to make conical party hats. The conical hats are as shown in the diagram below, where *h* equals the height of the cone and *r* is equal to the radius of the base of the cone. All measurements are in centimetres.



a. Express *r* in terms of *h* and hence show that the volume of the cone in terms of its height *h*, is $V = \frac{1}{3}\pi(100h - h^3)$. (Volume of a cone is $\frac{\pi r^2 h}{3}$) 3 marks

b. Find the volume of the cone if the height is 7.5 cm, rounding off to 2dps. 1 mark

Alisha wants the volume of the hats to be as large as possible.

c. Find the value of h that gives the maximum volume for the cone, and hence find the corresponding value of r, rounding off to the nearest centimetre. 3 marks

d. Find the maximum volume of the cone (to the nearest cubic centimetre). 1 mark

e. Hence sketch the graph of V against h over an appropriate domain. Label the turning points and axes intercepts. 2 marks