Mathematical Methods SAC 1 (2016)

APPLICATION TASK

WENDY'S WEDDING Booklet 1 - May 23rd

10 minutes reading time

120 minutes writing time

This task is to be completed in one session of duration 130 minutes.
During this task, you may use your calculator and refer only to your Application SAC Preparation Booklet. No other pieces of paper may be used.
You must work silently and independently for the duration of this task.
All answers are to be written within this booklet.
When drawing graphs, ensure that a pencil is used and that significant features of the graph are clearly indicated in pen.
Exact values are expected throughout, unless otherwise stated and units must be included.
Your CAS calculator will be collected at the conclusion of the SAC so that all memories can be cleared, and will be returned to you in your first Mathematical Methods class.
No electronic devices (such as mobile phones) may be brought into the examination room.

Total : 53 marks

Student Name : _____

Teacher Name: _____

The grade awarded to this SAC is subject to statistical moderation

by the VCAA and is likely to change.

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, \ n \neq -1$
$\frac{d}{dx}\left((ax+b)^n\right) = an(ax+b)^{n-1}$	$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, n \neq -1$
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e(x) + c, \ x > 0$
$\frac{d}{dx}\left(\sin(ax)\right) = a\ \cos(ax)$	$\int \sin(ax) dx = -\frac{1}{a}\cos(ax) + c$
$\frac{d}{dx}\left(\cos(ax)\right) = -a\ \sin(ax)$	$\int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$
$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\ \sec^2(ax)$	

product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$		

Wendy's Wedding

Question 1

Wendy and Peter decided that they wanted to get married. To highlight the importance of this event in their lives, they decided to hold the wedding ceremony at the local cathedral. On the day of the wedding, after exhausting encounters with a beautician and a hairdresser, Wendy was running late. Upon finally disembarking from the wedding car near the cathedral, Wendy was located at point A (see Diagram 1) and wanted to rendezvous with her father standing at the main cathedral door located at point B and anxiously waiting to give her away. In spite of the fact that the gardens around the cathedral were considerable and she wanted to meet up with her father in the least possible time so that the ceremony could start as promptly as possible. Wendy's speed on the smooth concrete path, v_c, was 2 ms^{-1} whilst her speed on the grass, v_g, was 1 ms^{-1}. Leaving A, Wendy headed for point X on BC which was x m away from B.

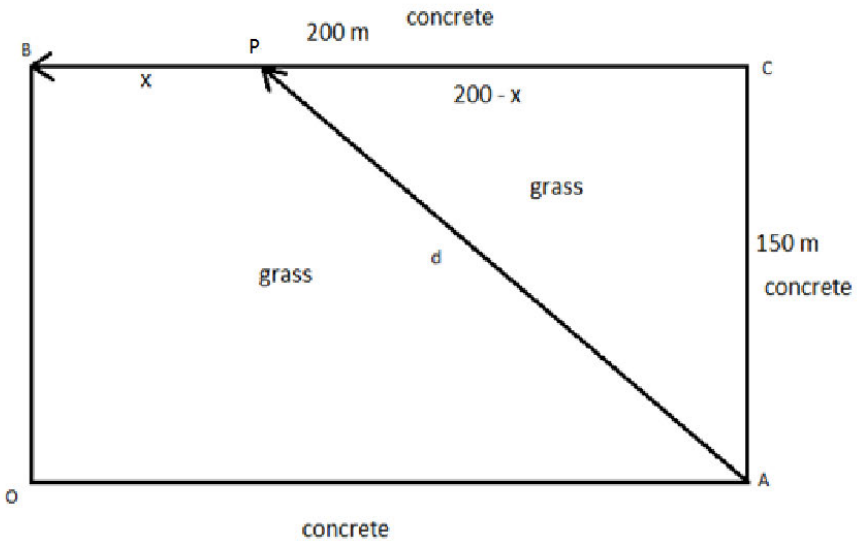

Diagram 1 (AP = d, PB = x, PC = 200 – x)

Using the measurements given in Diagram 1, determine the following:

(a) The distance, d m, travelled by Wendy **on the grass** in term of x.

(2)

(b) The time taken to travel from A to P in seconds in terms of x.
[hint: speed=distance/time]

(2)

(c) Show that the total time, $T(x)$, taken by Wendy to rendezvous with her father in terms of x is given by:

$$T(x) = \sqrt{x^2 - 400x + 62500} + \frac{x}{2} \text{ s}$$

(1)

(d) Using interval notation, state the domain of $T(x)$.

(1)

(e)

i. **Using CAS**, find $\frac{dT}{dx}$.

(1)

ii. Determine the value of x to two decimal places which corresponds to $T(x)$ being minimized.

(1)

(f) Determine the minimum value of $T(x)$ in seconds to two decimal places.

(1)

(g) Verify that this value of x results in $T(x)$ being minimized.

(1)

(h) How long would it have taken Wendy had she followed the path ACB, whilst travelling on **concrete only**?

(1)

(i) How long would it have taken Wendy had she gone from A to B directly, **travelling on grass only**?

(2)

(j) How long would it have taken Wendy if she had travelled to a point midway between B and C, to two decimal places?

(2)

(k)

i. Sketch the graph of $y = T(x)$ and give the endpoints and the turning point in co-ordinate form, to two decimal places where necessary.

(5)

ii. State the range of $T(x)$.

(1)

(l) Using interval notation:

i. Over what set of values does the inverse function $T^{-1}(x)$ exist?

(1)

ii. Over what set of values is T(x) continuous?

(1)

iii. Over what set of values is T(x) differentiable?

(1)

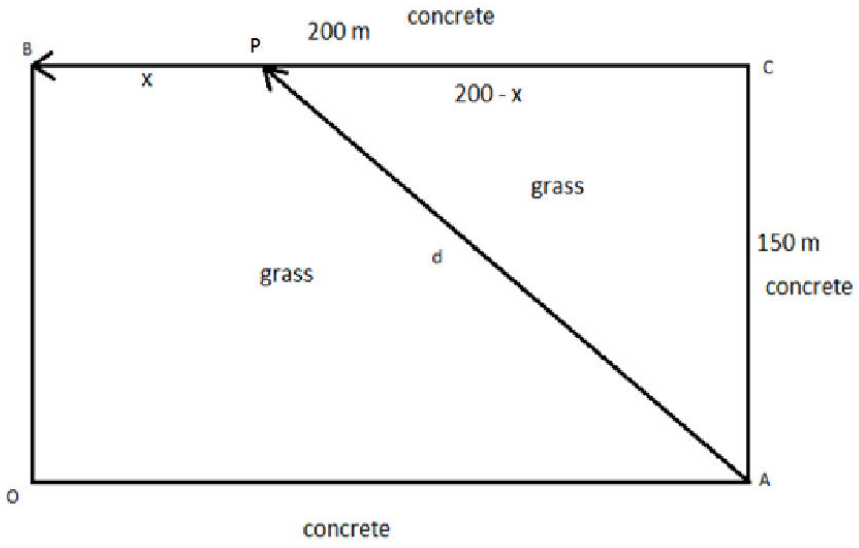

Diagram 1 (AP = d, PB = x, PC = 200 – x)

(m) In Diagram 1, O = O(0,0), A = A(200,0), B = B(0,150) and C = C(200,150). How far along the route AP, to two decimal places, did Wendy travel before she was closest to the point C, given that she was following the pervious path of minimum time?

(6)

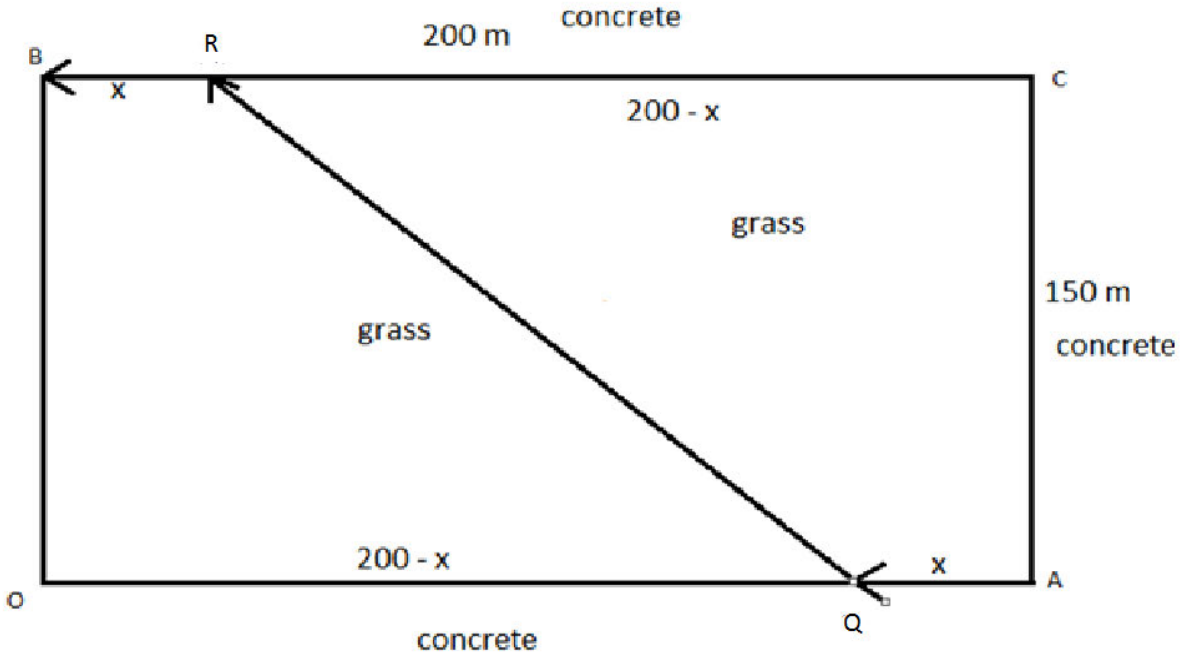

Diagram 2 (AQ = x, RB = x, RC = 200 – x)

(n) If Wendy had decided to travel along the path AQRB as indicated in Diagram 2:

i. What would be the new time function $T(x)$ for this case?

(2)

ii. What would be the domain of this function?

(1)

iii. What would be the exact value of x for which this time function is minimized in fully factorized form?

(1)

iv. What would be the exact minimum value of $T(x)$ in fully factorized form?

(1)

Question 2

After meeting her father at the cathedral door, Wendy slowly walked with him along the main aisle heading towards the altar to the sound of Mendelssohn's wedding march. Just as the organist was reaching a crescendo, Wendy noticed immediately to her right a magnificent leadlight stained glass window which had the upper perimeter given by the following equation:

$$f(x) = ax^2 + bx + c$$

This window is shown in the following diagram. All coordinates are in metres.

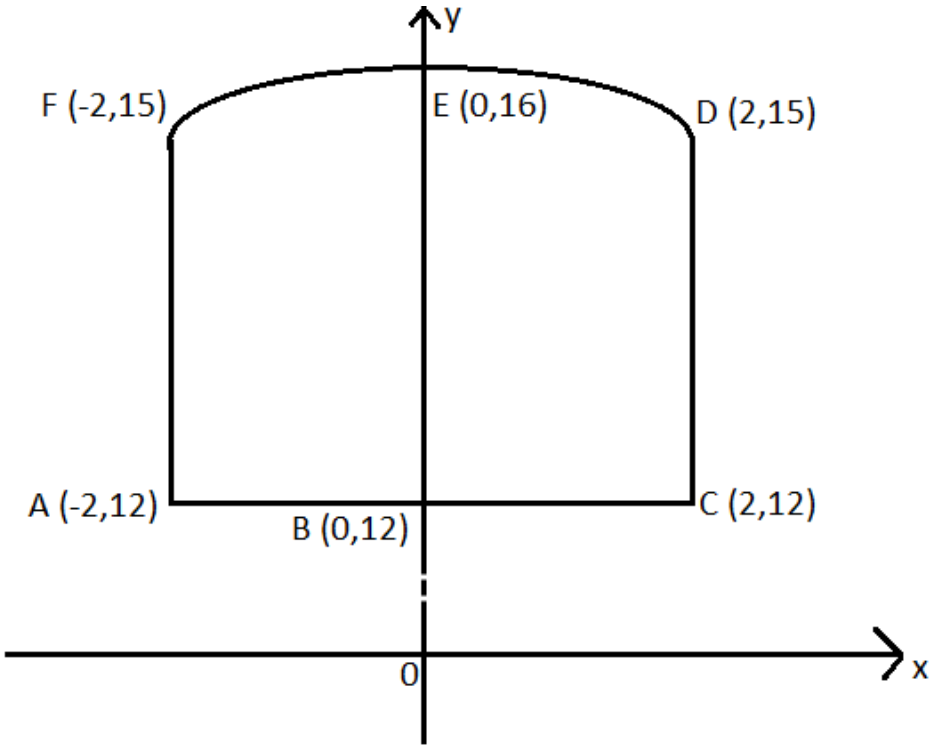

Diagram 3

(a) Using Diagram 3, determine the values of *a*, *b* and *c* and hence express *f(x)* with numerical coefficients.

(3)

(b) What is the domain and range of *f*(*x*).

(2)

(c) What is the gradient of $f(x)$ at $x = 0, 1, 2$?

(3)

(d) What do you observe about the gradient as you move from E to D?

(2)

(e) Using CAS, calculate the area of this window.

(1)

(f) Given that the distance along the parabola FED (in Diagram 3) is given by the arc length *s* where *s* is obtained using the following integral:

$$s = \int_{-2}^{2} \sqrt{1 + [f'(x)]^2}\, dx$$

and where $f(x) = 16 - \frac{1}{4}x^2$ and $f'(x) = \frac{d}{dx} f(x)$, calculate the **full** perimeter of this window, ***using a calculator*** and giving your answer correct to two decimal places.

(4)

(g) Given that the thickness of this glass is 6.5 mm and that its density is 2600 kgm^{-3}, calculate the total mass of this glass window in kilograms, to two decimal places.
[hint: Density=Mass/Volume]

(1)

(h) Given that the cost of leadlight stained glass windows is \$855.60 per m^2 installed, what would be the cost for this particular window?

(1)

(i) Based on your answers to parts (g) and (h), what is the cost per kilogram of this window?

(1)

End of SAC Pt 1

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