APPLICATION TASK

Mathematical Methods SAC 1 (2016)

APPLICATION TASK

WENDY'S WEDDING Booklet 2 – May 25th

10 minutes reading time

120 minutes writing time

This task is to be completed in one session of duration 130 minutes.
During this task, you may use your calculator and refer only to your Application SAC Preparation Booklet. No other pieces of paper may be used.
You must work silently and independently for the duration of this task.
All answers are to be written within this booklet.
When drawing graphs, ensure that a pencil is used and that significant features of the graph are clearly indicated in pen.
Exact values are expected throughout, unless otherwise stated.
Your CAS calculator will be collected at the conclusion of the SAC so that all memories can be cleared, and will be returned to you in your first Mathematical Methods class.
No electronic devices (such as mobile phones) may be brought into the examination room.

Total : 40 Marks

Student Name : _____

Teacher Name: _____

The grade awarded to this SAC is subject to statistical moderation

by the VCAA and is likely to change.

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$		$\int x^n dx=\frac{1}{n+1}x^{n+1}+c,\ n\neq -1$	
$\frac{d}{dx}\left((ax+b)^n\right)=an(ax+b)^{n-1}$		$\int (ax+b)^n dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c, n\neq -1$	
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$		$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$	
$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$		$\int \frac{1}{x}dx=\log_e(x)+c,\ x>0$	
$\frac{d}{dx}\left(\sin(ax)\right)=a\ \cos(ax)$		$\int \sin(ax)dx=-\frac{1}{a}\cos(ax)+c$	
$\frac{d}{dx}\left(\cos(ax)\right)=-a\ \sin(ax)$		$\int \cos(ax)dx=\frac{1}{a}\sin(ax)+c$	
$\frac{d}{dx}\left(\tan(ax)\right)=\frac{a}{\cos^2(ax)}=a\ \sec^2(ax)$			
product rule	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$		

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Question 3

After advancing further down the aisle, Wendy looked upwards observing that the shape of the right hand side of the main arches supporting the roof structure above the aisle was given by the following equation:

$$g(x) = ax^3 + bx^2 + cx + d, x \geq 0$$

where x is the horizontal distance from the centre of the arch.

(a) Given that $g(0) = \frac{33}{2}, g(2) = 14, g(4) = \frac{23}{2}$,g$(7) = \frac{11}{2}$, determine the values of *a, b, c, d* in rational form. Then express $g(x)$ as a cubic polynomial with known rational coefficients.

(5)

(b) Give an expression for the reflected image of $g(x)$ in the y-axis.

(2)

(c) Find the coordinates of the Y-intercept of g(x) .

(1)

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(d) Draw the graph of:

$$y = \begin{cases} g(-x), x < 0 \\ g(x), x \geq 0 \end{cases}$$

Expressing the x and y-intercepts in co-ordinate form.

(5)

(e) Discuss the differentiability of:

$$y = \begin{cases} g(-x), x < 0 \\ g(x), x \geq 0 \end{cases}$$

(2)

(f) What is the gradient of $g(x)$ when $(x,y) = (8.6380,0)$ to four decimal places?

(1)

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Question 4

The music abruptly stopped. Although all of Wendy's thoughts were focused on Peter, her husband-to-be, who was now standing beside her, she could not help noticing that the arch immediately above the main altar was not Gothic in character. This arch had the shape of an inverted catenary having an equation given by:

$$h(x) = 4\left[3 - \cosh\left(\frac{x}{5}\right)\right]$$

where

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \quad \text{and} \quad \sinh(x) = \frac{e^x - e^{-x}}{2}$$

are known as the hyperbolic cosine and sine functions.

(a) Show that $h(x) = 12 - 2\,e^{\frac{x}{5}} - 2\,e^{-\frac{x}{5}}$

(2)

(b) Show that the derivative function, $\frac{d}{dx}h(x) = \frac{2}{5}e^{\frac{-x}{5}} - \frac{2}{5}e^{\frac{x}{5}}$

(2)

(c) Draw the graph of $y = h(x)$ and state the x-intercepts (to 2 decimal places) and the y-intercept expressed in co-ordinate form.

(4)

(d) Using your answer to (b) and your CAS calculator, determine the value of x when $\frac{d}{dx}h(x) = 0$.

(1)

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(e) Determine the first positive value of x to four decimal places such that $h(x)=g(x)$ where

$$g(x) = -\frac{3}{140}x^3 + \frac{9}{70}x^2 - \frac{199}{140}x + \frac{33}{2}, x \geq 0$$

(2)

(f) On the same set of axes sketch both $h(x)$ and $g(x)$. Clearly label the point of intersection for the x value found in (e).

(5)

(g) Use your CAS to calculate the area bound by the x-axis, the y-axis, the curve with rule *g(x)* and the vertical line through the point of intersection with the x coordinate found in (e) to four decimal places.

(2)

(h) Use your CAS to calculate the area bound by the x-axis, the y-axis, the curve with rule h(x) and the vertical line through the point of intersection with the x coordinate found in (e) to four decimal places.

(2)

(i) **Using your answers from the previous two parts** calculate the area enclosed by y=g(x), y=h(x) and the y-axis to four decimal places.

(1)

The end!

Even though Wendy's mind did allow itself to stray mathematically during the course of the wedding ceremony, it should be noted that Wendy and Peter were happily wedded and lived together thereafter in a state of married bliss.

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