Student NAME:

Teachers name:

MATHEMATICAL METHODS UNIT 4

School Assessed Coursework 3 (SAC 3):
Problem-Solving Task 2 (Probability and Statistics)

Reading Time: 15 minutes
Writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
2	2	72

- **SAC 3 Problem-Solving Task 2 consists of two extended-response questions.**
- Students are permitted to bring into the assessment room: a **bound book** of notes, pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aides for curve sketching, one bound reference, one approved CAS calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator.
- Students are NOT permitted to bring into the assessment room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book, sheet of miscellaneous formulae.
- Working space is provided throughout the book.

Instructions

- Write your **name** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the assessment room.

Mathematical Methods formulae

Probability

$\Pr(A) = 1 - \Pr(A')$	$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$
$\Pr(A\|B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$	

mean	$\mu = \mathrm{E}(X)$	variance	$\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X - \mu)^2) = \mathrm{E}(X^2) - \mu^2$

	Probability distribution	Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x - \mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \displaystyle\int_a^b f(x)dx$	$\mu = \displaystyle\int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \displaystyle\int_{-\infty}^{\infty} (x - \mu)^2\, f(x)dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$\mathrm{E}(\hat{P}) = p$
standard deviation	$\mathrm{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}} \right)$

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Instructions

Answer all questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

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Question 1 (16 marks)

Cholesterol is a type of fat found in the blood of all animals and is essential for many of the body's metabolic processes. However, while cholesterol is essential for life, too much cholesterol is recognised as a risk factor for cardiovascular disease. There are two types of cholesterol:

• high-density lipoprotein (HDL) - the 'good' cholesterol.

• low-density lipoprotein (LDL) - the 'bad' cholesterol.

High levels of blood LDL-cholesterol place the body at a greater risk of cardiovascular disease, while high levels of blood HDL-cholesterol may help to protect the body from adverse cardiovascular events (such as heart attack or stroke).

Cholesterol levels depend on a number of factors including genetic make-up, weight, level of physical activity and the types of food that you eat. Research shows that if you lower your blood LDL-cholesterol level you will lower your risk of heart disease and stroke.

The blood LDL-cholesterol concentration of an adult is a **normally distributed random variable** with a mean of 2.9 mmol/L and a standard deviation of 1 mmol/L. [mmol/L is a measure of the concentration]

a. Find the probability, ***correct to four decimal places***, that the blood LDL-cholesterol level of an adult is

    i. between 2.0 mmol/L and 3.1 mmol/L.   1 mark

    ii. less than 3.6 mmol/L.   1 mark

A blood LDL-cholesterol level greater than 4.1 mmol/L is considered high, a level less than *a* mmol/L is considered optimal and levels in between are considered acceptable (although there is always room for improvement). The probability of a blood LDL-cholesterol level being optimal is equal to 0.3786.

b. i. Find the value of *a*. Give your answer in mmol/L and correct to 1 decimal place.   1 mark

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ii. Given that a patient is an adult **with a high level of blood LDL-cholesterol**, find the probability that such an adult will have a level greater than 4.3 mmol/L. Give your answer ***correct to four decimal places.***   3 marks

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c. The probability that an adult has a blood LDL-cholesterol level between b mmol/L and 3.46 mmol/L is less than 0.5. Find, ***correct to two decimal places***, the smallest possible value of b.   3 marks

d. Find the probability, ***correct to four decimal places***, that in a random group of nine adults

   i. exactly five of the adults will have a blood LDL-cholesterol level between 2.0 mmol/L and 3.6 mmol/L.   2 marks

   ii.   <u>*more*</u> **than three** of the adults will have a high blood LDL-cholesterol level.   2 marks

The blood HDL-cholesterol level of an adult is a normally distributed random variable. The probability of a blood HDL-cholesterol level being optimal is equal to 0.3085. The probability of a blood HDL-cholesterol level being poor is equal to 0.1587.

e. Find the mean and standard deviation of the blood HDL-cholesterol level of an adult. Give your answers in mmol/L and *correct to one decimal place*.   3 marks

Question 2 (35 marks)

Galaxian Highscore, the intrepid adventurer, buys all his strawberries in packets from the *Moles* supermarket. At *Moles* the number X of strawberries in a **large** packet is a discrete random variable with probability distribution as shown in the following table:

x	24	25	26	27
$\Pr(X = x)$	$\dfrac{1}{12}$	$\dfrac{1}{2-k}$	$\dfrac{5}{21}$	$\dfrac{1}{4}+k$

a.   Find all possible values of k, ***correct to 4 decimal places***   2 marks

At the *Moles* supermarket the number Y of strawberries in a **small** packet is a discrete random variable with probability distribution as shown in the following table:

y	14	15	16	17
$\Pr(Y = y)$	0.21	0.38	0.23	0.18

b.   Galaxian buys **two small** packets of strawberries.

  i.   Find the probability, ***correct to four decimal places***, that each packet has 14 strawberries.   1 mark

Question - continued

   ii. Find the probability, ***correct to four decimal places***, that the total number of strawberries in both packets is equal to 32.   2 marks

   iii. Find the probability, ***correct to four decimal places***, that one of the packets has 15 strawberries **given** that the total number of strawberries in both packets is equal to 32   3 marks

c. On another occasion Galaxian buys **seven** **small** packets of strawberries.
   i. Find the probability, ***correct to four decimal places***, that **at least** three of the packets have 16 strawberries.   2 marks

Max Hardcase, the steadfast partner-in-adventure of Galaxian Highscore, also buys all his strawberries from the *Moles* supermarket. He buys a number of **small** packets of strawberries in readiness for a new adventure.

d. A regular strawberry has a width of at least 2 cm. One of the small packets that Max buys has 15 strawberries in it and 12 of those strawberries are regular.

   i. Max selects a random sample of 11 strawberries from this packet **without replacement**.
   Find the probability that *more* than 80% of the strawberries in the sample are regular.

3 Marks

Moles supermarket claims that 90% of all the strawberries it sells are regular.
Galaxian Highscore collects a random sample of 51 strawberries after buying two large packets.

e. i.   Find, ***correct to three decimal places***, the expected value and standard deviation of the
         sampling distribution of the proportion of regular strawberries in his sample.   2 marks

     ii. Find, ***correct to four decimal places***, the probability that at least 85% of the strawberries
         in his sample are regular. **Do not use a normal approximation.**   2 marks

     iii. Use the ***normal approximation*** to find the approximate probability that at least 80% of the
         strawberries in his sample are regular. ***Give your answer to three decimal places.***   2 marks

Galaxian counts 47 regular strawberries in his sample.

   iv. Find a 98% confidence interval for the proportion of regular strawberries sold by
   Moles. Show working.   1 mark

Galaxian Highscore has millions of followers on Twitter. Twitter is an online social networking and micro-blogging service that enables its users to send and read short text-based messages known as *tweets*. On any given day, the number T of tweets that Galaxian makes about his upcoming adventures is a random variable with probability distribution given by

t	0	1	2	3
$Pr(T = t)$	0.1	0.3	0.4	0.2

f. Galaxian tweets on both Thursday and Friday.
   i. What is the probability that he tweets on both Thursday and Friday   1 mark

   ii. What is the probability that he makes a total of four tweets given that he tweets over both Thursday and Fridays?

                                                                                        2 marks

Galaxian and Max know that in their next adventure they will need strawberries. They also know that they will need to correctly answer problems set by the feared *Stochastica* tribe, a tribe of natives that worships probability. They have heard rumours that Tasmania Jones, a rival adventurer who recently disappeared, met this tribe, tried to answer their problems and failed (and in failing, died a horrible death). It has been said that many problems set by the *Stochastica* tribe are similar to the following.

g.   For events A and B from a sample space, $\Pr(A) = \dfrac{1}{5}$ and $\Pr(B) = \dfrac{2}{3}$.

   If A' denotes the complement of A, calculate $\Pr(A' \mid B)$ when

   i. A and B are **independent.**   2 marks

   ii. A and B are **mutually exclusive**.   1 mark

   iii. $\Pr(A \cup B) = \dfrac{3}{4}$.   3 marks

End of Question 2-
END OF QUESTION AND ANSWER BOOK

h. Let the random variable X be normally distributed with mean 1.5 and standard deviation 0.4.

Let Z be the standard normal random variable, such that $Z \sim N(0, 1)$.

i. Find a such that $\Pr(X < a) = \Pr\left(Z > \dfrac{a}{3}\right)$. ***Give answer to 4 decimal places***   3 marks

i. Find all possible values of b so that the function
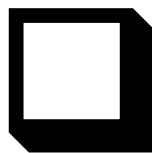
$$f(x) = \begin{cases} 4bx^3 + 3x^2 - b^2 & \text{if } 0 < x \leq 1 \\ \\ 0 & \text{otherwise} \end{cases}$$

is a probability density function.   3 marks

END OF QUESTION

Assessment Grade for Task

This grade is subject to statistical
moderation at the Victorian
Curriculum and Assessment
Authority (VCAA) and is likely to
change.