XAVIER COLLEGE

SAC / Assessment Conditions

Date: **Time:**

MATHEMATCAL METHODS APPLICATIONS SAC 1 **BOOKLET 1** **Total marks 38**	**June 7 2017**	**2.45 – 4.55**

- *Listen carefully to the supervisor's instructions.*
- *Permissible items include: 1 bound book*
  CAS calculator
  pens, pencils, highlighters, erasers, sharpeners, rulers.
- *You are not permitted to use white out (liquid paper).*
- *You have 10 minutes reading and 2 hrs writing to complete this part.*
- *Complete this task in the spaces provided.*
- ***Exact values are expected throughout unless otherwise stated.***
- ***Units are required where applicable***
- *A number of questions are consequential in nature. You are advised to show all working, even for questions worth one mark.*
- ***In questions worth more than 1 mark, working is required to gain full marks.***
- *You must work silently and independently for the duration of the task.*

PLEASE NOTE: *Students are NOT permitted to have mobile phones or any other unauthorised electronic devices in their possession during a SAC/examination*

COMPULSORY STUDENT DECLARATION

I, (*print your name neatly*) _____
acknowledge that I have read the SAC/examination conditions and understand which items/materials I am permitted to use and have in my possession.

****If you have any doubts as to what is permitted, raise your hand and DO NOT sign this declaration****

Student's Signature: _____

Student's Name: _____

Teacher's Name: _____

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

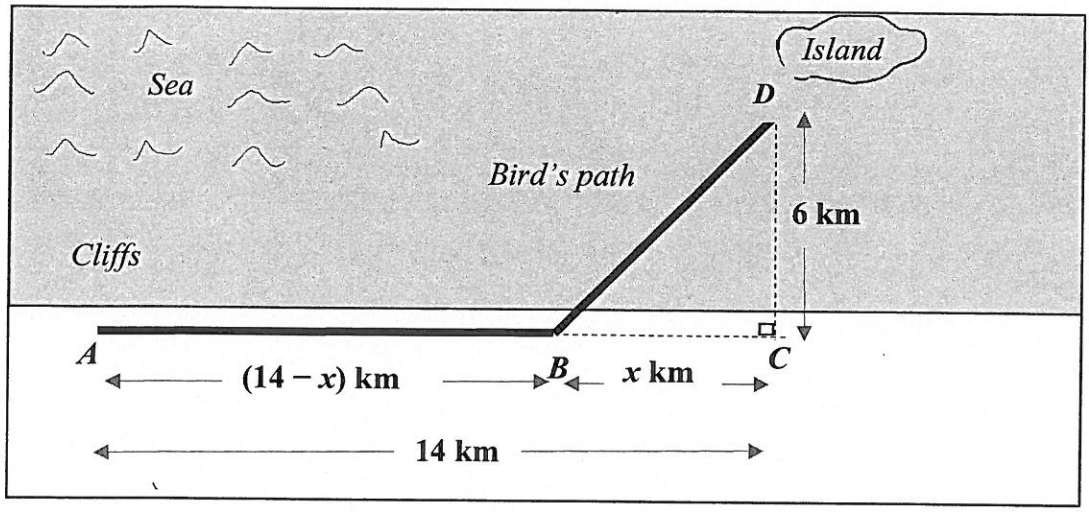
Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$	$\int x^n dx=\frac{1}{n+1}x^{n+1}+c,\ n\neq -1$
$\frac{d}{dx}\left((ax+b)^n\right)=an(ax+b)^{n-1}$	$\int (ax+b)^n dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c, n\neq -1$
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$	$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$
$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$	$\int \frac{1}{x}dx=\log_e(x)+c,\ x>0$
$\frac{d}{dx}\left(\sin(ax)\right)=a\ \cos(ax)$	$\int \sin(ax)dx=-\frac{1}{a}\cos(ax)+c$
$\frac{d}{dx}\left(\cos(ax)\right)=-a\ \sin(ax)$	$\int \cos(ax)dx=\frac{1}{a}\sin(ax)+c$
$\frac{d}{dx}\left(\tan(ax)\right)=\frac{a}{\cos^2(ax)}=a\ \sec^2(ax)$	

product rule	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$		

QUESTION ONE Habits of Seabirds

Miriam, a biologist, is studying the habits of a species of seabird.
The diagram below shows the flight path taken by adult birds when flying from the cliffs at point A to the island at point D.
Points A, B and C are on a straight shore and D is 6 km from the shore.
$AC = 14$ km, $BC = x$ km and $AB = (14 - x)$ km.



a. Find the exact length of BD in terms of x

1 mark

b. Suppose that a particular bird can travel at an average speed of 20 km/h over the land and at 12 km/h over the sea.

i. Write an expression, in terms of x, for the time, T hours, that it will take the bird to fly from A to B to D.

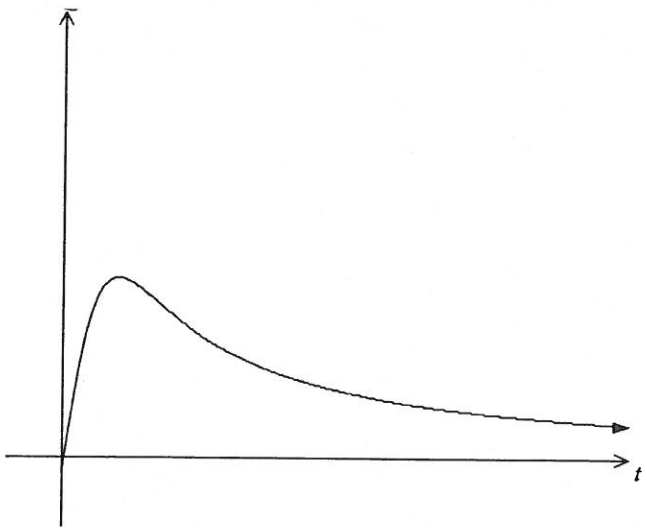
ii. Find the value of x for which the time taken by the bird to reach the island is a minimum.

iii. Find the minimum time, in hours, correct to one decimal place, required for the bird to fly to the island.

2 + 2 + 1 = 5 marks

Miriam is also treating an injured sea lion at the base of the cliffs. She administers a dose of an analgesic (pain reliever) at $t = 0$. The concentration, Q units/cm^3, of analgesic in the animal's bloodstream, t hours after it is administered, is modelled by the function.

$Q:[0,\infty) \to R,\ Q(t) = \dfrac{6t}{t^2+1}$. The graph of this function is shown.



c. i. What is the maximum concentration of the analgesic, and how long after the dose is administered does the maximum occur?

ii. On the graph of Q above, label the stationary point with its exact coordinates.

2 + 1 = 3 marks

d. The analgesic will provide pain relief when the concentration is above 1.25 units/cm^3. For what length of time, in hours, correct to two decimal places, will the animal experience relief from the pain?

2 marks

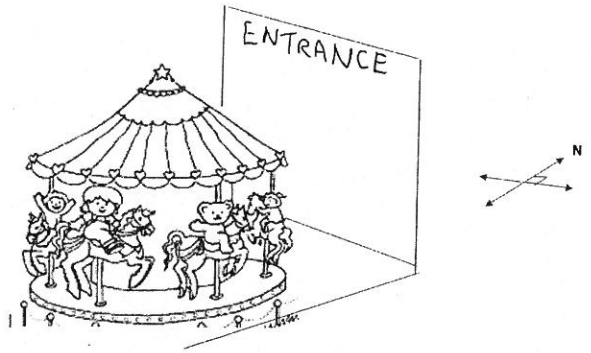
$\frac{dQ}{dt}$ is a measure of the rate of at which the analgesic is being absorbed into the bloodstream (when the rate is positive) or expelled from the bloodstream (when the rate is negative).

Consider the function $S:(0,\infty)\rightarrow R,\ S(t)=\frac{dQ}{dt}$

e. The set of axes above part **c. i.** shows the graph of Q.
On the same set of axes, sketch the graph of S. Label the t-axis intercept. Label the local minimum with its coordinates, correct to two decimal places. Label any asymptote with its equation.

3 marks
TOTAL 14 marks

QUESTION TWO Carousel



On a waterfront there is an information building containing a carousel. The distance, measured in metres, north of a particular child on the carousel from the entrance to the building at t seconds after the carousel starts, can be modelled by the equation $D(t) = 4\sin\left(\frac{\pi t}{30}\right) + 12$.

a. At the point the carousel starts, what is the child's initial distance north from the entrance?

_____

1 mark

b. What is the child's maximum distance north from the entrance?

_____

_____

1 mark

c. If the ride goes for 5 minutes, how many revolutions does the carousel make?

_____

_____

_____

_____

_____

_____

2 marks

d. Find the first four times that the child is level with their father, who is standing 14m from the entrance to the building.

3 marks

e. The child's sibling is also on the carousel, but on the opposite side and 1metre further out from the centre of the carousel. Find the equation that can be used to model the sibling's distance north from the building entrance.

2 marks

f. In the **5 minutes**, how much further does the sibling travel than the original child ?
Give your answer correct to the nearest metre.

3 marks

TOTAL 12 marks

QUESTION THREE Radioactive Substances

Carbon-14 is a radioactive substance that decays exponentially. It is known to have a half-life of 5730 years. If the decay can be represented by the equation $C = C_0e^{kt}$, where t is time in years

a. show that the value of k can be approximated as -0.000121.

2 marks

Use this value of k for the remaining questions

b. How many years does it take a substance to decay to 25% of its original mass. Give answer to the nearest year.

2 marks

c. A fossil containing 1.23×10^{-66} grams of carbon-14 was dated 1.2 million years old. How much carbon-14 did the fossil originally contain. . Give answer correct to 5 decimal places.

2 marks

d. Find the rate of change of decay with respect to t.

1 mark

e. After how many years is the rate of change of decay half of the initial rate of change. Give answer to the nearest year.

2 marks

Another radioactive substance that decays exponentially is plutonium-240. It has a half-life of 6563 years

f. If the fossil contained 4.05×10^{-6} grams of carbon-14 and 3.28×10^{-7} grams of plutonium; how many years until the fossil contained an equal amount of both substances, to the nearest year?

3 marks

END OF SAC PART ONE

Total 12 marks