XAVIER COLLEGE

SAC / Assessment Conditions

	Date:	Time:
MATHEMATCAL METHODS APPLICATIONS SAC 1	June 19 2017	2.45 – 4.55
BOOKLET 2 **Total marks 46**		

- *Listen carefully to the supervisor's instructions.*
- *Permissible items include: 1 bound book*
  CAS calculator
  pens, pencils, highlighters, erasers, sharpeners, rulers.
- *You are not permitted to use white out (liquid paper).*
- *You have 10 minutes reading and 2 hrs writing to complete this part.*
- *Complete this task in the spaces provided.*
- ***Exact values are expected throughout unless otherwise stated.***
- ***Units are required where applicable***
- *A number of questions are consequential in nature. You are advised to show all working, even for questions worth one mark.*
- ***In questions worth more than 1 mark, working is required to gain full marks.***
- *You must work silently and independently for the duration of the task.*

PLEASE NOTE: *Students are NOT permitted to have mobile phones or any other unauthorised electronic devices in their possession during a SAC/examination*

COMPULSORY STUDENT DECLARATION

I, (*print your name neatly*) ______________________________
acknowledge that I have read the SAC/examination conditions and understand which items/materials I am permitted to use and have in my possession.

******If you have any doubts as to what is permitted, raise your hand and DO NOT sign this declaration******

Student's Signature: ______________________________

Student's Name: ______________________________

Teacher's Name: ______________________________

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

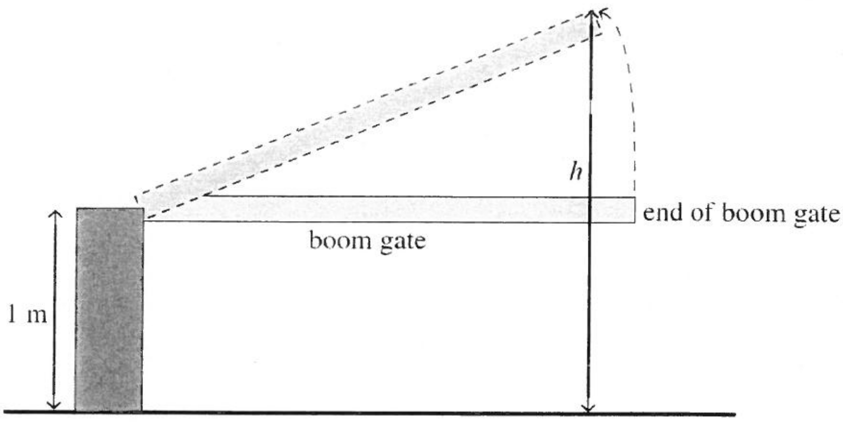
Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$	$\int x^n dx=\frac{1}{n+1}x^{n+1}+c,\ n\neq -1$
$\frac{d}{dx}\left((ax+b)^n\right)=an(ax+b)^{n-1}$	$\int (ax+b)^n dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c, n\neq -1$
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$	$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$
$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$	$\int \frac{1}{x}dx=\log_e(x)+c,\ x>0$
$\frac{d}{dx}\left(\sin(ax)\right)=a\ \cos(ax)$	$\int \sin(ax)dx=-\frac{1}{a}\cos(ax)+c$
$\frac{d}{dx}\left(\cos(ax)\right)=-a\ \sin(ax)$	$\int \cos(ax)dx=\frac{1}{a}\sin(ax)+c$
$\frac{d}{dx}\left(\tan(ax)\right)=\frac{a}{\cos^2(ax)}=a\ \sec^2(ax)$	

product rule	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$		

QUESTION ONE BOOM GATES

The boom gate at the entrance to an industrial estate is malfunctioning. It is not responding to manual operation, but periodically opening and closing in a pattern that **repeats every hour**.



The height above the ground, h metres, of the end of the boom gate at time t minutes is given by the **continuous function**

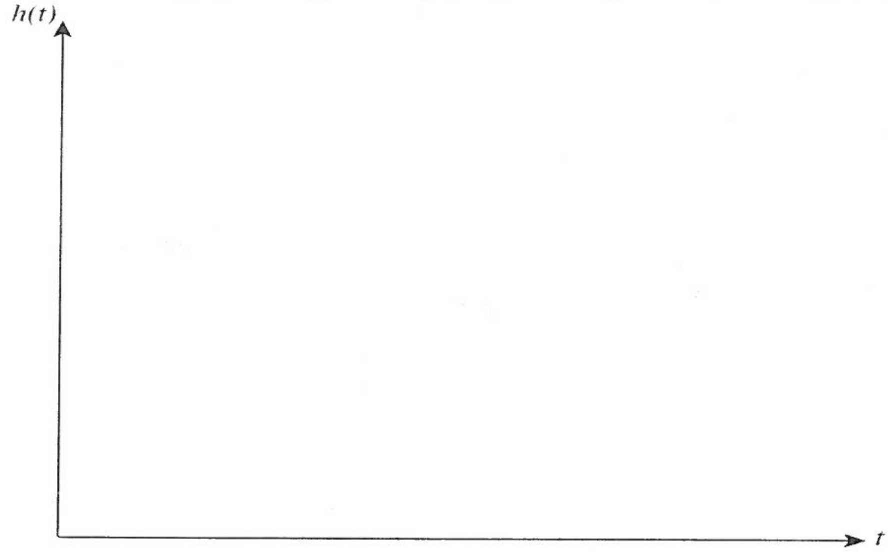
$$h(t) = \begin{cases} 3\sin(\frac{\pi}{20}(t-10))+1 & t \in [10,30] \\ 3\cos(\frac{\pi}{20}(t-40))+1 & t \in [30,50] \\ k & t \in [0,10) \cup (50,60] \end{cases}$$

Where $t = 0$ corresponding to 6:00 am on Saturday

a. Find the value of k.

1 mark

b. Sketch the graph of $h(t)$ for $t \in [0, 60]$, labelling the endpoints with coordinates



3 marks

c. Find at what time(s) during the first hour the gate reaches its maximum height above the ground.

2 mark

A van has come to make a delivery to one of the warehouses in the estate. It can only gain access when the boom gate is at least 3 metres above the ground. The driver of the van arrives at the boom gate at 9.10 am.

d. How long does the driver have to wait to gain access through the boom gate and into the estate? Express your answer in minutes, correct to two decimal places.

2 marks

e. How many minutes of each hour would the driver be able to drive the van through the boom gate? Express your answer in minutes, correct to two decimal places.

2 marks

The time, T minutes, that the driver takes to enter through the boom gate, unload his van and return to the boom gate is dependent on the mass, m kg, of his load, where

$$T = e^{0.005m} + 5 \quad 0 \leq m \leq 1000$$

Assume that the driver enters through the boom gate at the first possible opportunity after his arrival at 9:10 am.

f. Find the total time, in minutes, that elapsed between the driver arriving and exiting the gate, given that the load he is delivering is 200 kg. Give your answer correct to two decimal places.

2 marks

A second delivery van arrives at 9.28 am, carrying a load of mass 500 kg.

g. What is the earliest time this van will be able to leave the estate after having delivered its load to the warehouse? Give your answer correct to the nearest minute.

3 marks

Total 15 marks

QUESTION TWO

Consider the functions below

$f(x) = x^2 + 5$ $g(x) = \sqrt{4-x}$ $h(x) = \dfrac{3}{x-2}$

a. State, with reasons, whether the following composite functions exist

i. $f(g(x))$

ii $g(h(x))$

iii $h(f(x))$

b. Give the rules, domain and range for the composite functions that exists in **a.**

7 marks

c i. Find $h^{-1}(x)$ such that $g \circ h^{-1}$ exists

c ii. hence define the composite function $g \circ h^{-1}$

4 marks

A transformation $T: R^2 \rightarrow R^2$ is defined by

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -2 & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

Another transformation S, has the following sequence:

- Reflection in the y-axis
- Dilation of factor $\frac{1}{2}$ from the x-axis
- Translation 4 units in the negative direction of the x-axis and 2 units in the positive direction of the y-axis

d i. Find the image of function g after the transformation T

d ii. Find the image of function f after the transformation S

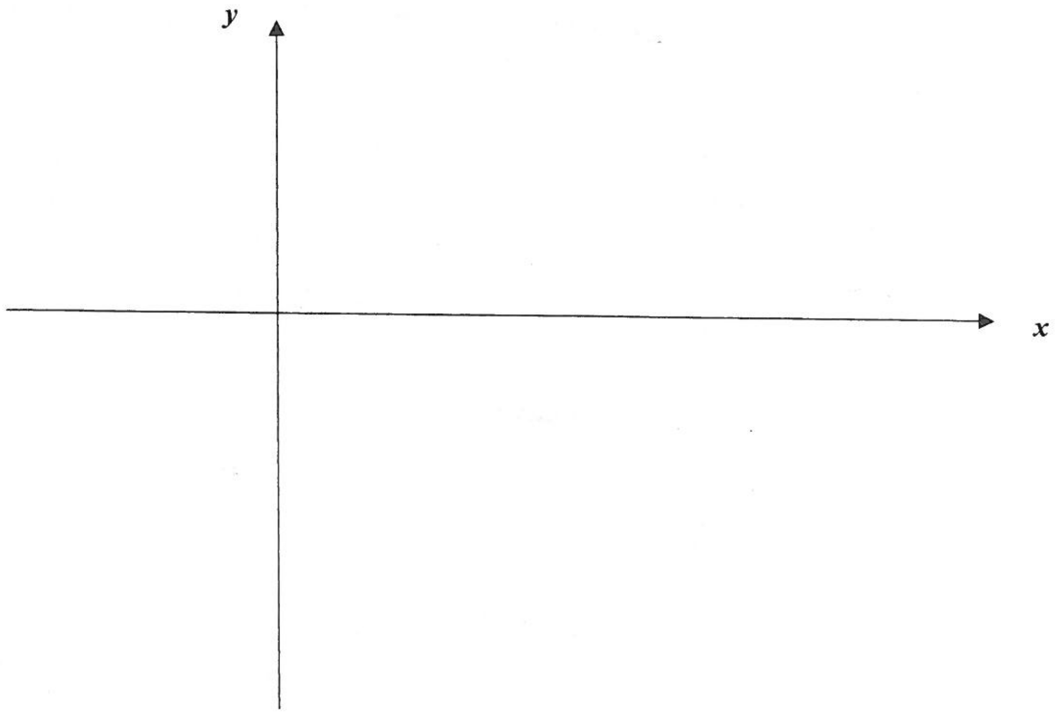
2 + 2 = 4 marks

Total 15 marks

QUESTION THREE Function

Consider the function $f: R \to R, f(x) = \frac{1}{2}x^2(x-8)+12$

a. Sketch the graph of $y = f(x)$ on the axes below.
Label axes intercepts and stationary points with their **exact** coordinates.



3 marks

b. Find the exact area of the region enclosed by the graph of $y = f(x)$ and the x axis.

2 marks

Given g: $R \to R$, $g(x) = ax^2(x - 8)$ where $a > 0$

c. Show that the equation of the tangent to the curve $y = g(x)$, in terms of a, at the point where $x = 1$ is $y = 6a - 13ax$

3 marks

d i. Find the coordinates, in terms of a, of where the tangent meets the curve $y = g(x)$.

d ii. Hence, find, in terms of a, the area of the region bounded by the curve $y = g(x)$ and the tangent to the curve $y = 6a - 13ax$.

4 marks

e i. Find $g'(1)$ in terms of a

e ii. Find the gradient of the curve at the second point of intersection of the curve and the tangent. Give answer in terms of a

e iii Hence or otherwise find the value of a so that the tangent to the curve $y = g(x)$ at $(1, g(1))$ is also the normal to the curve at the second point of intersection of the tangent with the curve.

3 marks

Total 15 marks

END OF SAC PART TWO