XAVIER MATHS

Mathematical Methods SAC 1 (2018)

APPLICATION TASK

The Maytime Fair

Booklet 1 – 8th June

9.00 – 11.10 a.m.

10 minutes reading time

120 minutes writing time

This task is to be completed in one session of duration 130 minutes.
During this task, you may use your calculator and refer only to your Application SAC Preparation Booklet and one other set of bound notes. No other pieces of paper may be used.
You must work silently and independently for the duration of this task.
All answers are to be written within this booklet.
When drawing graphs, ensure that a pencil is used and that significant features of the graph are clearly indicated in pen.
Exact values are expected throughout, unless otherwise stated and units must be included.
Your CAS calculator may be collected at the conclusion of the SAC so that all memories can be cleared, and will be returned to you in your first Mathematical Methods class.
No electronic devices (such as mobile phones) may be brought into the examination room.

Total : 50 marks

Student Name : ______________________________

Teacher Name: ______________________________

The grade awarded to this SAC is subject to statistical moderation by the VCAA and is likely to change.

COMPULSORY STUDENT DECLARATION

I, (*print your name neatly*) ____
acknowledge that I have read the SAC/examination conditions and understand which items/materials I am permitted to use and have in my possession.

****If you have any doubts as to what is permitted, raise your hand and DO NOT sign this declaration****

Student's Signature: ____

Student's Name: ____

Teacher's Name: ____

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

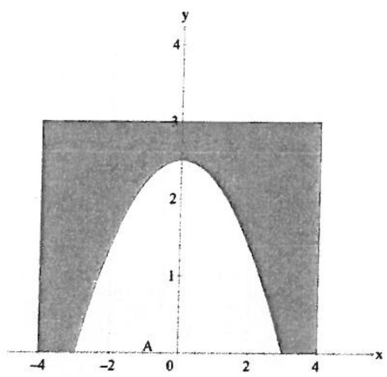
Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$		$\int x^n dx=\frac{1}{n+1}x^{n+1}+c,\ n\neq -1$	
$\frac{d}{dx}\left((ax+b)^n\right)=an\left(ax+b\right)^{n-1}$		$\int (ax+b)^n dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c, n\neq -1$	
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$		$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$	
$\frac{d}{dx}(\log_e(x))=\frac{1}{x}$		$\int \frac{1}{x}dx=\log_e(x)+c,\ x>0$	
$\frac{d}{dx}(\sin(ax))=a\ \cos(ax)$		$\int \sin(ax)dx=-\frac{1}{a}\cos(ax)+c$	
$\frac{d}{dx}(\cos(ax))=-a\ \sin(ax)$		$\int \cos(ax)dx=\frac{1}{a}\sin(ax)+c$	
$\frac{d}{dx}(\tan(ax))=\frac{a}{\cos^2(ax)}=a\sec^2(ax)$			
product rule	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$		

The Maytime Fair

Question One (11 marks)

The planning for the Maytime Fair for 2019 is well under way. The committee is considering erecting a security fence around the carnival area and have patrons enter the area through an elaborate arch way. The design of the arch is given below but the dimensions need to be checked.



The height of the arch is $\frac{5}{2}m$ and the width is *6m*. The equation is of the form $y = ax^2 + b$, taking the axes as shown.

a) Show that the values of *a* and *b are* $-\frac{5}{18}$ and $\frac{5}{2}$ respectively.

(2 marks)

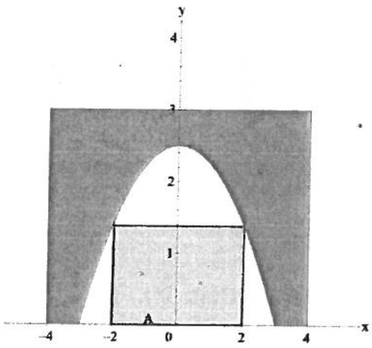
b) The point A is located *1m* left of centre of the arch. A man with a height of *2m* stands at point A. How far above his head is the arch?

(2 marks)

c) A horizontal steel rod is to be fixed into the centre of the arch at a height of *2m* from the arch floor. Find the length the rod to the nearest cm.

(2 marks)

A large box needs to pass through the arch.



i) Given the end area of the box is given by $A = 2xy$ write an equation for area in terms of *x* only.

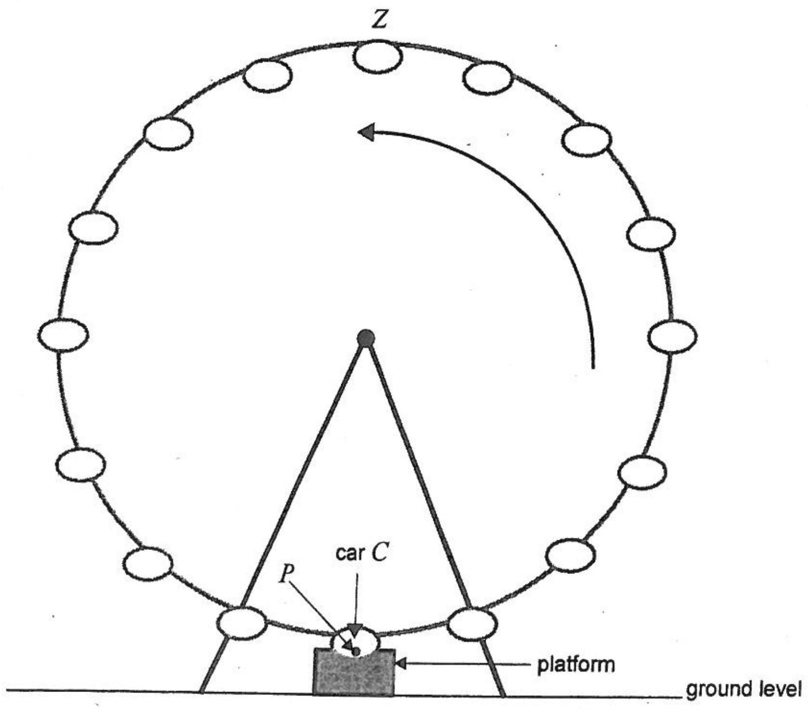
ii) Find the maximum end area of a box that will be able to pass through the arch. The box is permitted to touch the arch.

iii) What is the height and width of this maximum end area box.

(1 + 2 + 2 = 5 marks)

Question Two (17 marks)

Once patrons have passed through the arch the first ride they encounter is the Ferris Wheel. This Ferris Wheel rotates in an anticlockwise direction at a constant rate. People enter the cars of the Ferris Wheel from a platform which is above ground level. The Ferris Wheel does not stop at any time. The Ferris Wheel has 16 cars, spaced evenly around the circular structure.



A spider attached itself to the point P on the side of car C when the point P was at its lowest point at 1.00 pm.

The height, h metres, of the point P above ground level, at time t hours after 1.00 pm., is given by

$$h(t) = 62 + 60\sin\left[\frac{(5t-1)\pi}{2}\right]$$

a) Write down the maximum height, in metres, of the point P above ground level.

(1 mark)

b) Write down the minimum height, in metres, of the point P above ground level.

(1 mark)

c) At what time, after 1.00 pm. does point P first return to its lowest point?

(1 mark)

d) How may revolutions does car C do in 3 hours?

(1 mark)

e) Find the time, after 1.00 pm., when the point P first reaches a height of 92 metres above ground level. (Correct to the nearest minute)

(2 marks)

f) Find the number of minutes during one rotation when the point P is at least 92 metres above ground level. (Correct to the nearest minute)

(2 marks)

g) Find the first four times (correct to the nearest minute) that the point P is level with the height of the launch of the Speed slide which is 20 metres above ground level.

(2 marks)

h) Write an expression, in terms of t, for the rate of change of h with respect to time.

(1 mark)

i) At what rate in metres/hour is h changing when t = 1 hour?

(1 mark)

j) What is the distance travelled by car C in three hours? (correct to the nearest metre)

(1 mark)

When point *P* first reaches position *Z*, the highest point, the spider becomes frightened. It drops down from the car on a thread at a rate of 5 metres per minute until it reaches the ground.

As it drops, the spiders height *s(t)* metres above the ground at time *t* (where *t* is the time in hours after 1.00 pm.) is given by

$$s(t) = h(t) - 300(t - 0.4)$$

k) Find, to the nearest minute, the time taken from when the spider leaves car C to when it reaches the ground.

(2 marks)

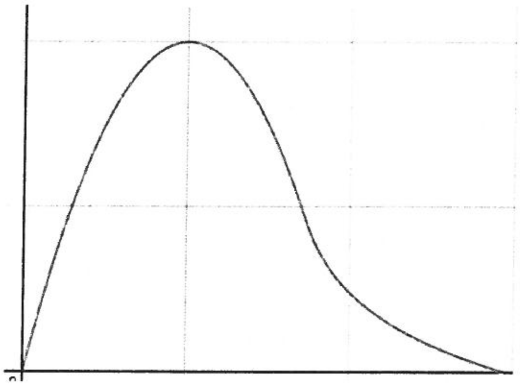
l) Find the maximum height reached by the spider.

(2 marks)

Question Three (8 marks)

The committee is also considering a new rollercoaster ride. Part of the rollercoaster can be modelled by the function $f(x)$, where *x* is the horizontal distance in metres from the start of the rollercoaster and *f(x)* represents the height above the ground.

$$f(x)=\begin{cases} -0.4x(x-10) & x\le 8.679 \\ -2\log_e(x-8)+c & x>8.679 \end{cases}$$



a) Find the value of '*c*' correct to 3 decimal places.

(2 marks)

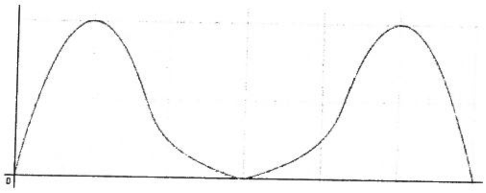
b) Express the first part of *f(x)* in turning point form.

(2 marks)

c) Given the rollercoaster does not go below ground level, state the domain and range of the function, correct to 3 decimal places.

(2 marks)

The second part of the rollercoaster is a mirror reflection of the first part of the rollercoaster.



d) State a series of transformations required to obtain the graph of the second part of the rollercoaster, $g(x)$.

(2 marks)

Question Four (10 marks)

Maths Whizz, Gim Trant, puts forward that he wishes to run a Mathletics competition, knowing that it will be a sure fire money spinner for the Fair. He believes he has Problem Impossible – and anyone cracking this problem will be a Problem Impossible Champion.

His problem is multi-faceted and he requires participants to get all components correct. Your task is to take on Problem Impossible and make it Problem Possible.

He first states that $f{:}[0,\infty) \rightarrow R \text{ where } f(t) = 2\,e^{-t}$

He then asks

a) State the range of *f*.

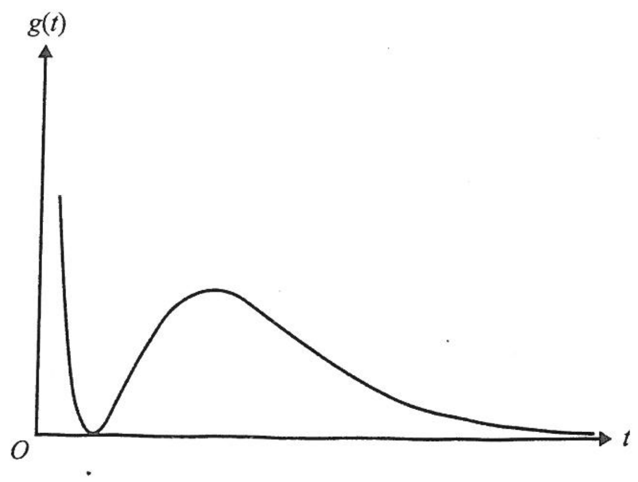
(1 mark)

b) Find the rule of the inverse of *f* and state its domain.

(2 marks)

He then states that $g{:}[0,\infty) \rightarrow R \text{ where } g(t) = (t-1)^2 e^{-t}$.

Part of the graph of *g* is shown below.



c) The rule for the derivative of g may be expressed in the form

$$g'(t) = (-t^2 + bt + c)\, e^{-t}$$

Find the exact values of b and c.

(2 marks)

d) The graph of $y = g(t)$ has stationary points $(1,p)$ and (m,n). Find the exact values of p, m and n.

(3 marks)

e) Now Gim is just mucking with you. For the function

$$q{:}[0,\infty) \rightarrow R \text{ where } q(t) = 2\,g(t) - 5,$$

state the exact coordinates of the stationary points of $y = q(t)$

(2 marks)

Question Five (4 marks)

Super student, Nitram Yrogerg, solved all of Gim's problems and then threw one straight back at him. Can you solve Nitram's problem for Gim?

The function

$h: R \to R$ where $h(t) = (t^2 + at + 10)\, e^{-t}$, where a is a real constant,

has derivative

$h'(t) = (-t^2 + (2 - a)t + (a - 10))\, e^{-t}$.

Find the values of a such that

a) The graph of $y = h(t)$ has exactly one stationary point

(4 marks)

b) $h'(t) < 0$ for all $t \in R$

(2marks)

END OF QUESTION BOOKLET