XAVIER MATHS

Mathematical Methods SAC 1 (2018)

APPLICATION TASK

The Maytime Fair

Booklet 2 – 20th June

10 minutes reading time

120 minutes writing time

This task is to be completed in one session of duration 130 minutes.
During this task, you may use your calculator and refer only to your Application SAC Preparation Booklet and one other set of bound notes. No other pieces of paper may be used.
You must work silently and independently for the duration of this task.
All answers are to be written within this booklet.
Questions worth more than one mark require some explanation of working or process.
When drawing graphs, ensure that a pencil is used and that significant features of the graph are clearly indicated in pen.
Exact values are expected throughout, unless otherwise stated and units must be included.
Your CAS calculator may be collected at the conclusion of the SAC so that all memories can be cleared, and will be returned to you in your first Mathematical Methods class.
No electronic devices (such as mobile phones) may be brought into the examination room.

Total : 50 marks

Student Name : ______________________________

Teacher Name: ______________________________

The grade awarded to this SAC is subject to statistical moderation

by the VCAA and is likely to change.

COMPULSORY STUDENT DECLARATION

I, (*print your name neatly*) ____
acknowledge that I have read the SAC/examination conditions and understand which items/materials I am permitted to use and have in my possession.

****If you have any doubts as to what is permitted, raise your hand and DO NOT sign this declaration****

Student's Signature: ____

Student's Name: ____

Teacher's Name: ____

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$		$\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$	
$\frac{d}{dx}((ax+b)^n) = an(ax+b)^{n-1}$		$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, n \neq -1$	
$\frac{d}{dx}(e^{ax}) = ae^{ax}$		$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$	
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$		$\int \frac{1}{x} dx = \log_e(x) + c,\ x > 0$	
$\frac{d}{dx}(\sin(ax)) = a\ \cos(ax)$		$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$	
$\frac{d}{dx}(\cos(ax)) = -a\ \sin(ax)$		$\int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$	
$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a\ \sec^2(ax)$			
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$		

The Maytime Fair

Question One (14 marks)

Gim (from the first part) smashed Nitram's problem so he was sent into the Room of Hobert. Hobert has in his room several problems but he also allows people to phone a friend to assist with their solutions.

He knows he will struggle – so he is checking in with you guys as his phone buddies. Your responses to assist Gim need to be written in the spaces provided below.

Hob Rume presents three separate situations

a) i) Let $f(x) = x^2 + 1$ and $g(x) = 2x + 1$. Write down the rule for $f(g(x))$ and state the maximal domain and range.

(2 marks)

ii) For the function $h(x) = 3e^{2x} + 1$ find the inverse function $h^{-1}(x)$ and state the domain and range of $h^{-1}(x)$

(2 marks)

b) Consider the function $f: [0,2] \rightarrow R \; where \; f(x) = ax^3 + bx^2 + cx$.

The graph of $y = f(x)$ has a turning point at the point with coordinates (1,1)

i) Find the values of a and b in terms of c only.

(2 marks)

ii) If also $f(2) = 0$, find the value of c.

(1 mark)

c) OK - Then Professor Rume presented the following. Now consider the function

$$f: [0,2] \rightarrow R \; where \; f(x) = (x-1)^2(x-2) + 1$$

i) Find *f '(x)*

(1 mark)

(2

ii) The coordinates of the turning point of the graphs of $y = f(x)$ occur at $(m, 1)$ and $(n, \frac{23}{27})$. Find the values of m and n.

(3 marks)

iii) What are the absolute maximums and minimums of this function over this domain.

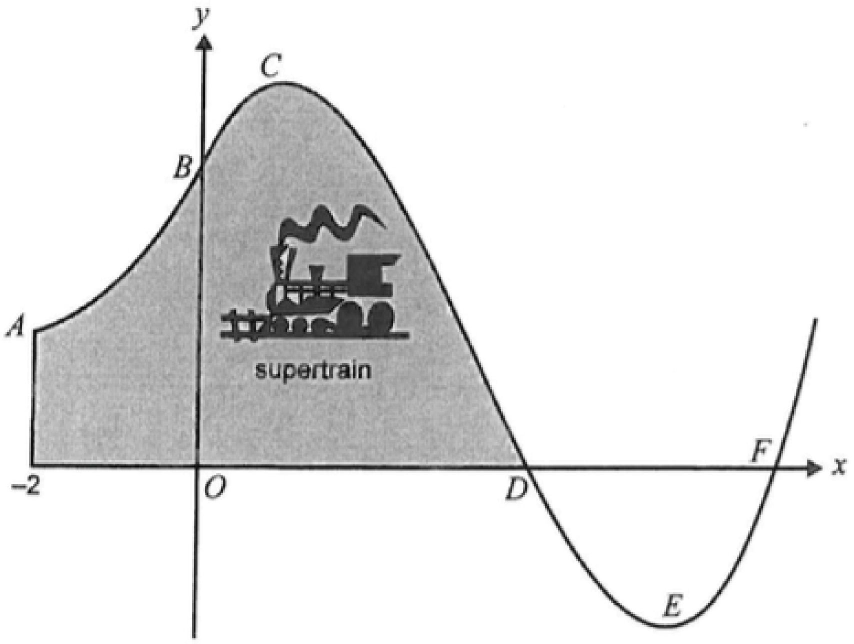
(2 marks)

iv) Find the real values of p for which the function $f(x) = p$, where $x \in [0,2]$, has exactly one solution.

(2 marks)

Question Two (16 marks)

A major attraction at the Maytime Fair was the model train which follows the curve passing through the points *A, B, C, D, E* and *F* shown below. (units are in metres)



The designer completed the track after putting axes on the drawing as shown. The track is made up of two different curves, one to the left of the *y*-axis and the other to the right.

B is the point (0,7).

The curve from B to F is part of the graph of $f(x) = px^3 + qx^2 + rx + s$ where p, q, r and s are constants and $f'(0) = 4.25$

a) i) Show that $s = 7$

ii) Show that $r = 4.25$

(1 + 1 = 2 marks)

The furthest point reached by the track in the positive y direction occurs when $x = 1$. Assume $p>0$.

b) i) Use this information to find q in terms of p.

ii) Find $f(1)$ in terms of p.

iii) Find the value of a in terms of p for which $f'(a) = 0$ where $a > 1$

iv) If $a = \frac{17}{3}$, show that p = 0.25 and q = - 2.5.

(2 + 1 + 2 + 2 = 6 marks)

For the following assume $f(x) = 0.25x^3 - 2.5x^2 + 4.25x + 7$.

c) Find the exact coordinates of *D* and *F*.

(3 marks)

d) Find the greatest distance that the track is from the *x*-axis, when it is below the *x*-axis, correct to two decimal places.

(2 marks)

The curve from *A* to *B* is part of the graph with equation $g(x) = \frac{a}{1-bx}$, where *a* and *b* are positive real constants. The track passes smoothly from one section of the track to the other at *B* (that is, the gradients of the curves are equal at *B*).

e) Find the exact values of *a* and *b*.

(3 marks)

Question Three (9 marks)

To more important problems! (sorry – Yrogerg, Gim and Hobert).

If the rubbish is not cleaned up quickly The Maytime Fair area can have a mouse problem. A particular population of mice can be represented by the equation

$$f(t) = \frac{17}{4} log_e\left(\frac{t+2}{2}\right)\sin\left(\frac{\pi t}{3}\right) + 10$$

where t is the time in days.

a) Find the size of the initial population.

(1 mark)

b) At what time, in days and hours (correct to the nearest hour), does the population first double?

(2 marks)

c) Assuming that the population cannot recover from zero, state the maximal domain for the function, correct to one decimal place.

(2 marks)

d) Find the rate of change of the population with respect to time.

(2 marks)

e) Find the average rate of change of the population during the maximal domain, correct to two decimal places.

(1 mark)

The number of another population of mice present locally can be represented by

$$g(t) = 4e^{0.05t} + 2$$

f) When are the two populations first equal? Give your answer in days correct to two decimal places.

(1 marks)

Question Four (11 marks)

Another problem for the Maytime fair committee to consider is the mosquito population. The mosquito numbers can be represented by the function.

$$N(t) = N_0 e^{kt}$$

N represents the number of mosquitoes and t is the time in days.

The numbers increase by 5% every day.

a) Show that the value of k is 0.04879 correct to five decimal places.

(2 marks)

b) Given that after 10 days the number of mosquitoes is 8000, find N_0 correct to the nearest whole number.

(2 marks)

c) If the mosquito population continued to grow in this way find the number of mosquitoes that would be in the population after 8 weeks, correct to the nearest whole number.

(1 mark)

d) If the mosquito population continued to grow in this way, how long will it take for the number of mosquitoes to reach 40000? Give your answer to the nearest whole day.

(1 mark)

After 12 days, the food source begins to wane and the mosquito population begins to fall.

e) Show that there were 9260 mosquitoes present after the 12 days?

(1 mark)

The number of mosquitoes and times is given in the following table.

Day (after 12^{th} day)	Number of Mosquitoes
0	9260
13	4544
26	2599
39	789

f) Find an equation in the form $N(t) = at^3 + bt^2 + ct + d$ that represents this information. (where N = mosquito population and t = time after the 12^{th} day). Give the values of *a, b, c* and *d.* (*a* to one decimal place and *b, c* and *d* to the nearest integer value)

(3 marks)

g) How long will it take for all the mosquitoes to die off?

(1 mark)

END OF QUESTION BOOKLET