Student NAME:______________________________

2018

XAVIER COLLEGE

MATHEMATICAL METHODS UNIT 4

School Assessed Coursework 3

SAC 3:Problem-Solving Task 2

(Probability and Statistics)

Reading Time: 15 minutes
Writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
5	**5**	**60**

SAC 3 Problem-Solving Task 2 consists of 5 extended-response questions.

- Students are permitted to bring into the assessment room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aides for curve sketching, one bound reference, one approved CAS calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator.
- Students are **NOT** permitted to bring into the assessment room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book, sheet of miscellaneous formulas.
- Working space is provided throughout the book.

Instructions

- Write your **name** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the assessment room.

Mathematical Methods formulae

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A\|B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = \mathrm{E}(X)$	variance	$\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x-\mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$\mathrm{E}(\hat{P}) = p$
standard deviation	$\mathrm{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

COMPULSORY STUDENT DECLARATION:

I, ____, acknowledge that I have read the SAC/examination conditions and understand which items/materials I am permitted to use and have in my possession.
****If you have any doubts as to what is permitted, raise your hand and DO NOT sign this declaration****

Student's signature: ___________________________

Teacher's name: ____

Instructions

Answer ***all*** questions in the spaces provided.
In all questions where a numerical answer is required, an **exact value** must be given ***unless otherwise specified.***
In questions where more than **one mark** is available, appropriate working **must** be shown.

Question 1 **[Total - 12 marks]**

Two divisions of a Norwegian organisation compete in an annual soccer match for bragging rights at the company's Christmas party. The Accounting division of the organisation (ACC) is favourite to defeat their colleagues from the Maintenance division (MAN).

Statisticians have found that if ACC win one year the probability that they win the following year is 0.82, but if MAN win one year the probability that MAN win the following year is 0.6. ACC won last year's game.

a i Represent the possible outcomes for this year's match *and* next year's match on a tree diagram. Include all probabilities. **(2 marks)**

a ii. Find the probability that MAN win this year's game. *Answer required correct to 2 decimal places* **(1 mark)**

a iii. Find the probability that ACC wins next year's match. *Answer required correct to 4 decimal places* **(1 mark)**

Darius is the ACC team's most aggressive player and has a poor reputation for playing outside the rules of the competition. The probability of the number of fouls that Darius commits each game is given in the table below:

f	0	1	2	3	4	5	6
$\Pr(F = f)$	0.1	0.12	a	b	0.3	0.1	0.08

b i Find the probability, *correct to 2 decimal places*, that Darius commits *at least* 2 fouls in the game. **(1 mark)**

b ii. Find the probability, *correct to 3 decimal places,* that Darius will commit a total of 10 fouls in the next 2 years (ie the next 2 games). **(2 marks)**

b iii. If the expected number of fouls, E*(F)*, that Darius commits is 3.1, find the values of *a* and *b*. **(2 marks)**

Statisticians identified that the chances of ACC winning are dependant on the number of fouls that Darius incurs. If Darius controls his aggression and commits *less than* 4 fouls, the probability that ACC wins the game is 0.72. If Darius commits 4 or more fouls, the probability that MAN win the game is 0.8.

c. Find the probability that MAN wins the game, given that Darius commits 3 fouls. Give answer *correct to 2 decimal places.* **(1 mark)**

d. Find the probability, *correct to 3 decimal places*, that Darius committed 4 fouls or more, given that ACC won the game. **(2 marks)**

Question 2 **[Total 7 marks]**

The MAN team is highly tactical and puts a lot of emphasis on how far their players can 'head' a soccer ball. The distance that the MAN players can head the ball is normally distributed with a mean of 18 metres and standard deviation of 3 metres.

a. Find the probability, *correct to 3 decimal places,* that a MAN player can header *at least* a distance of 20 metres. **(1 mark)**

Use your answer to part a for each of the following parts.

b. At a training session 8 of the MAN players are selected at random and asked to head the ball.

Find the probability *correct to 3 decimal places* that:

b i. Only the first 2 players head the ball *at least* 20 metres. **(1 mark)**

b ii Exactly 2 of the players head the ball *at least* 20 metres. **(1 mark)**

b iii. At least 4 of the players head the ball *at least* 20 metres, given the first player chosen head the ball *at least* 20 metres. **(2 marks)**

c. Find the minimum number of MAN players required to ensure that the probability that at least one of them can head the ball at least 20 metres is at least 0.98.

(2 marks)

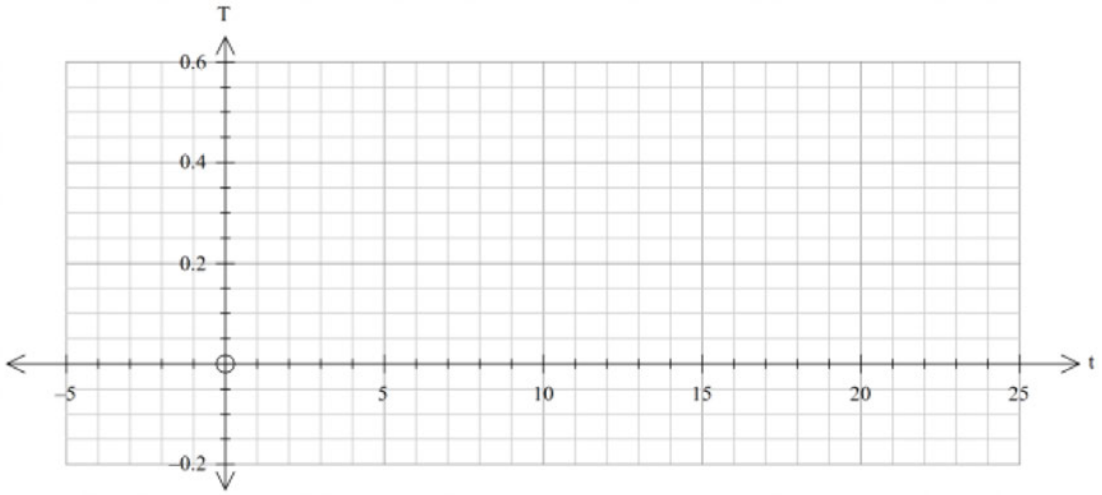
Question 3 **[Total - 10 marks]**

During the 80 minute game, the amount of time in which MAN is **leading** ACC is a continuous random variable described by the probability density function where *t* is in minutes

$$T(t) = \begin{cases} a(8t^2 - t^3), t \in [0,8] \\ 0 \quad elswhere \end{cases}$$

a. show that the exact value of *a* is $\frac{3}{1024}$ **(2 marks)**

b. Sketch the graph of on the axes below, labelling intercepts and turning points correct to 2 decimal places as appropriate. **(3 marks)**



c. Find the probability that MAN is in the lead for between 5 and 8 minutes during the match, *correct to 4 decimal places*. **(1 mark)**

d. Find the mean length of time that MAN lead ACC in these matches, *correct to the nearest second.* **(2 marks)**

e. Find the median length of time that MAN lead ACC, correct to the nearest second. **(2 marks)**

Question 4 **[Total - 18 marks]**

The volume of a particular peptide injection the ACC players take is normally distributed with mean 12 ml and variance 4 ml. The distribution of the volume of these peptides injections is denoted by V.

a. If $Pr(V < 15) = Pr(z < a)$, then find the value of a. **(2 marks)**

b. Find $Pr(9 < V < 13)$ correct to 3 decimal places. **(1 mark)**

c. If one of these peptide injections is selected at random, find the probability that it's volume is less than 13 ml, correct to 3 decimal places. **(1 mark)**

d. If one of these peptide injections is selected at random, find the probability that it's volume is less than 13 ml given that is volume is more than the mean. Answer to 3 decimal places. **(2 marks)**

e. If 5 peptide injections are selected at random, find the probability correct to 3 decimal places that:

i. None will have a mass less than 13 ml **(2 marks)**

ii. Exactly one will have a mass less than 13 ml **(1 mark)**

iii. Exactly one will have a mass less than 13ml given at least one has a mass less than 13 ml **(2 marks)**

200 peptide injections of a different variety are analysed, it is known that 60% of this variety have a volume of less than 13 ml.

f. Use a Normal approximation to the Binomial distribution to calculate the probability that between 55% and 65% of the peptide injections from this group of 200 have a volume of less than 13 ml. *Answer to 2 decimal places*.
(2 marks)

A third variety of peptide injections is considered. The volume of this variety is normally distributed with mean 15 ml and the variance is unknown.

g. If the probability of a randomly selected peptide injection from this group having a volume greater than 12 ml is 0.7, find the value of σ correct to two decimal places. **(2 marks)**

A sample of 50 peptide injections is randomly selected from the population of **all** varieties of peptide injections. The sample proportion $\hat{p}$ of peptide injections which have a volume of more than 12 ml is found to be 0.8.

h. Find an approximate 95% confidence interval for the proportion p of peptide injections which have a volume of more than 12 ml based on this sample. Answer correct to 4 decimal places. **(2 marks)**

i. Calculate the margin of error for the 95% confidence interval for this estimate of p. Express your answer as a percentage to the nearest percent. **(1 mark)**

Question 5 **[Total - 13 marks]**

The MAN team is concerned about their ability to perform in a penalty shootout. The coach puts all of his players through a training session where they take 20 penalty shots.

a. Amir is one of the MAN team weaker players and it is known that the probability that he scores on any given penalty kick is 0.4.

Let **G** be the number of goals that Amir kicks in his training session.

i. Find the number of goals that Amir is expected to kick **(1 mark)**

ii. Find the standard deviation in the form $\frac{a\sqrt{b}}{c}$ where a, b and c are integers. **(2 marks)**

b. Amir has 3 other teammates with the same probability of scoring a penalty goal.

i. Find the total number of goals that this group, including Amir, is expected to kick **(1 mark)**

ii. Find the exact variance for the total number of goals that this group kicks. **(1 mark)**

c. Find the probability that Amir kicks **9 goals** at the training session, *answer correct to 3 decimal places.* **(1 mark)**

d. The coach considers that a satisfactory pass result on the training session as getting *at least* 15 out of the 20 goals. Find that probability that Amir gets a satisfactory pass result. *Answer correct to 4 decimal places*.
(1 mark)

e. Find the least number of kicks that Amir needs to take to ensure that the probability of him getting ***at least*** two goals is ***at least*** 95%. **(3 marks)**

f. Another player Taufik has an even chance of kicking the penalty goal or missing it. Find the expected number of penalty kick training sessions (*to the nearest whole number*) he would need to go to in order to get a satisfactory pass result from the coach. **(3 marks)**

END OF TASK BOOK

Additional Working Space (*if needed*)

Assessment Grade for Task

This grade is subject to statistical moderation at the Victorian Curriculum and Assessment Authority (VCAA) and is likely to change.