XAVIER MATHS

Mathematical Methods SAC 1 (2019)

APPLICATION TASK

The Roller Coaster Ride

Booklet 1 – 20th May

3:45 – 5:55

10 minutes reading time

120 minutes writing time

This task is to be completed in one session of duration 130 minutes.
During this task, you may use your calculator and refer only to your Application SAC Preparation Booklet and one other set of bound notes. No other pieces of paper may be used.
You must work silently and independently for the duration of this task.
All answers are to be written within this booklet.
When drawing graphs, ensure that a pencil is used and that significant features of the graph are clearly indicated in pen.
Exact values are expected throughout, unless otherwise stated and units must be included.
Your CAS calculator may be collected at the conclusion of the SAC so that all memories can be cleared, and will be returned to you in your first Mathematical Methods class.
No electronic devices (such as mobile phones) may be brought into the examination room.

Total : 70 marks

Student Name : ______________________________

Teacher Name: ______________________________

The grade awarded to this SAC is subject to statistical moderation

by the VCAA and is likely to change.

COMPULSORY STUDENT DECLARATION

I, (*print your name neatly*) ________________________________
acknowledge that I have read the SAC/examination conditions and understand which items/materials I am permitted to use and have in my possession.

****If you have any doubts as to what is permitted, raise your hand and DO NOT sign this declaration****

Student's Signature: ________________________________

Student's Name: ________________________________

Teacher's Name: ________________________________

Mathematical Methods formulas

Mensuration

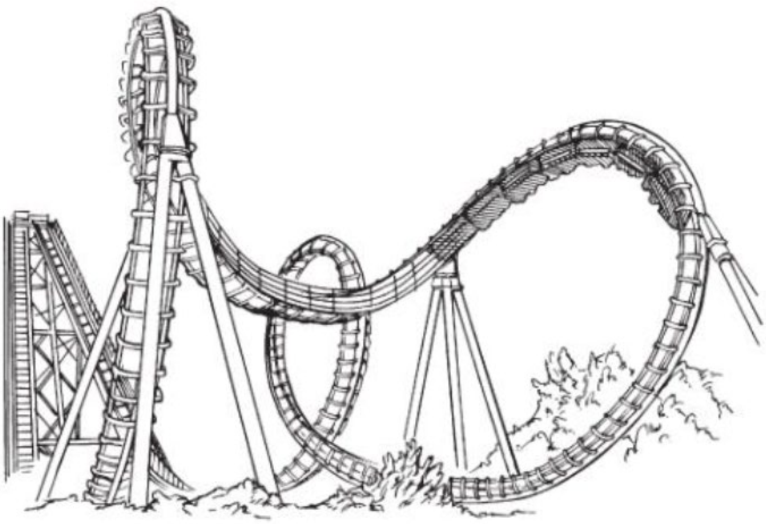
area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$		$\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$	
$\frac{d}{dx}\left((ax+b)^n\right) = an\left(ax+b\right)^{n-1}$		$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, n \neq -1$	
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$		$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$	
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$		$\int \frac{1}{x} dx = \log_e(x) + c,\ x > 0$	
$\frac{d}{dx}\left(\sin(ax)\right) = a\ \cos(ax)$		$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$	
$\frac{d}{dx}\left(\cos(ax)\right) = -a\ \sin(ax)$		$\int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$	
$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\ \sec^2(ax)$			
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$		

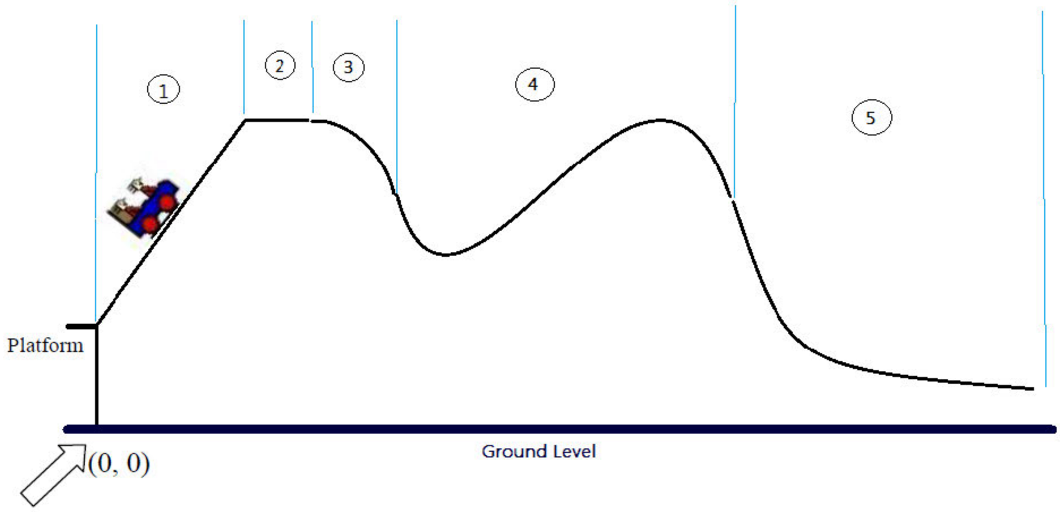
SECTION 1

A Gold Coast "theme park" is going to develop a popular amusement ride – roller coaster. Two likely lads from Xavier, Linus and Ollie, have offered to investigate the design and proposal of the roller coaster project.



roller coaster

The following sketch represents a number of sections of the design for the track, which is going to be constructed in this project.



Each labelled section of the track can be modelled as a mathematical function. For safety purpose, all sections from section 2 need to be **gapless and jointed** smoothly which means the gradient on both sides is the same. When describing the domain of the different sections the first point is to be included but the last point should be excluded. The first and last points of the entire track are to be included.

Question 1

The roller coaster train starts at the platform, which is 5 metres high above the ground, and will climb up to a height of 25 metres through the straight track as labelled, section 1, in the diagram. When the train reach the highest point of the slope, it has a horizontal distance of 16 metres from the starting point.

Assume the ground level is **x-axis** and the height of the train above ground level is described by **y-coordinate.** The point of origin has been given.

a. Determine the coordinates of the starting point and the highest point of the slope.

2 marks

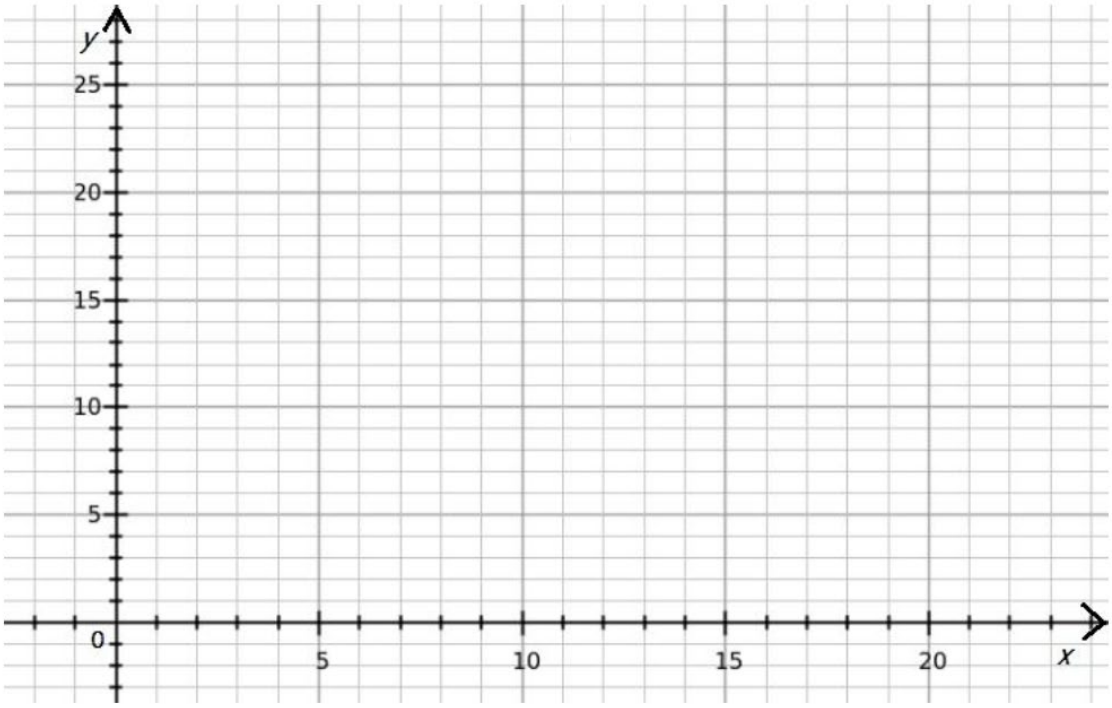
b. If the section 1 of the track can be modelled as a linear function, $y = mx + c$, find the positive constant m and c, and specify the restricted domain of the function.

3 marks

c. Find the angle of this section of the track to the horizontal line. Give the answer in degrees correct to one decimal place.

2 marks

d. Sketch the graph of the function found in ***b.*** Label the coordinates of the starting point and the end point.



2 marks

e. If the average speed during the train climbing up the slope is 4.5 m/s, how long does it take for the train to climb up the top from the starting point? Give your answer in seconds correct to one decimal place.

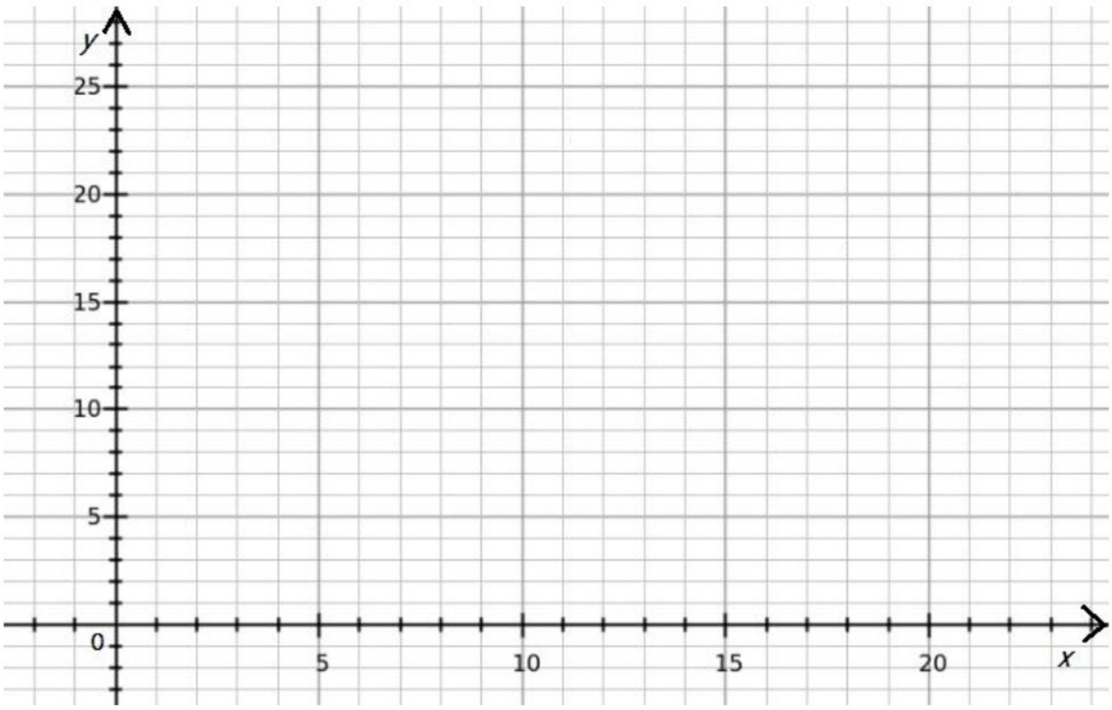
3 marks

Section 2 in the diagram refers to a flat track of the roller coaster project. It gives the passengers a short calm period, before the scary and exciting experience in the rest of the ride. The flat section joints the slope with a sharp corner, but because of the slow speed during these sections, it is still safe, even though the train would still bump when it moves onto the flat track. The length of the flat section is 5 metres.

f. Give a function describing the flat track, and specify its domain.

2 marks

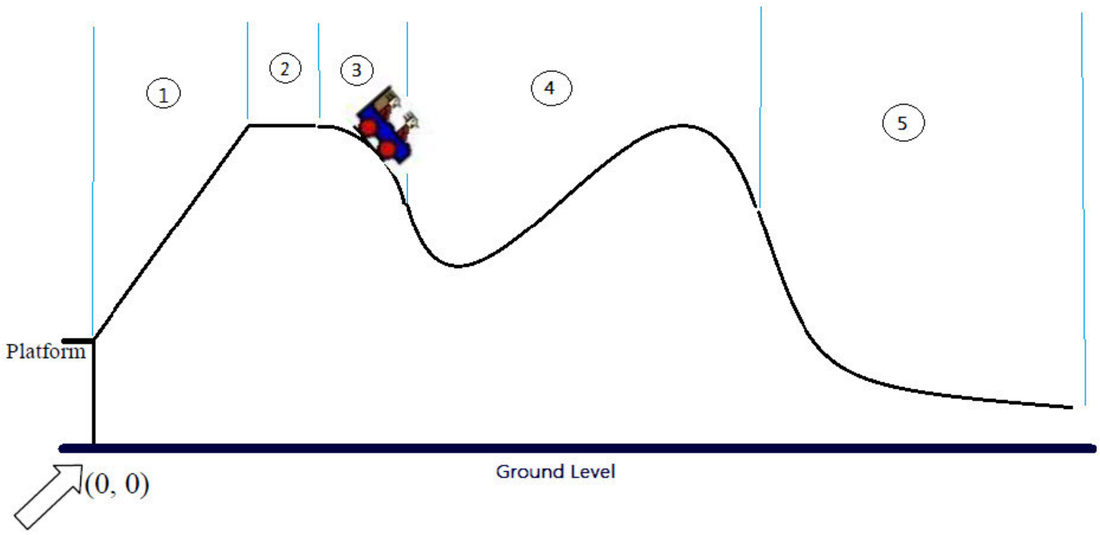
g. Sketch the graph of the function found in ***f.*** Label the coordinates of the starting point and the end point.



2 marks

Question 2

After the end of ride in section 2, the roller coaster train will accelerate and travel in a very fast manner. In order to be safe, there is **no gap** between each section of the track, and they join each other **smoothly.**



Research shows that the shape of the track in section 3 can be modelled as a quadratic function. It passes and also finishes at a point (26, 18).

a. Find the quadratic function, *f(x)*, in the turning point form.

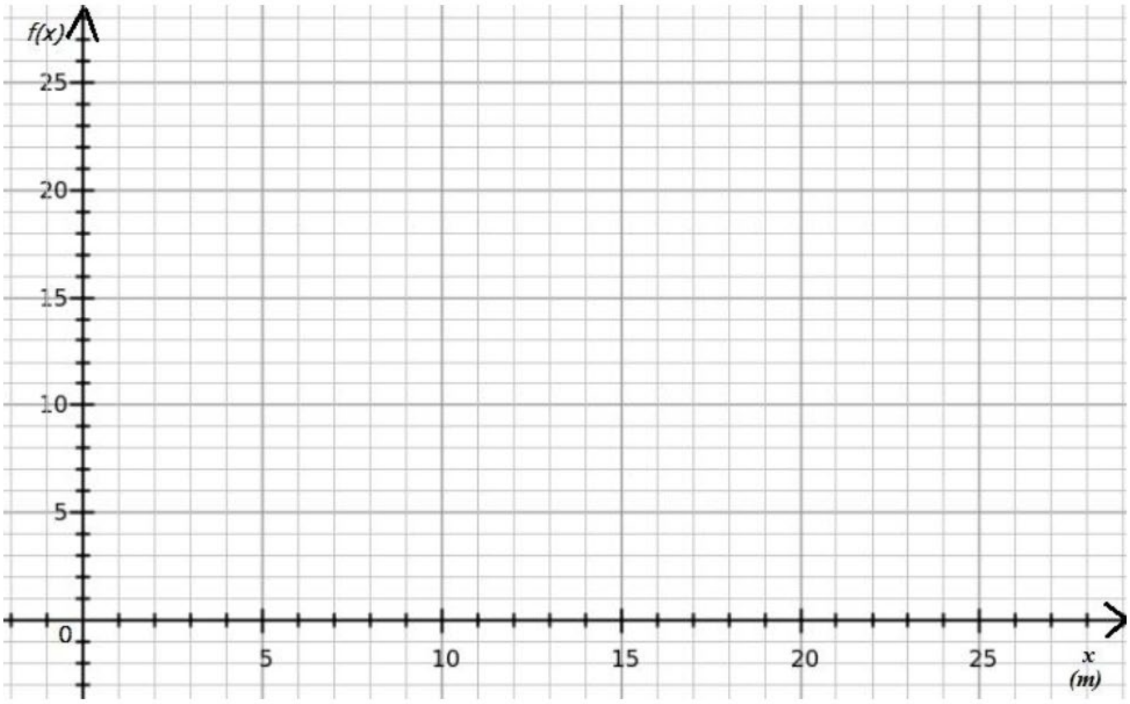
3 marks

b. Write the general equation for the quadratic function found in part **a**,

as $y = ax^2 + bx + c$

2 marks

c. Sketch the graph of the function found in ***part a, within the domain.*** Label the coordinates of the starting point and the end point.



2 marks

d. Find the gradient of the track when passing through a position, which has a horizontal distance of 24 metres. Give your answer in degrees measured from the positive direction of the horizontal axis. Give your answer correct to one decimal place.

3 marks

e. When comparing the function *f(x)* found in part ***a***, to the function $y = x^2$. Describe the sequence of transformations required to obtain *f(x)*.

3 marks

f. Use matrix form to describe the transformations as stated in part ***f.***

3 marks

The section 4 of the track passes through positions: J (m, 18), P (36, 24), and Q (40, 19).

g. State value of m:

1 mark

h. The track of the roller coaster in section 4 can be represented by the function

$$g(x) = ax^3 + bx^2 + cx + d$$

Show that the values are $a = \frac{-661}{19600}$, $b = \frac{4052}{1225}$, $c = \frac{-10641}{100}$ and $d = \frac{279633}{245}$

(Remember the track sections join smoothly.)

4 marks

i. Find the maximum and minimum height the roller coaster train will pass through within the section 4 of the ride. Answer given to two decimal places.

2 marks

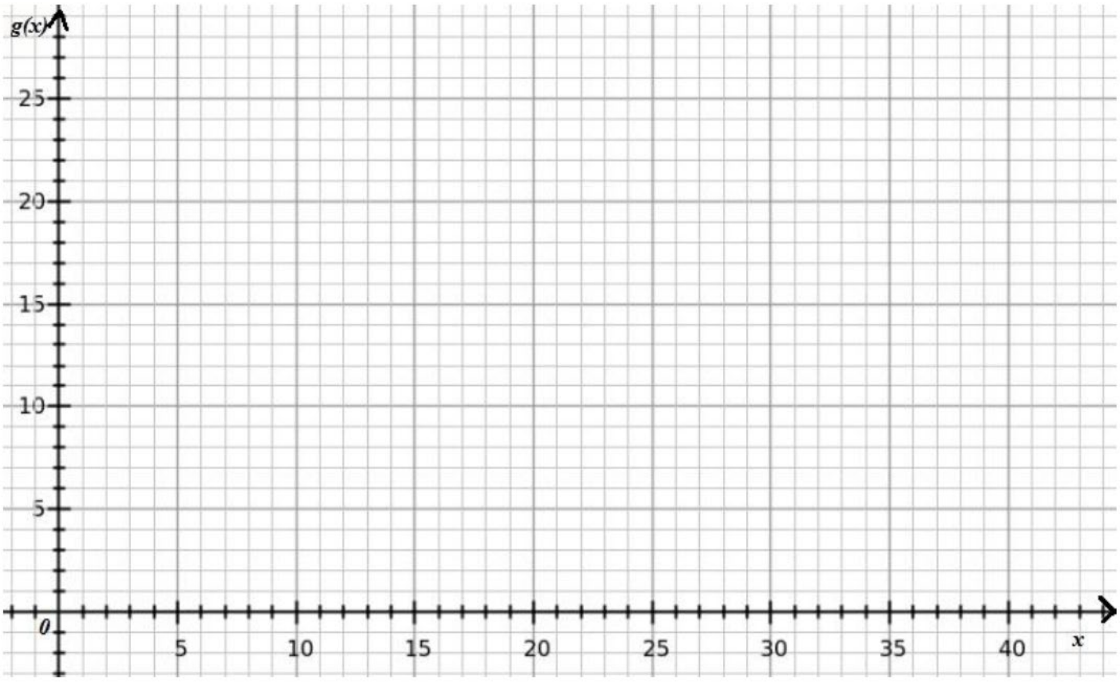
j. Find the value of n, the point where section 4 and section 5 meet, that is, $(41, \text{n})$. Give your answer to two decimal places.

1 marks

k. Specify the domain and range of the function $g(x)$.

2 marks

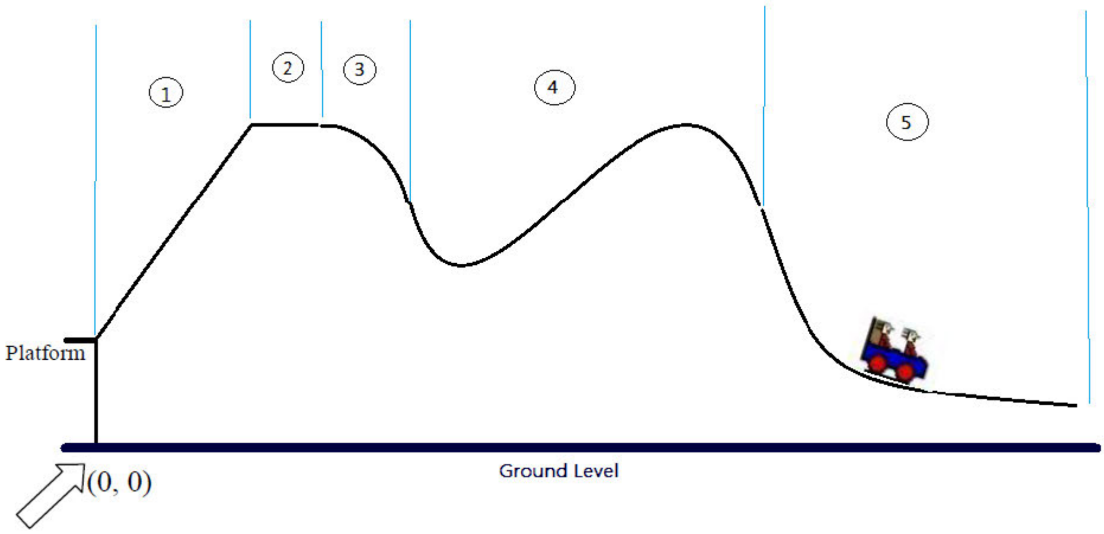
l. Sketch the graph of the function *g(x)*, ***within the domain.*** Label the coordinates of turning points, the starting point and the end point.



3 marks

Question 3

Section 5 of the track, as shown in the following schematic diagram, is the last one to be discussed in this project.



Another student, Brodie, suggests that this part of the track should be modelled as a hyperbolic function,

$$h(x) = \frac{40.4}{x - 38.225}$$

a. If the section 5 ends at the position, when it is 3 metres above the ground, find the horizontal distance of the end point from the origin. Give your answer correct to 2 decimal places.

2 marks

According to a scientific research, the roller coaster cart can be safely stopped only when the gradient of the track is less than 5° to the ground level.

b. If the cart needs to stop safely at the end point of the section 5, then we need to extend the track, when it still follows the hyperbolic model. How much further in horizontal axis does the section need to be extended? Give your answer correct to 2 decimal places.

3 marks

The boys, Linus, Ollie and Brodie are proud of their achievements but they are reminded by another student, Darcy, that they must complete the following questions.

Question 4

Consider the simultaneous linear equations

$$ax + 3y = b - 3$$
$$3x + (a + 8)y = -b$$

where *a and b* are real constants.

a. If there is no real solution for these simultaneous equations, find values of *a* and *b*.

4 marks

b. Find the value of *a and b,* where there is unique solution.

1 marks

c. Find the value of *a and b,* where there are infinite solutions.

2 marks

Question 5

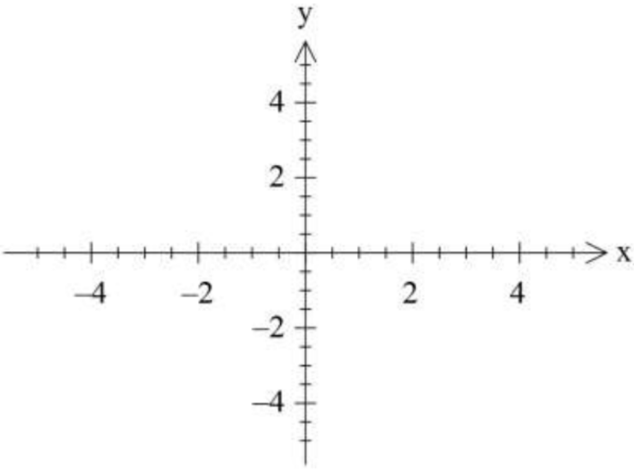
Define functions $p:[2,\infty)\to R, p(x)=(x-2)^2+1$

and $q:R\setminus\{1\}\to R, q(x)=\dfrac{1}{x-1}+3$

a. Give the rule and the domain of the function p^{-1} .

2 marks

b. Sketch p **and** p^{-1} on the axis provided below. Use co-ordinates to label the endpoints and any point(s) where the two graphs intersect. Round to 2 decimal places.



3 marks

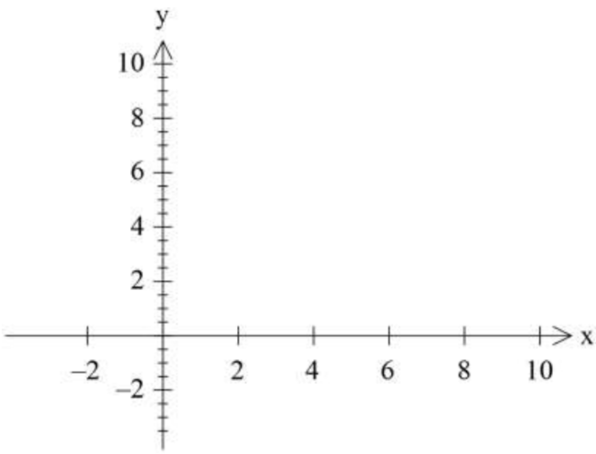
c. Show that $p(q(x))$ is **not** defined

2 marks

d. Restrict the domain of q to the largest possible domain such that $p(q(x))$ is defined, and state the rule for $p(q(x))$.

3 marks

e. Sketch the composite function on the axis provided below. Clearly label any vertical asymptotes, axis intercepts and endpoints.



3 marks

End of Part 1