Student NAME:______________________________

2019

XAVIER COLLEGE

MATHEMATICAL METHODS UNIT 4

School Assessed Coursework 3

Reading time concludes	
Writing time Commenced	
End of writing time	

SAC 3: Problem-Solving Task 2

(Probability and Statistics)

Reading Time: 10 minutes

Writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
5	**5**	**60**

SAC 3 Problem-Solving Task 2 consists of 5 extended-response questions.

- Students are permitted to bring into the assessment room: one bound reference booklet and SAC Preparation booklet, pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aides for curve sketching, one bound reference, one approved CAS calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator.
- Students are NOT permitted to bring into the assessment room: loose blank sheets of paper and/or correction fluid/tape.
- You must work silently and independently for the duration of this task.
- All answers are to be written within this booklet in the spaces provided.
- Questions worth more than one mark require some explanation of working or process.
- **Exact values** are expected throughout, **unless otherwise stated**.
- You will be required to clear all **CAS** memory at the conclusion of the SAC
- No electronic devices (such as mobile phones, smart watches or any other unauthorised electronic devices) may be brought into the examination room.
- Questions seeking clarification may occur in reading time only

Materials supplied

- Question and answer book, sheet of miscellaneous formulas.
- Working space is provided throughout the book.

Mathematical Methods formulae

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A \mid B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = \mathrm{E}(X)$	variance	$\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x-\mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$\mathrm{E}(\hat{P}) = p$
standard deviation	$\mathrm{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

COMPULSORY STUDENT DECLARATION

I, (*print your name neatly*) ____

acknowledge that I have read the SAC/examination conditions and understand which items/materials I am permitted to use and have in my possession, and will undertake to **clear the memory of my CAS calculator at the conclusion of the SAC.**

****If you have any doubts as to what is permitted, raise your hand and DO NOT sign this declaration****

Student's Signature: ____

Student's Name: ____

Teacher's Name: ____

Instructions

Answer ***all*** questions in the spaces provided.
In all questions where a numerical answer is required, an **exact value** must be given ***unless otherwise specified.***
In questions where **more** than **one mark** is available, appropriate working **must** be shown.

Question 1

Simon and Gavin play tennis each week. In any one game, Simon has the probability of winning of 0.7. The outcome of any one game between the two is independent of the outcome of the previous games played.

a. If they play 8 games, what is the probability, to 4 decimal places, that Simon wins 5 out of the 8 games?

2 marks

b. What is the probability, to 4 decimal places, that Gavin wins **at most** 6 of the 8 games?

2 marks

c. What is the probability, to 4 decimal places, that Simon only wins the second, fourth, fifth, seventh and eighth games?

2 marks

d. Compare the value obtained in part (a) with that of part (c). Explain the result.

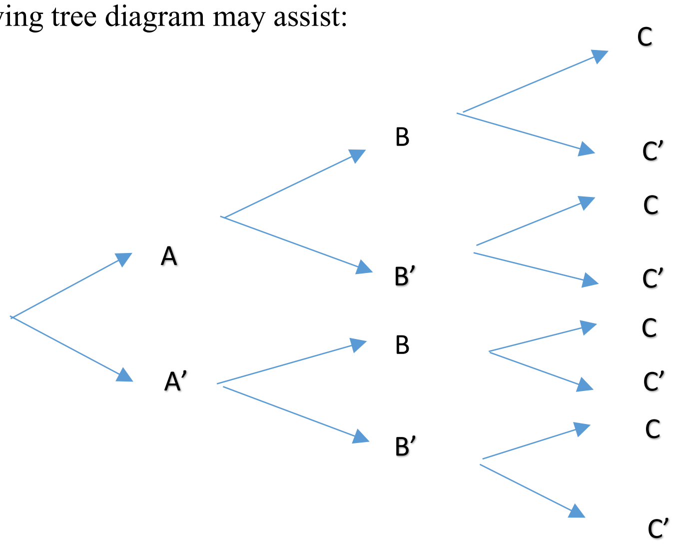
1 mark

e. How many games do they need to play so that Gavin has at least 80% chance of winning **at least** one game?

3 marks
Total 10 marks

Question 2
There are three ovals located within a large school; Alphington Oval, Bennett Oval and Cordner Oval. These ovals are sought for the scheduling of training and competition games but may be actually unused on any particular day. The probability that Alphington Oval is unused is 0.3, that Bennett Oval is unused is 0.4 and that Cordner Oval is unused is 0.5.

The following tree diagram may assist:



a. What is the probability, to 2 decimal places, that on any particular day:

(i) no oval is **unused**

1 mark

(ii) one oval is **unused**

2 marks

(iii) two ovals are **unused**

2 marks

(iv) three ovals are **unused**

1 mark

b. Let X represent the discrete random variable, which counts how many of the three grounds are **unused** on a particular day.

(i) Specify, correct to 2 decimal places, the probability distribution of X.

x				
$\Pr(X = x)$				

1 mark

(ii) Find the mean of X, correct to 1 decimal place.

2 marks

(iii) Find the standard deviation of X, correct to 4 decimal places.

3 marks

(iv) Find the mode of X.

1 mark

c. Find the probability, correct to **2 decimal places**, that on a particular day:

(i) at least one ground is **unused**

2 marks

(ii) if Alphington Oval is unused, it is the only oval **unused**

2 marks

(iii) exactly one oval is unused, given at least one oval is **unused**

2 marks

Total 19 marks

Question 3
The resistance of heating elements produced by a well-known major electrical firm, Sparks Limited, are found to be normally distributed with mean 50 ohms and standard deviation of 4 ohms.

a. If the required specifications demand that acceptable elements shall have a resistance between 45 and 55 ohms, find the probability, to 4 decimal places, that a randomly selected element has these specifications.

2 marks

b. Find the **least** resistance, correct to 2 decimal places, that 85% of heating elements would have?

2 marks

c. The profit on an acceptable element, that is, one whose resistance is within the specific limits, is \$2, while unacceptable elements result in a loss of \$0.50.

If P dollars is the profit in a randomly selected element produced by the firm, find the **mean** and **variance** of P. Give answers to 2 decimal places.

3 marks

d. A student in the class has left his calculator at home and therefore must make use of the established confidence limit rules to find the nearest whole percentage of the heating elements that would be expected to have resistances between 42 ohms and 54 ohms.

2 marks
Total 9 marks

Question 4

ONOnet is an new internet provider company, that currently has two available internet access plans:

- NBN plan
- ADSL 2 plan

80% of customers have the NBN plan and 20% of customers have the ADSL 2 plan.

a. If 10 customers of **ONOnet** are selected at random,

i. what is the probability, correct to 4 decimal places, that exactly eight customers are on the NBN plan?

2 marks

ii. How many customers would be expected to be on the NBN plan?

1 mark

b. The marketing for ONOnet has decided that the monthly hours used by NBN customers is a random variable with a probability density function given by

$$f(t) = \begin{cases} 0.05e^{-0.05t} & 0 \le t \le 720 \\ 0 & \text{elsewhere} \end{cases}$$

i. Find the mean number of hours used in a month by a randomly selected NBN plan customer.

2 marks

ii. Find the median number of hours, correct to two decimal places, used in a month by a randomly selected NBN plan customer.

2 marks

iii. What percentage, correct to the nearest per cent, of NBN customers uses more than 30 hours in the month?

2 marks

iv. What is the probability, correct to three decimal places, that a randomly chosen NBN plan customer uses less than 40 hours per month, given that the customer uses more than 30 hours per month?

2 marks

Total 11 marks

Question 5

A new soft drink is about to be launched by the **Columbia Cola Company**, and it is taste tested by a random sample of people. It was found that 81% of people sampled said that they preferred it over the other soft drinks made by the company. It was also found that the standard deviation of this sample was 5.4%.

a. Find the size of the sample.

2 marks

b. Give a 99% confidence interval, correct to 2 decimal places, for the proportion of people who prefer the new soft drink.

2 marks

A second sample of 500 people is taken, for which the standard deviation is found to be 1.91%.

c. What is the sample proportion, correct to 2 decimal places, assuming it is over 50%?

2 marks

d. Assuming that $\hat{p} = 0.76$, if the upper limit of a confidence interval is 0.7914, find the percentage confidence interval

2 marks

e. By how much would the company need to increase the size of the sample in order to achieve a margin of error of 1.9% for the confidence interval and proportion found previously? Give answer to the nearest 10.

2 marks
Total 10 marks

End of Component 2

END OF TASK BOOK

Additional Working Space (*if needed*)

AT THE CONCLUSION OF THE SAC IT IS YOUR REPONSIBILITY TO CLEAR YOUR CAS CALCULATOR'S MEMORY

FAILURE TO DO SO IS A BREACH OF THE SAC CONDITIONS

STEP-WISE PROCEDURE

Press on

Select 2 My Documents

Press menu

Select C:Delete All

Choose OK

Assessment Grade for Task

This grade is subject to statistical moderation at the Victorian Curriculum and Assessment Authority (VCAA) and is likely to change.