Mathematical Methods Unit 3 2020

APPLICATION TASK
SAC

Part 1

10 minutes reading time, plus 120 minutes writing time.

- During this task, you may use your calculator and refer to your own bound prepared notes and the preparation booklet only
- You must work independently for the duration of this task.
- All answers are to be written within this booklet
- Questions worth more than one mark **require** adequate working out
- When drawing graphs, ensure that a pencil is used and that significant features of the graph are clearly indicated in pen.
- **Exact values are expected throughout, unless otherwise stated and units must be included.**
- No electronic devices (such as mobile phones) may be used during the task

Total : 50 marks

Student Name : ________________________________

Teacher Name: ________________________________

The grade awarded to this SAC is subject to statistical moderation by the VCAA and is open to change.

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$		$\int x^n dx=\frac{1}{n+1}x^{n+1}+c,\ n\neq -1$	
$\frac{d}{dx}\left((ax+b)^n\right)=an\left(ax+b\right)^{n-1}$		$\int (ax+b)^n dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c, n\neq -1$	
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$		$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$	
$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$		$\int \frac{1}{x}dx=\log_e(x)+c,\ x>0$	
$\frac{d}{dx}\left(\sin(ax)\right)=a\ \cos(ax)$		$\int \sin(ax)dx=-\frac{1}{a}\cos(ax)+c$	
$\frac{d}{dx}\left(\cos(ax)\right)=-a\ \sin(ax)$		$\int \cos(ax)dx=\frac{1}{a}\sin(ax)+c$	
$\frac{d}{dx}\left(\tan(ax)\right)=\frac{a}{\cos^2(ax)}=a\ \sec^2(ax)$			
product rule	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$		

Question One (7 marks)

Ruby runs an electric car company. She finds that the cost of running one electric car at a constant speed of v km/hr follows the following equation

$$C(v) = 30 - \frac{195}{238}v + \frac{13}{29750}v^3$$

where C is the cost in dollars per hour.

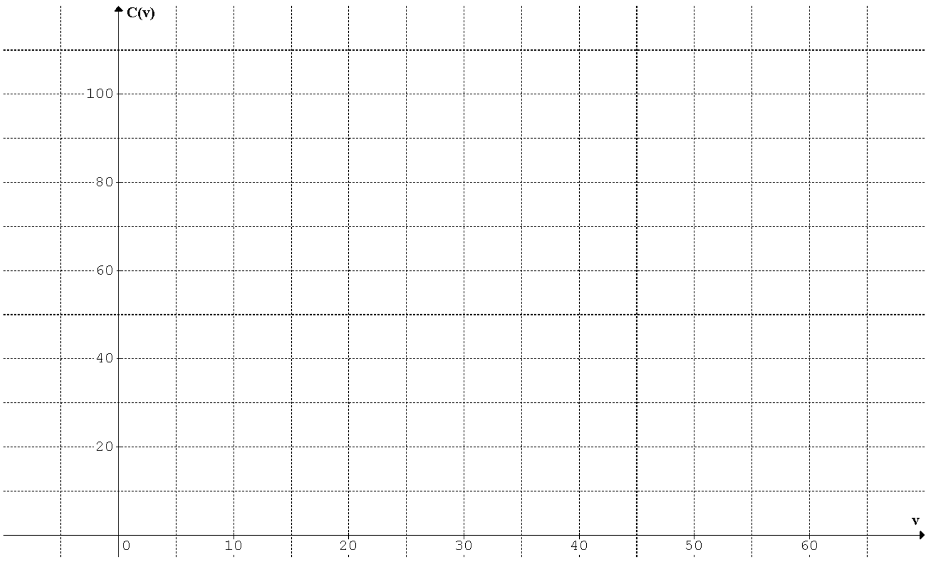
a. Find the cost of running an electric car at a constant speed of 50 km/ hr for 3 hours. Give your answer correct to the nearest cent. **1 mark**

b. Find the cost of running an electric car for a journey of 200 km travelling at a constant speed of 40 km/ hr. Give your answer correct to the nearest cent. **2 marks**

c. **i.** What speed gives the least cost per hour? **1 mark**

ii. What is this cost per hour? Give the cost correct to the nearest cent. **1 mark**

d. Sketch the graph of $C(v)$ for $0 \leq v \leq 60$. Include the coordinates of any axes intercepts, endpoints or stationary points, correct to two decimal places. **2 marks**



Question Two (11 marks)

The wind turbine pictured is situated on Jim Trant's farm in Western Victoria. At its maximum speed the blade rotates at a speed of one full rotation every 6 seconds. At this speed it generates 3.4 megawatts of energy from each rotation. Its blade lengths are 50 metres long and it is built on a pillar that is 110 metres high – hence the central point is 110 metres of the ground. The tip of one blade is marked with a dot in red paint.

Assume the initial position of the blade with the red dot is its highest vertical position and t is time in seconds. Assume the bottom of the wind turbine pillar is at height 0 metres. Also assume the turbine is working at its maximum speed.

a. Write an equation relating the height of the red dot from the ground, h, to t in the form $h = a\cos(nt) + b$ **2 marks**

b. Write down the maximum and minimum heights of the red dot **2 marks**

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c. How much energy, in megawatts, does the turbine produce in one hour? **1 mark**

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d. Jim has installed a camera at a point on each pillar which films the rotors as they turn. The camera detects when the red dot is less than 85 metres above the ground. For how many seconds does the camera detect the red dot being below 85 metres in the first two rotations? **2 marks**

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e. Find the speed of the red dot in **km/ hr** when the turbine is spinning at maximum speed. **2 marks**

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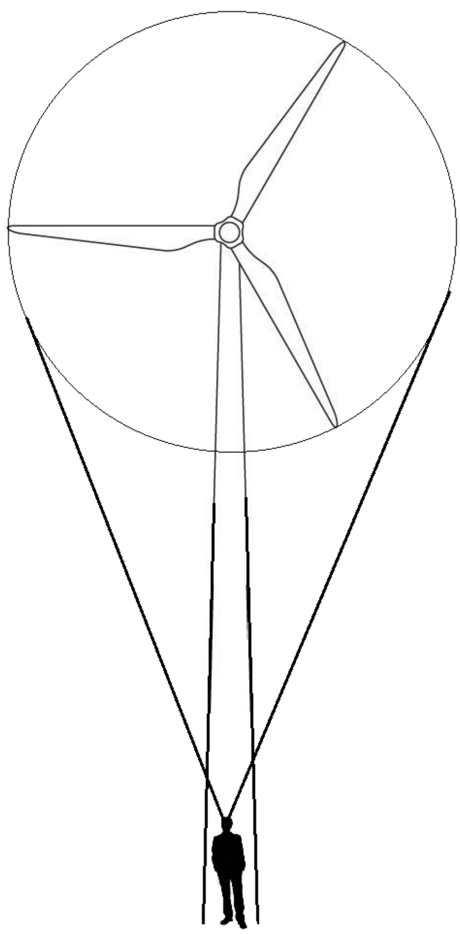
f. Jim, whose eyes are 2 metres above the ground, is situated at point Y, directly at the base of the pillar. For what percentage of the rotation of the turbine can Jim see the red dot on the end of the blade? Give your answer as a percentage correct to two decimal places. **2 marks**



Question Three (8 marks)
The population of mosquitoes has started to increase at Lake Pertobe. Local scientists note that the number of Mosquitoes, M, has been increasing exponentially according to the rule $M = M_0 e^{kt}$, where M_0 is the number of mosquitoes at Lake Pertobe at the start of 2015, t is time in years and k is a constant. 2 years after they began measuring the population, the scientists estimated there were 22 000 mosquitoes.

a. **i.** Show that $k = \frac{1}{2}\log_e\left(\frac{22\,000}{M_0}\right)$ **1 mark**

ii. If the number of mosquitoes at the Lake was measured at 35 000 halfway through 2018, find the value of k, giving your answer correct to 4 decimal places and the value of M_0 correct to the nearest whole number. **Use these values of M_0 and k for the remainder of this question**. **2 marks**

iii. How many years after the beginning of 2015 will the population have risen to 100 000? Give your answer in years correct to two decimal places **1 mark**

b. At a certain point in time, the mosquito population has increased to M_1 mosquitoes. Local environmentalists, led by the active Bel Perk, take it upon themselves to try to control the mosquito population. They introduce a new species of newt that feed on mosquitoes. It is expected that the introduction of newts will cause the mosquito population to decrease by 10% per year

i. Using this model, how long after the introduction of newts will the mosquito population be halved? Give your answer in years correct to two decimal places **2 marks**

On introduction, the number of newts is $\frac{1}{20}$th of the mosquito population, M_1, and the newt population will increase by 8% per year.

ii. Using this model, how long after the introduction of the newts will the population of newts be greater than the population of mosquitoes? Give your answer in years, correct to two decimal places, **2 marks**

Question Four (12 marks)

Well known Maths analyst, Nancy Frigro, has been studying carbon emissions for a long time. She believes that Australia has adopted a policy starting in 2021 where carbon emissions per capita on average follow the trend which is modelled by

$$E(t) = 9\boldsymbol{e}^{0.2(1-t)} + 4$$

where E is the carbon emissions per capita in metric tonnes per year and t is time in years since January 1 2021.

a. **i.** What was the predicted level of carbon emissions per capita on January 1 2021 (i.e $t = 0$)? Give your answer correct to two decimal places. **1 mark**

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ii. After how many years were the predicted level of carbon emissions per capita reduced by 30%? Give your answer in years correct to two decimal places. **2 marks**

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Saul Pallis has been working on his own model for a number of years and he was intrigued by Nancy's work. However, he argued that the inverse function, $E^{-1}(t)$, became a better model of the carbon emission trend from January 1 2026.

b. **i.** Find the *rule* for the function that Saul is using as his model. **2 marks**

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ii. Using Saul's model what were the predicted carbon emissions per capita, correct to two decimal places, on January 1 2026? **2 marks**

iii. Using Saul's model, how many years after January 1 2026 were the predicted carbon emissions reduced by 80% from the original figure predicted in 2026? Give your answer in years correct to two decimal places. **2 marks**

c. When do Saul's model and Nancy's model predict the same figure for carbon emissions per capita? Give your answer correct to the nearest month. **2 marks**

d. Old Mr Croft studied both models and explained that Saul's model had a major long term fault when compared to Nancy Frigro's model. Explain the logic that Old Mr Croft was using to explain his findings. **1 mark**

Question Five (12 marks)

Hobert Rume is designing a series of logos for a Mathematical company. He decides on the graph of $f(x) = -3\sqrt{2x - 4} + 1$ as a starting point.

a. State a series of transformations that would take the graph of $g(x) = \sqrt{x}$ onto $f(x)$ **3 marks**

b. Hobert needs to use some advanced software in order to improve his design. In order to gain access to the software, Hobert is asked a riddle by a colleague in his office, "Consider the simultaneous equations

$$mx + y = 2$$
$$2x + (m-1)y = -4$$

find the value(s) of *m* for which this system of equations would have…

i. A unique solution **3 marks**

ii. No solution **1 mark**

iii. Infinitely many solutions **1 mark**

c. Once he has gained access, Hobert tries to print his logo, but when he goes to the printer, it says he needs to solve the following mathematical problems in order to print.

A function *f* is defined as $f: (-a, \infty) \rightarrow \mathbb{R}, f(x) = k \log_e(x + a) + c$ where *k*, *a* and *c* are **positive** constants. The graph of *f* has a vertical asymptote at $x = -3$ and has a *y*-intercept at (0, 1) There is a point on the graph with coordinates (*d*, 11)

i. State the value of *a*. **1 mark**

ii. Find the value of *c* in terms of *k*. **1 mark**

iii. Find *k* in terms of *d*. **1 mark**

iv. If $d = 2\boldsymbol{e} - 1$, find the value of k, correct to four decimal places. ($\boldsymbol{e}$ is Euler's number in this case) **1 mark**

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END OF PART ONE