Mathematical Methods Unit 3 2020

APPLICATION TASK
SAC

Part 2

10 minutes reading time, 120 minutes writing time

- During this task, you may use your calculator and refer to your own bound prepared notes and the preparation booklet only
- You must work independently for the duration of this task.
- All answers are to be written within this booklet
- Questions worth more than one mark **require** adequate working out
- When drawing graphs, ensure that a pencil is used and that significant features of the graph are clearly indicated in pen.
- **Exact values are expected throughout, unless otherwise stated and units must be included.**
- No electronic devices (such as mobile phones) may be used during the task

Total : 50 marks

Student Name : ____

Teacher Name: ____

The grade awarded to this SAC is subject to statistical moderation by the VCAA and is open to change.

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$		$\int x^n dx=\frac{1}{n+1}x^{n+1}+c,\ n\neq -1$	
$\frac{d}{dx}\left((ax+b)^n\right)=an\left(ax+b\right)^{n-1}$		$\int (ax+b)^n dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c, n\neq -1$	
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$		$\int e^{ax} dx=\frac{1}{a}e^{ax}+c$	
$\frac{d}{dx}(\log_e(x))=\frac{1}{x}$		$\int \frac{1}{x}dx=\log_e(x)+c,\ x>0$	
$\frac{d}{dx}(\sin(ax))=a\ \cos(ax)$		$\int \sin(ax)dx=-\frac{1}{a}\cos(ax)+c$	
$\frac{d}{dx}(\cos(ax))=-a\ \sin(ax)$		$\int \cos(ax)dx=\frac{1}{a}\sin(ax)+c$	
$\frac{d}{dx}(\tan(ax))=\frac{a}{\cos^2(ax)}=a\ \sec^2(ax)$			
product rule	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$		

Question One (12 marks)

Due to the changes in environmental conditions Daz, the fruit and vegetable farmer, has designed a new greenhouse that maximises production. The temperature in his greenhouses is given by the function

$$H(t) = 28 + 2.5\sin\left(\frac{\pi}{6}\left(t - \frac{5}{3}\right)\right)$$

Where H is in degrees Celsius and t is the number of hours after midnight.

a. **i.** Write down the minimum temperature in the greenhouse **1 mark**

____

____

ii. Find the smallest value of t such that the temperature in the greenhouse is a minimum. **Give an exact answer.** **2 marks**

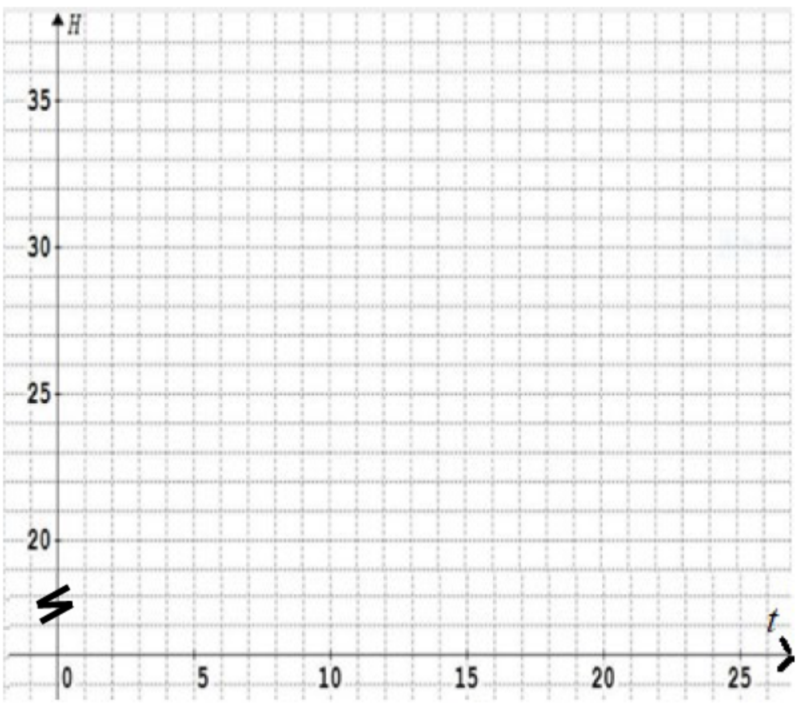
____

____

____

____

b. Sketch the graph of $H(t)$ for $0 \le t \le 24$. Labelling end points, axes intercepts and any maximum and minimum points with their coordinates correct to two decimal places **3 marks**



c. Daz has found that the optimal conditions for fruit and vegetable growing occur when the temperature is above 29.25°. For what times of the day are conditions for fruit and vegetable growing optimal? **3 marks**

d. Daz finds that the greenhouse is inefficient when the rate of change of temperature with respect to time is more than +1° Celsius per hour. Find the times of day for which the greenhouse is inefficient during the course of a day. Give your answer correct to the nearest minute. **2 marks**

e. Daz wishes to make sure his greenhouse is never inefficient (as described in **d**.). Write down a new model for $H(t)$ that would make this possible. **1 mark**

Question Two (5 marks)

The spread of a mutated virus through a population is such that the proportion $M(t)$ of the population which has the virus *t* days after its introduction into the population is given by

$$M(t) = 0.3 - 0.2e^{-t} + 0.15e^{-\frac{t}{10}}, t \geq 0$$

a. Write down the proportion of the population which has the virus 10 days after its introduction. Give your answer correct to three decimal places **1 mark**

---

b. Find the long term proportion of the population that contract the virus. i.e. Find the value of *M* as *t* approaches infinity **1 mark**

---

c. Use calculus to find the maximum proportion of the population that contract the virus correct to 2 decimal places, and the exact value of *t* for which this maximum occurs. **3 marks**

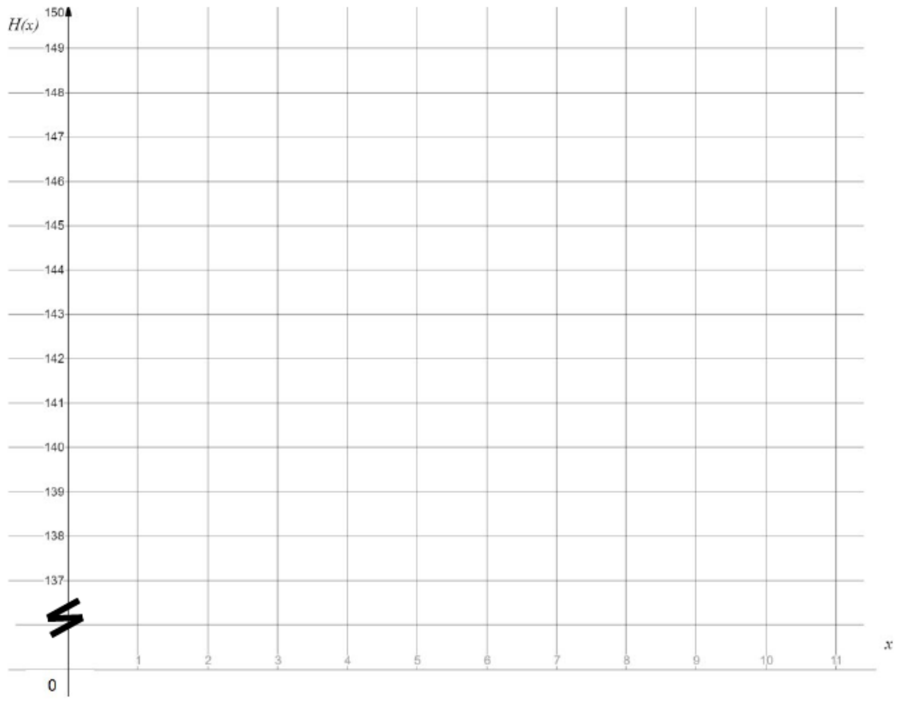
Question Three (18 marks)
Well regarded (and highly credentialed) engineer, Gretel Margaroy has been employed to design a treetop trainline so visitors to Victoria can experience a view of the beautiful Great Otway National Park. Gretel has to produce a trainline that travels 50 km. with restrictions on energy use whilst ensuring visitors get a wonderful experience.

Gretel decides on the following model

$$H(x) = \begin{cases} 140 + 0.1(x-2)^2, & 0 \le x \le 10 \\ 1.6x + 130.4, & 10 < x \le 20 \\ ax^3 + bx^2 + cx + d, & 20 < x \le 50 \end{cases}$$

where x is the distance in km from the start of the trainline and H is the height in metres above sea level.

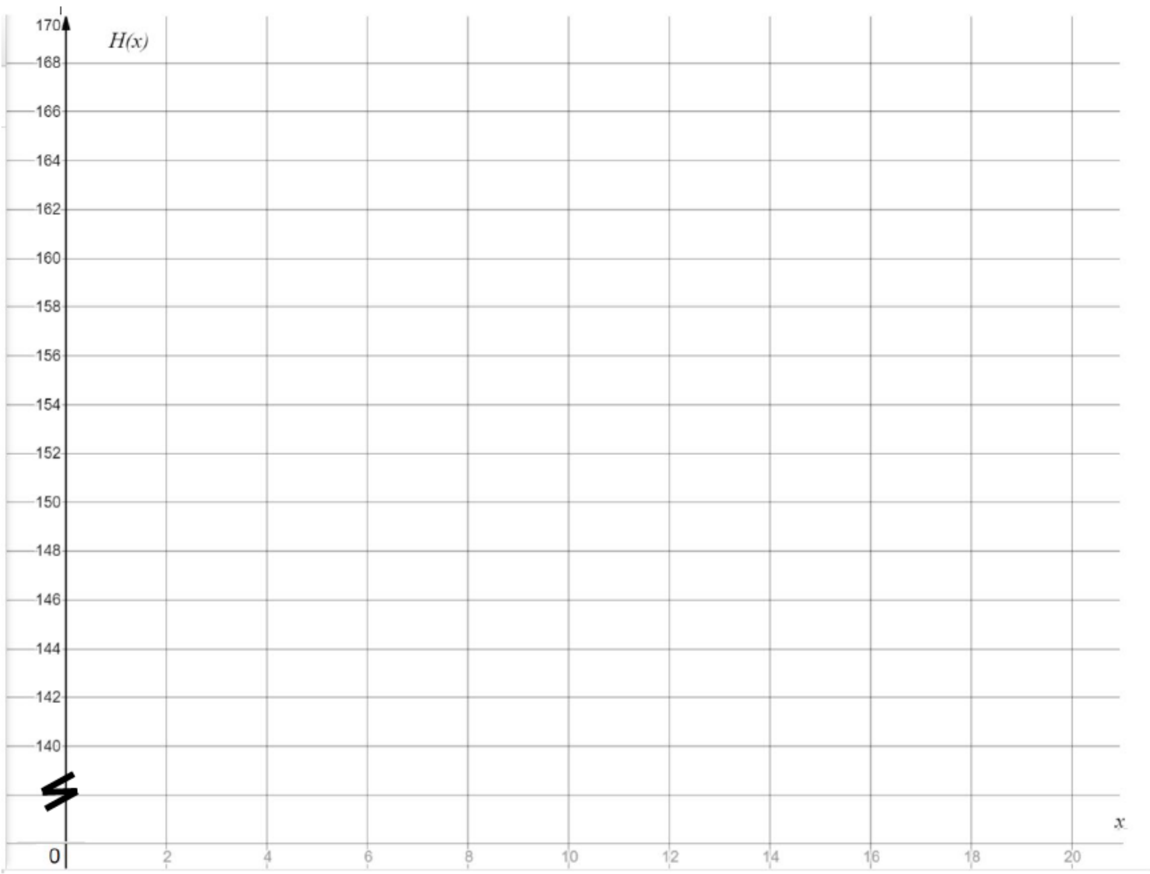
a. **i.** Sketch the graph of stage one ($0 \le x \le 10$) of Gretel's trainline showing all important features. **2 marks**



ii. What is the gradient of the trainline at the $x = 0$? **2 marks**

iii. Show that stage one and stage two of the trainline meet at the same point and that the function is **smooth** at this point. **3 marks**

iv. Sketch the graph of stages one **and** two of Gretel's trainline showing all important features. **2 marks**



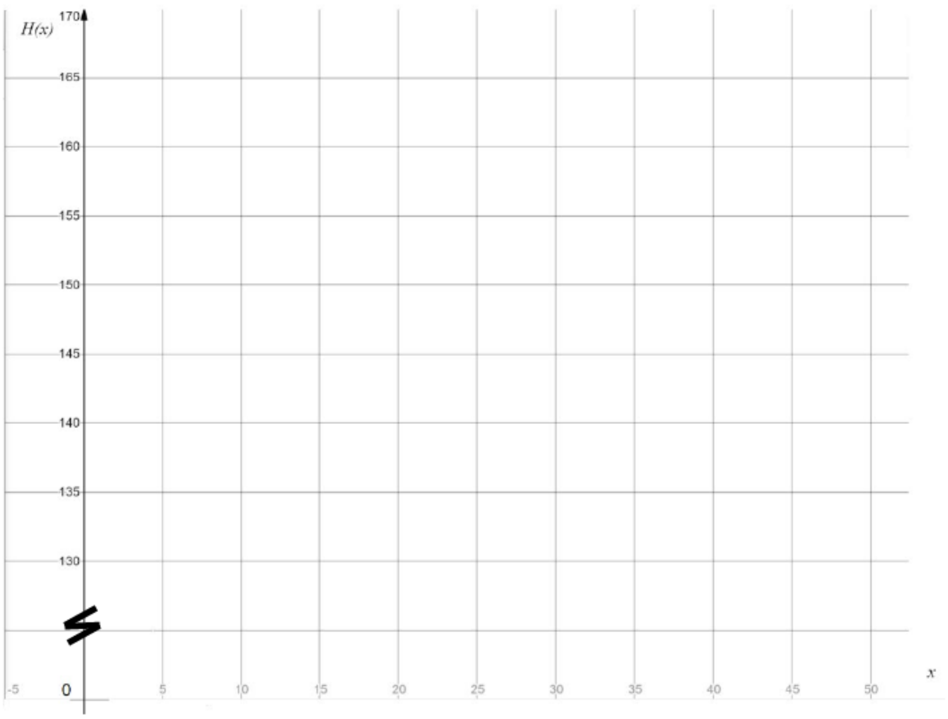
v. Given that Gretel's trainline remains **smooth** and safe for the passengers and it passes through a height of 140 metres above sea level after travelling 35 km as well as finishing at a height of 140.4 metres above sea level, find the values of a, b, c, and d that give Gretel the final stage of her model. Write down the 4 equations that can be solved simultaneously to show that **4 marks**

$a = 0.0085643$, $b = -0.8484$, $c = 25.26$, $d = -71.97$

(you are not required to solve the system of equations)

These values are to be used for the remainder of this question.

vi. Sketch the graph of stages one, two and three of Gretel's trainline showing coordinates of endpoints, turning points and axes intercepts correct to two decimal places. **3 marks**



b. Old Mr Croft again comes to spoil the party. He suggests that sections of the track need support structures as the gradient exceeds a magnitude of 2m/km. On which section(s) of the track would Gretel have to supply extra support to meet these regulations? Give your answer using interval notation correct to two decimal places. **2 marks**

Question Four (15 marks)

A rip is a strong, localized, and narrow current of water which moves directly away from the shore, cutting through the lines of breaking waves like a river running out to sea.
A rip current is strongest and fastest nearest the surface of the water. Rip currents can be hazardous to people in the water.

Henry is a lifeguard, and rips are a major issue as he must attend any such emergencies in the quickest possible time. Fortunately, Warrnambool beach has a straight coastline. Henry has found that he can run on the beach at a pace of 7 m/s and he can swim in the ocean at a pace of 4 m/s.

Assume Henry is posted at a point O on the coastline at Warrnambool beach.

Tilly, Henry's sister, is normally a reasonably strong swimmer. Today, however, she was caught in a rip. Now she has grabbed a stationary buoy, and can't move. She is currently 50 metres directly off-shore and 120 metres east of Henry's position on the beach at O. She is in trouble and Henry needs to act.

a. If Henry jumped straight into the water at O and swims directly to Tilly's position how long would it take him to get there? **2 marks**

____

____

____

____

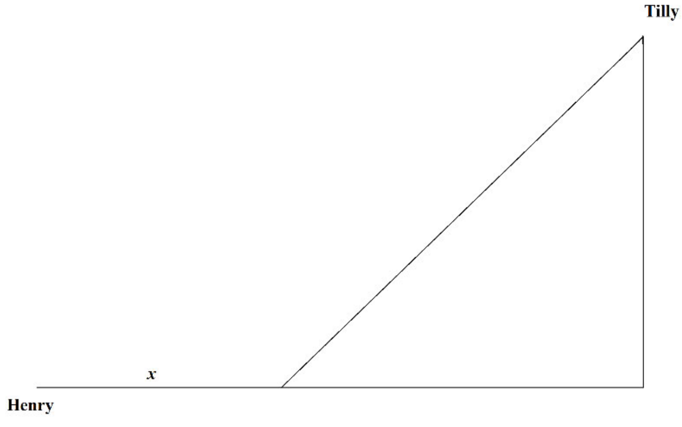
____

(space for diagram)

b. If Henry ran along the beach until he was adjacent with Tilly's position in the water and then swam out to her, how long would it take for him to get to her position? Give answer correct to two decimal places. **2 marks**

c. If Henry ran *x* metres along the beach and then entered the water and swam the required distance to get to Tilly, show that the equation relating the time taken, *T*, in terms of *x* is: **2 marks**

$$T(x) = \frac{x}{7} + \frac{\sqrt{16900 - 240x + x^2}}{4}$$



d. **i.** How far should Henry run along the beach before he enters the water to reach Tilly in the minimum time? Give your answer correct to two decimal places. Verify it is a minimum. **3 marks**

ii. What is the shortest amount of time Harry can reach Tilly in? Give your answer correct to two decimal places. **1 mark**

But Henry is a very smart man – his father is a maths teacher and so is his mother! He realises that the quicker he gets to Tilly the more likely he is to save her. As he starts running along the beach he notices a surf ski on the beach and thinks it is 20 metres further along the beach than the point adjacent to Tilly. Henry knows he can run on the beach at a pace of 7 m/s and ski at a pace of 15 m/s.

e. **i.** If Henry ran along the beach until he got to the surf ski and then skied out to Tilly, how long would it take for him to get to her position? Give answer correct to two decimal places. **2 marks**

____

____

____

____

(space for diagram)

ii. Henry was guessing the position of the surf ski along the beach. It is, in fact, further away than he first thought. How much further along the beach than initially thought could it be situated before running to the surf ski is not the quickest way to reach Tilly? Give your answer correct to one decimal place. **3 marks**

____

____

____

____

____

____

____

END OF APPLICATION TASK