READING START:

WRITING START:

WRITING END:

Mathematical Methods SAC 2 Part 1

Modelling/Problem Solving

Wednesday 26th August 2020

10 minutes reading time

90 minutes writing time

- This task is to be completed in one session of duration 100 minutes.
- During this task, you may use a CAS calculator and refer to one set of bound notes and SAC preparation material.
- You must work silently and independently for the duration of this task.
- All answers are to be written within this booklet.
- Questions worth more than one mark require some explanation of working or process.
- **Exact values** are expected throughout, unless otherwise stated.
- You will be required to clear all CAS memory at the conclusion of the SAC
- No electronic devices (such as mobile phones) may be brought into the examination room.
- Questions seeking clarification may occur in reading time only
- There are THREE questions.

Total : 51 marks

Student Name : ___________________________________

Teacher Name: ___________________________________

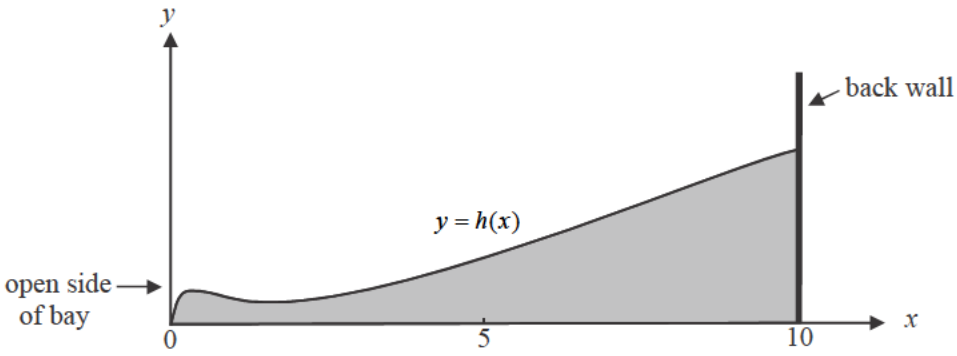
The grade awarded to this SAC is subject to statistical moderation

by the VCAA and is likely to change.

Question 1 [11 Marks]

At a garden supply business different soils and stones are stored in rectangular bays which have a depth of 10m and a width of 3m. Each bay has a back wall and two side walls with the remaining side left open to enable access by the bobcat.

One of these bays contains top soil. The cross-section of the top soil contained in this bay on a particular day is shown in the diagram below.



The height, in metres, of the top soil in this diagram is given by the function

$$h: [0,10] \rightarrow R, h(x) = \sqrt{\frac{x}{3}} + e^{-0.6} - 1$$

where x represents the horizontal distance, in metres from the front (open side) of the bay.

a. Find the maximum height of the top soil in this bay. Express your answer in metres correct to 2 decimal places. [1 mark]

b. Write down the interval over which the graph of h is strictly decreasing. Express the endpoints of the interval correct to 2 decimal places. [2 marks]

The gradient of the function h is equal to the actual height of the top soil at one point.

c. Find the height of the top soil at this point. Express your answer in metres correct to 2 decimal places. [2 marks]

Between the points $(1, \mathrm{h}(1))$, where *p* is a constant and $(\mathrm{p}, \mathrm{h}(\mathrm{p}))$, where p is a constant and $\mathrm{p} > 1$, the average rate of change in the height of the top soil is zero.

d. Find the value of *p*. Express your answer correct to 2 decimal places. [2 marks]

In order to get an estimate of the volume of top soil in the bay, it is to be assumed that the height of the cross section, as shown in the diagram, is constant across the 3 metre width of the bay.

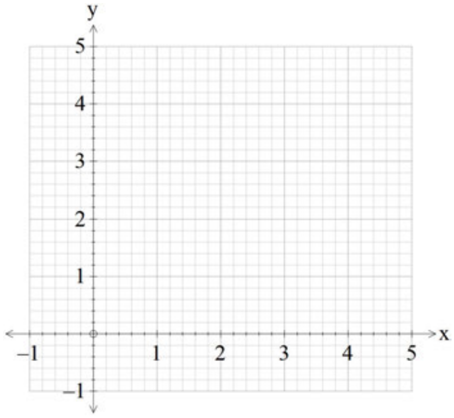
e. Find the volume of top soil in the bay using this estimate. Express your answer in cubic metres correct to 2 decimal places. [2 marks]

f. If the top soil in this bay were to be levelled flat, what would the height of the top soil be? Express your answer in metres correct to 2 decimal places. [2 marks]

Question 2. [21 marks]

Consider the region A bounded by the curve $f(x) = 4x - x^2$ and the x-axis for $x \in [0,4]$

a. Sketch the graph of f(x) on the axis below, labelling axis intercepts, turning point and shading the region A [3 marks]



b. Find the area of region A. [1 mark]

Region A can be subdivided into two regions, B and C, by the line $y = x$. Region B is above the line $y = x$ and region C is below.

c. Find the coordinates of the points of intersection of the line $y = x$ with $f(x)$. [1 mark]

d. Find the area of region B [2 marks]

e. Hence, find the area of region C [1 mark]

Now consider the equation $y = mx$, where m is a real constant such that $0 < m < 4$. This equation defines a whole family of lines. For each value of m the line will intersect $f(x)$ at the origin and at one other point, P_m. As before, the line $y = mx$ will divide region A into two regions, B_m (above the line) and C_m (below the line).

f. Find the co-ordinates of the point P_m in terms of m. [2 marks]

g. Find the area of region B_m in terms of m. [2 marks]

h. Hence, find an expression for the area C_m in terms of m. [2 marks]

i. Find the exact value of m for which the areas B_m and C_m are equal. [2 marks]

j. If value of m can be expressed in the form $4(1 - 2^k)$, find the value of k. [1 mark]

Now consider the family functions where region A is bounded by the curve $f(x) = ax - x^2$, and the x-axis for $x \in [0, a]$, where a is a real constant. This region is to be divided into two equal regions two regions, B_m (above the line) and C_m (below the line) by the line, $y = mx$ such that $0 < m < a$

Your investigation is to determine the value of m in terms a. Your solution should be able to be expressed in a form similar to that in part j. [4 marks]

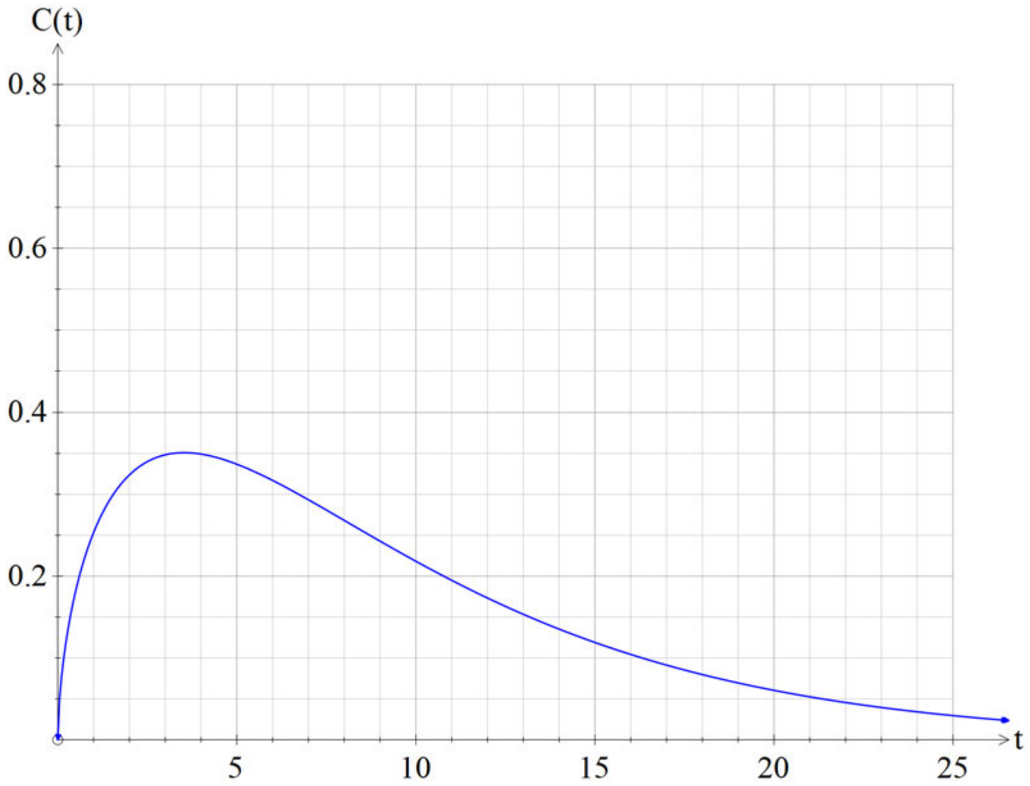
Question 3 [19 marks]

While smoking tobacco, the body absorbs many chemical compounds in addition to nicotine, including cyanide (which is highly toxic to humans).

Whilst smoking a single cigarette, the function:

$C_1(t) = 0.3t^{0.6}e^{-0.17t}, for\ t \in [0, \infty)$

is a reasonable model of the measured blood cyanide concentrations in mg/litre after t minutes. The graph is shown below.



a. Find the maximum concentration of blood cyanide, in mg/Litre, correct to three decimal places.

[1 mark]

b. Find when the maximum concentration occurs, in minutes to one decimal place.

[1 mark]

c. Find the concentration, correct to three decimal places, after 10 minutes.

[1 mark]

d. Find the average rate of change of concentration over the time interval $[0,10]$ correct to three decimal places. [2marks]

e. Find the average value of concentration over the time interval $[0,10]$ correct to three decimal places. [2 marks]

f. Find the rate of change of concentration at 10 minutes, correct to three decimal places. [2 marks]

Research shows that smokers will typically finish a cigarette in 5 minutes, which usually takes approximately ten puffs. Consider a chain smoker who begins a second cigarette straight after finishing the first, that is, when t =5 minutes. The concentration for the second cigarette can be represented by:

$$C_2(t) = \begin{cases} C_1(t-5), & for\ t \in [5, \infty) \\ 0 & for\ t \in [0,5) \end{cases}$$

The concertation of cyanide in the bloodstream from each cigarette consumed independently is shown in the graph below, with the second function modelled by:

g. On the axis above sketch the graph of $S_2(t) = C_1(t) + C_2(t), for\ t \in [0, \infty)$, which is the sum concentration of cyanide in the bloodstream from two cigarettes. [2 marks]

h. Find the maximum concentration of blood cyanide, in mg/Litre, correct to three decimal places.

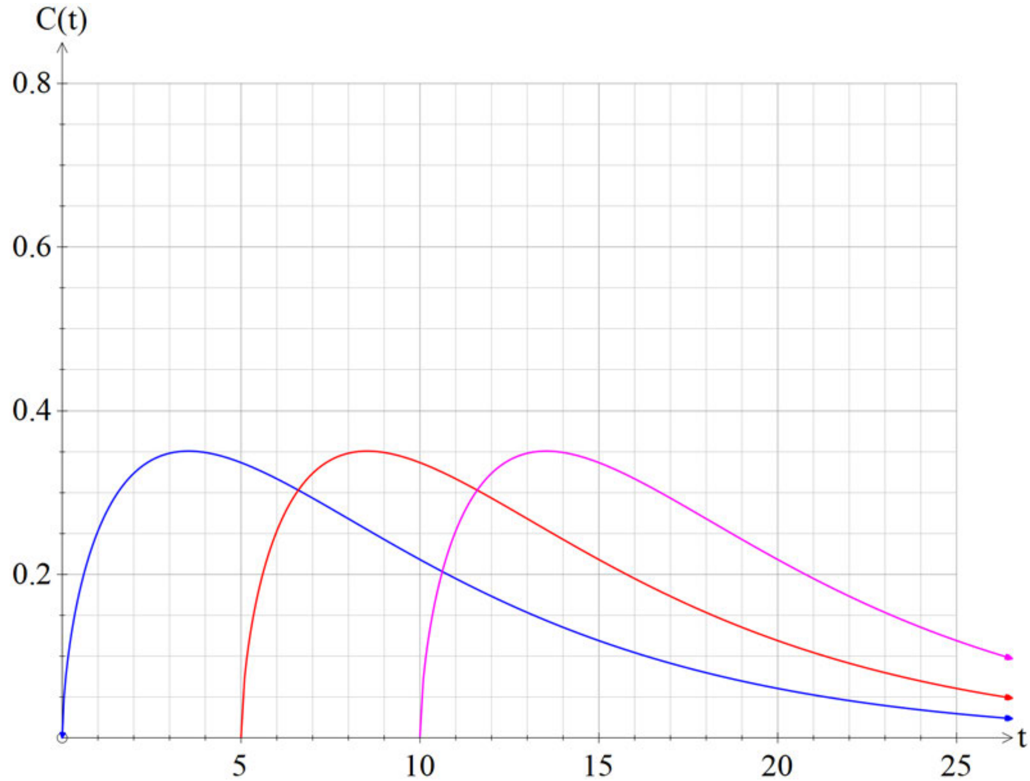
[1 mark]

i. Find when the maximum concentration occurs, in minutes to one decimal place.

[1 mark]

Consider a third cigarette, which is lit at t = 10 minutes:

$$C_3(t) = \begin{cases} C_1(t-10), & for\ t \in [10, \infty) \\ 0 & for\ t \in [0,10) \end{cases}$$



The graphs of $C_1(t)$, $C_2(t)$ and $C_3(t)$, $for\ t \in [0, \infty)$ is shown.

j. Sketch $S_3(t) = C_1(t) + C_2(t) + C_3(t), for\ t \in [0, \infty)$,

k. Find the maximum concentration of blood cyanide, in mg/Litre, correct to three decimal places.

[1 mark]

l. Find when the maximum concentration occurs, in minutes to one decimal place.

[1 mark]

Medical authorities provide the following advice regarding cyanide in the blood stream:

Levels of 0.5–1 mg/L are mild	**Level of 1–2 mg/L are moderate**	**Levels of 2–3 mg/L are severe**	**Level greater than 3 mg/L generally result in death.**

m. By considering the total concentration of cyanide in the blood stream for a chain smoker continuing to smoke a new cigarette every 5 minutes, comment on possibility of levels of the concentration exceeding the **mild** category. Use data to justify your view.

[4 marks]