XAVIER COLLEGE
SAC / Assessment Conditions

	Date:	Time:
MATHEMATICAL METHODS	**MONDAY 7TH JUNE**	**3.45 pm – 5.55 pm**

- Listen carefully to the supervisor's instructions.
- Permissible items include: pens, pencils, highlighters, erasers, sharpeners, rulers.
- You are not permitted to use white out (liquid paper).
- You have 10 minutes reading and 2 hrs writing to complete this part.
- Complete this task in the spaces provided.
- Give answers in exact form unless told otherwise.
- You can use your CAS calculator and a bound set of notes to complete this task.
- A number of questions are consequential in nature. You are advised to show all working, even for questions worth one mark. In questions worth more than 1 mark, working is required to gain full marks.
- You must work silently and independently for the duration of the task. Only questions of clarification can be asked of your teacher.
- It is not in your interest to talk about this task with students from other classes.

PLEASE NOTE: ***Students are NOT permitted to have mobile phones or any other unauthorised electronic devices in their possession during a SAC/examination***

COMPULSORY STUDENT DECLARATION

I, (*print your name neatly*) _____
acknowledge that I have read the SAC/examination conditions and understand which items/materials I am permitted to use and have in my possession.

****If you have any doubts as to what is permitted, raise your hand and DO NOT sign this declaration****

Student's Signature: _____

Student's Name: _____

Teacher's Name: _____

XAVIER MATHS

MATHEMATICAL METHODS APPLICATION SAC 2021

PART ONE

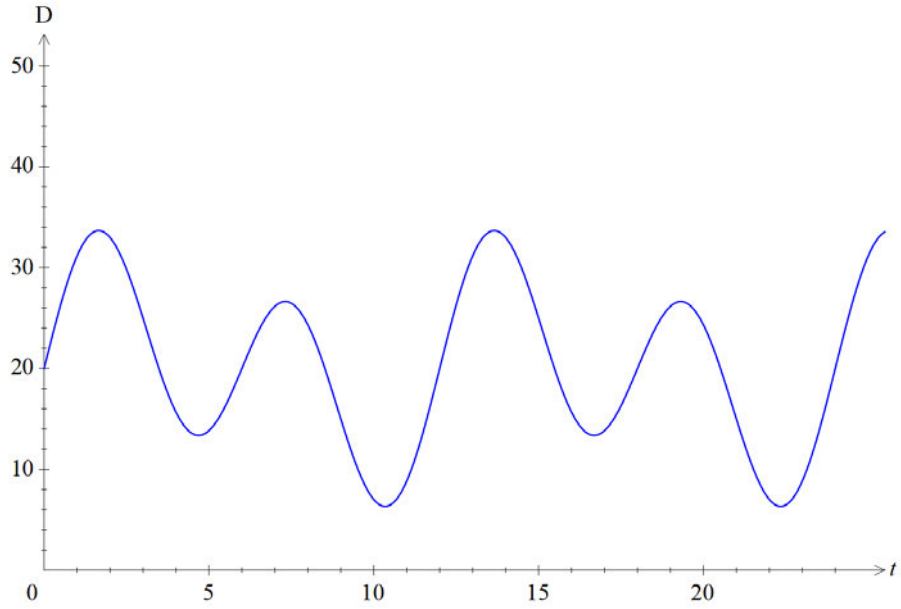
50 Marks

Question 1 (10 marks)

The asteroid TK-421 is present in our solar system. The distance of this asteroid from Earth is modelled by the function

$$D(t) = 10\sin\left(\frac{\pi}{3}t\right) + 5\sin\left(\frac{\pi}{6}t\right) + 20$$

Where D is the distance of the asteroid from the earth in millions of kilometres and t is time in years since December 31 2020. The graph of D(t) is shown below.



a Write down $D'(t)$, the derivative function of $D(t)$

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1 mark

b Find the maximum distance (in millions of kilometres) of the asteroid from Earth, and the t value for which this first occurs. Give your answers correct to two decimal places

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2 marks

c Find the minimum distance (in millions of kilometres) of the asteroid from Earth, and the t value for which this first occurs. Give your answers correct to two decimal places

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2 marks

d When the asteroid is within 10 million kilometres of the Earth, a warning system is engaged at NASA's control centre in Houston, Texas. For how long is the warning system engaged between Dec 31 2020 and Dec 31 2044? Give your answer in years correct to two decimal places.

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3 marks

e The control centre receives information that the orbit of the asteroid will be thrown out by an astronomical event on December 31 2044 (i.e. $t = 24$). The new function which models the distance of the asteroid from the Earth is given by;

$$D_2(t) = 10\sin\left(\frac{\pi}{3}(t-h)\right) + 5\sin\left(\frac{\pi}{6}(t-h)\right) + 10, t \geq 24$$

Where D is still the distance of the asteroid from the Earth in millions of kilometres and t is still time in years from December 31 2020.

Given that the graph of the distance of the asteroid from the earth remains *continuous*, find the value of h correct to two decimal places, given that $h > 0$.

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2 marks

Question 2 (10 marks)

Another asteroid is present in our solar system. The path of the asteroid P-31415 is not totally known by astrophysicists. It is theorised that this asteroid has the possibility of colliding with Earth in the future. The distance of this asteroid from the earth is modelled by a **quadratic** function. At time $t = 0$ (again December 31 2020), the asteroid is measured to be 91 million kilometres away from Earth. It is expected that the maximum distance of the asteroid from the earth will be 100 million kilometres and will occur on December 31 2050.

a Show that the distance of the asteroid from earth can be modelled by the equation;

$$D(t) = -\frac{1}{100}t^2 + \frac{3}{5}t + 91$$

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2 marks

b According to the model above, at what t value will this asteroid collide with the Earth?

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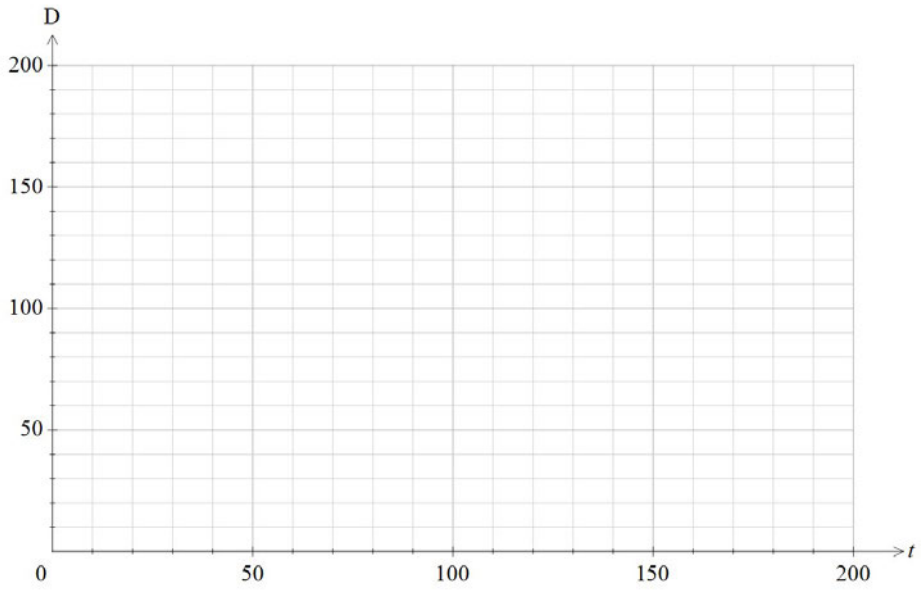
1 mark

c Sketch the graph of the distance of the asteroid P-31415 from the earth onto the axes below. Include the coordinates of the axis intercepts and the turning point.



2 marks

c The actual distance of the asteroid P-31415 from the earth is given by the function

$$D(t) = -\frac{1}{100}t^2 + \frac{3}{5}t + 91 + 15\sin\left(\frac{t}{6}\right)$$

Write down $D'(t)$

_____

1 mark

d **i** Find the maximum distance (in millions of kilometres) of the asteroid from the earth correct to two decimal places

_____

1 mark

ii At what t – value will the asteroid actually collide with the earth? Give your answer correct to two decimal places.

_____

1 mark

e Find the greatest speed of the asteroid for $t \in [0,100]$. Give your answer in millions of kilometres/year correct to two decimal places.

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2 marks

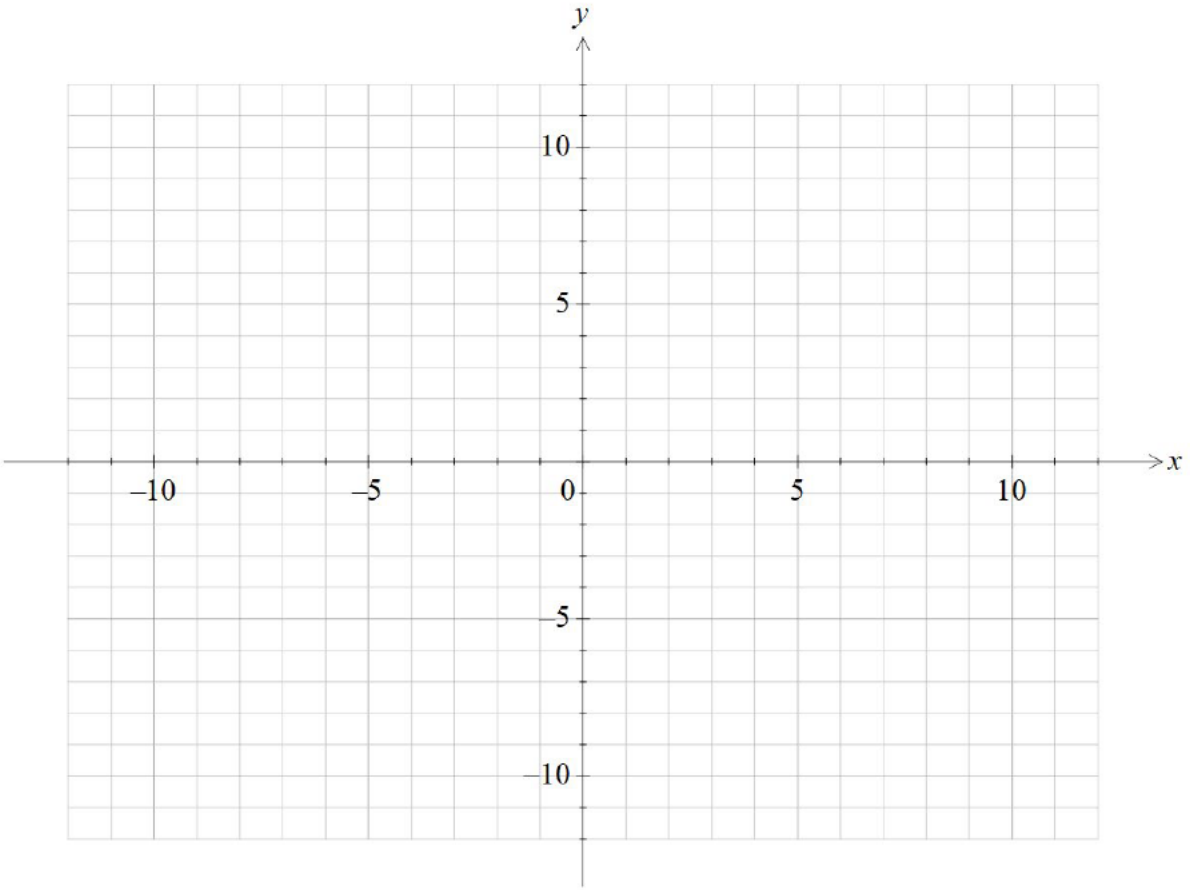
Question 3 (Open Ended – 10 Marks)

In order to stop the collision of asteroid P-31415 with the earth, a mission is devised by astrophysicists. They plan to land a shuttle on the asteroid, drill a nuclear device into the core of the asteroid, and then detonate the device, which will mean that the asteroid will not make an impact with the earth significant enough to cause any damage.

For various reasons, including the necessity to avoid space junk in orbit around the earth, the path of the shuttle through space is modelled by a cubic function, and for simplicity, it is modelled on a 2D Cartesian plane. The scientists are using a coordinate system where the earth is located at (-10, -10) and the asteroid is located at (10, 10). To ensure that the shuttle safely makes it to the asteroid, the scientists claim that the shuttle must follow the path roughly laid out by the table of values below.

x	-10	-5	-2	1	6	7	10
y	-10	1	6	3	0	4	10

a Plot the points in the table above onto the Cartesian Plane below, also include the position of Earth and the asteroid.



b Using the table of values on Page 9, select four of the points and find a cubic equation which is a possible path for the shuttle. Take into consideration that the shuttle must go through the points (-10, -10) and (10, 10). Give the equation of the path of the shuttle in the form $y_1 = ax^3 + bx^2 + cx + d$. Where *a*, *b*, *c* and *d* are fractions. Show your working. Sketch the graph of the path you have found onto the axes on page 9 for $x \in [-10,10]$. Include the coordinates of important points correct to two decimal places.

c Select a different set of points to the ones chosen in part **b** and devise a different path for the shuttle. The path must still include the points (-10, -10) and (10, 10). Give the equation of this path of the shuttle in the form $y_2 = ax^3 + bx^2 + cx + d$. Where *a*, *b*, *c* and *d* are fractions. Show your working. Sketch the graph of the path you have found onto the axes on page 8 for $x \in [-10,10]$. Include the coordinates of important points correct to two decimal places.

d Which one of the paths you devised best fits the table of values provided by the scientists? Give a reason for your answer.

Question 4 (10 marks)

In the end, the scientists settle on a different path for the shuttle. The path is given by the equation $g(x) = \frac{2}{75}x^3 - \frac{5}{3}x$, where $x \in [-10,10]$

a Find the exact coordinates of the stationary points of the path of the shuttle and state their nature,

3 marks

As part of the navigation system of the shuttle, the equations of tangents are used to triangulate the position of the shuttle in relation to the asteroid.

b Find the equation of the tangent to the path of the shuttle at $x = 1$

1 mark

c Find the equation of the line perpendicular to the tangent to the path of the shuttle at $x = 1$

1 mark

d Find the equation of the line perpendicular to the path of the shuttle at $x = p$. Give your answer in the form $y = q_1(p)x + q_2(p)$, where $q_1(p)$ and $q_2(p)$ are functions in terms of p.

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2 marks

e Find the equations of the two lines which are perpendicular to the tangent to the curve and pass through the position of the asteroid. Give your answers in the form $y = mx + c$, where m and c are decimal numbers correct to 3 decimal places.

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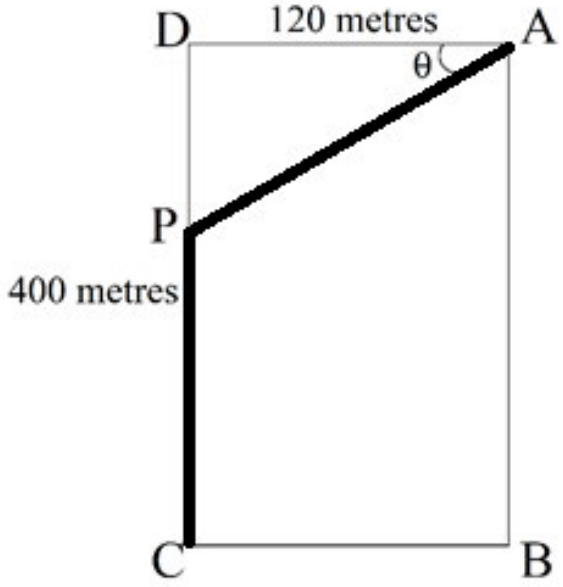
3 marks

Question 5 (10 marks)

Once the shuttle lands on the asteroid, the drilling will commence. The path of the drill into the asteroid will not be straight down as it needs to avoid certain anomalies found within the asteroid.

In the diagram below, the drill will follow the path from A to P and then from P to C. When drilling on a slant, the drill can move at 6 metres an hour. When drilling directly down, the drill can move at 10 metres an hour. θ is in degrees.



a Find the length of AP in terms of θ.

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1 mark

b **i** Find the length of DP in terms of θ.

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1 mark

ii Hence, write the distance PC in terms of θ.

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1 mark

c Show that the total time taken to travel from A to C via P in terms of θ can be expressed as $T = 40 + \dfrac{20 - 12\sin(\theta)}{\cos(\theta)}$, where T is in hours.

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3 marks

d Find $\dfrac{dT}{d\theta}$

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2 marks

e Find the value of θ that makes T a minimum. Give your answer in degrees correct to 2 decimal places

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1 mark

f Hence find the total time taken by the drill to reach its destination, correct to 2 decimal places.

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1 mark

END OF PART ONE