XAVIER COLLEGE

SAC / Assessment Conditions

PART TWO – TECHNOLOGY ENABLED

MATHEMATICAL METHODS	MONDAY 2ND AUGUST 3.45 pm – 5.55 pm

- Listen carefully to the supervisor's instructions.
- Permissible items include: pens, pencils, highlighters, erasers, sharpeners, rulers.
- You are not permitted to use white out (liquid paper).
- You have 1 hour writing time to complete this part.
- Complete this task in the spaces provided.
- Give answers in exact form unless told otherwise.
- You may use your CAS calculator and a bound set of notes to complete this part of the SAC.
- A number of questions are consequential in nature. You are advised to show all working, even for questions worth one mark. In questions worth more than 1 mark, working is required to gain full marks.
- You must work silently and independently for the duration of the task. Only questions of clarification can be asked of your teacher.
- It is not in your interest to talk about this task with students from other classes.

PLEASE NOTE: ***Students are NOT permitted to have mobile phones or any other unauthorised electronic devices in their possession during a SAC/examination***

COMPULSORY STUDENT DECLARATION

I, (*print your name neatly*) ____________________________________ acknowledge that I have read the SAC/examination conditions and understand which items/materials I am permitted to use and have in my possession.

****If you have any doubts as to what is permitted, raise your hand and DO NOT sign this declaration****

Student's Signature: ____________________________________

Student's Name: ____________________________________

Teacher's Name: ____________________________________

XAVIER MATHS

MATHEMATICAL METHODS MODELLING/PROBLEM SOLVING SAC 2021

(SAC 2)

PART TWO

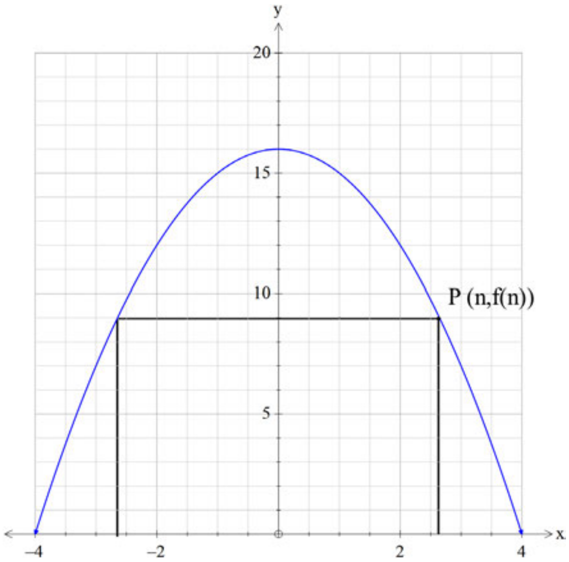
TECHNOLOGY ENABLED

39 Marks

Question 1 (10 marks)

Part of a new underground rail network involves a section where a tunnel is to be created by drilling a parabolic cross section. A rectangular concrete section is then installed, which will form the passageway for the trains.

The parabolic tunnel can be modelled by: $f(x) = 16 - x^2, x \in [-4, 4]$ as shown in the diagram below. The concrete rectangle is also shown and touches the parabolic tunnel at point P. Distance is measured in metres.



a Show the area of the concrete rectangular cross section can be determined by the expression:

$$A(n) = 2n(16 - n^2)$$

2 marks

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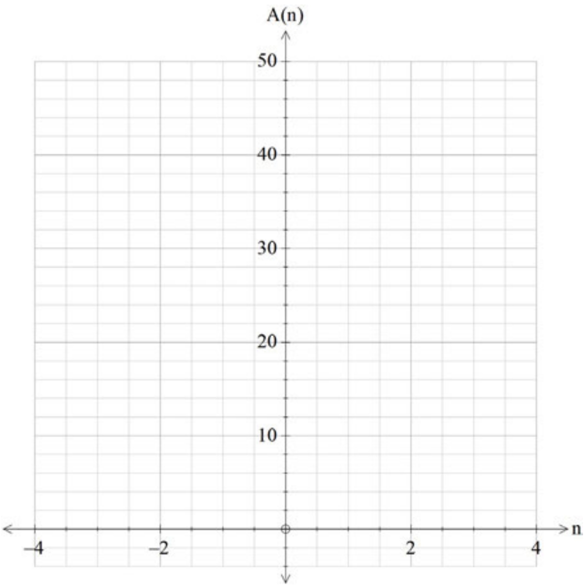
b Sketch the graph of $A = 2n(16 - n^2)$ over a valid domain for n. Indicate the coordinates of the axes intercepts. Give the coordinates of the maximum correct to two decimal places.

3 marks



c Find $A'(n)$ and determine the value of n for which $A'(n) = 0$ 2 marks

d Hence find the exact maximum value of the area of the rectangle, giving your answer in square metres 1 mark

e Health regulations require that area of the concrete rectangle must be at least 30 square metres.

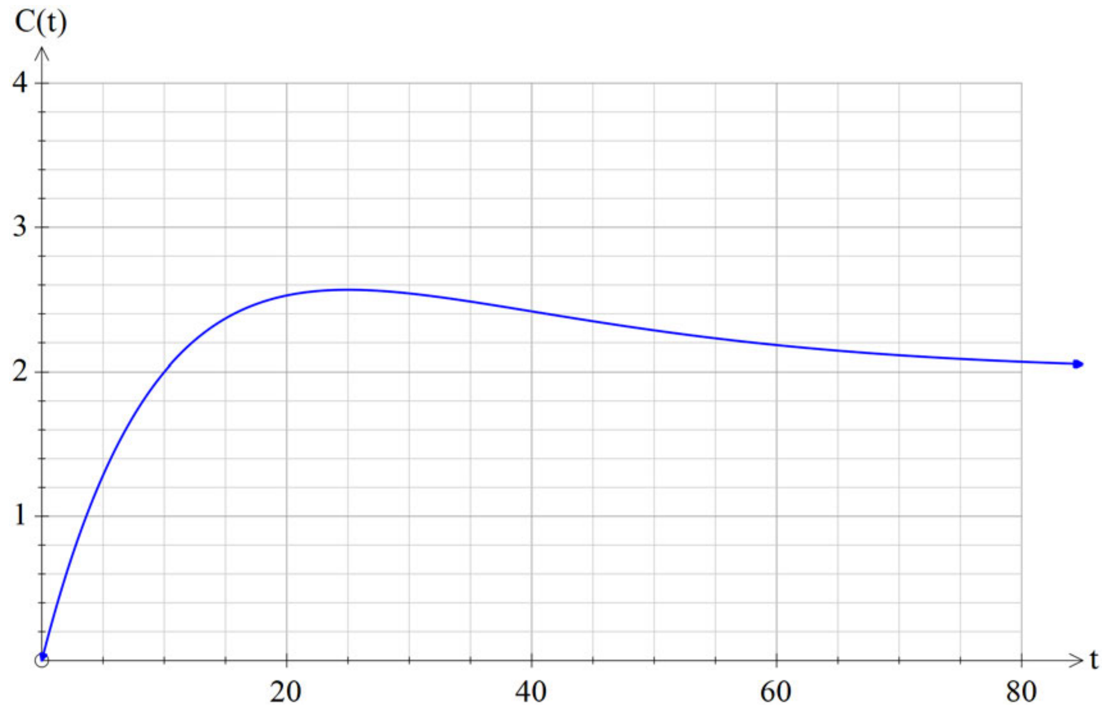
Find the values of n that satisfy this requirement. Answer to two decimal places. 2 marks

Question 2 (11 Marks)

An automatic pool cleaner is used to dispense chlorine into a small pool that contains pure water.
The concentration, C, in mg L^{-1}, of chlorine in the pool, *t* minutes after the cleaning process begins is given by:

$$C(t) = \frac{1}{5}(t-10)e^{\frac{-t}{15}} + 2$$

where $t \geq 0$. The graph of $y = C(t)$ is shown for $0 \leq t \leq 80$



a. Find $C'(t)$ 1 mark

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b. Find the set of values of t for which the concentration of chlorine is strictly increasing. 1 mark

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c. Find the maximum concentration of chlorine in the pool, in mg L^{-1}, correct to two decimal places. 1 mark

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d. At what concentration, in mg L^{-1}, does the chlorine ultimately stabilise? 1 mark

e **i.** Find the average rate of change of C for $t \in [0,25]$, in mg L^{-1} per minute, correct to two decimal places.

2 marks

ii. Find the value of t for which the instantaneous rate of change of the concertation of chlorine in the pool is equal to the average rate of change of C for $t \in [0,25]$, correct to two decimal places.

1 mark

f. Find the average concentration of chlorine in the pool for $t \in [0,25]$, in mg L^{-1} ,

to two decimal places.

2 marks

g. Find the rate of change of the concentration of chlorine in the pool when the concentration of chlorine is decreasing most rapidly, in mg L^{-1} per minute, correct to two decimal places.

2 marks

Question 3 (18 Marks)

Consider the function $f(x) = \frac{-1}{4}(x-2)(x+1)^2$ 2 marks

a **i.** Given that $g(x) = \int f(x)dx$ and $g(0) = 1$, show that $g(x) = \frac{-x^4}{16} + \frac{3x^2}{8} + \frac{x}{2} + 1$

[CAS syntax may be used in your working]

ii Find the x value of any stationary points of $g(x)$ and state their nature (no verification required).

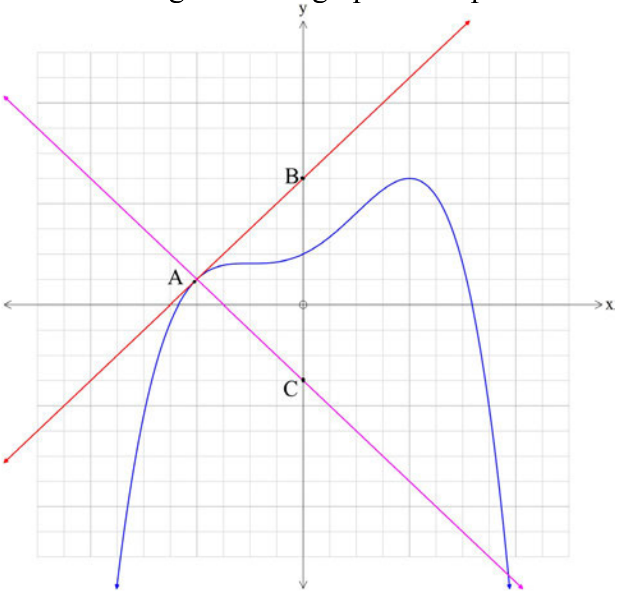
2 marks

b. Solve $g(x) = 0$, giving your answers correct to two decimals places. 2 marks

c. Find the average value of the function $g(x)$ over the interval between the x intercepts, giving your answers to two decimals places. 2 marks

The diagram below shows part of the graph of $y = g(x)$.

At the point A, where $x = -2$ the tangent to the graph is shown and a straight line drawn perpendicular to the tangent to the graph at the point A is also shown.



d **i.** Find the equation of the tangent at $x = -2$. 1 mark

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ii. Find the equation of the line that is perpendicular to the tangent at $x = -2$. 1 mark

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The tangent cuts the *y*-axis at *B*. The line perpendicular to the tangent cuts the *y*-axis at *C*.

e **i.** Give the co-ordinates of B 1 mark

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ii Give the co-ordinates of C 1 mark

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iii. Find the area of the triangle ABC. 2 marks

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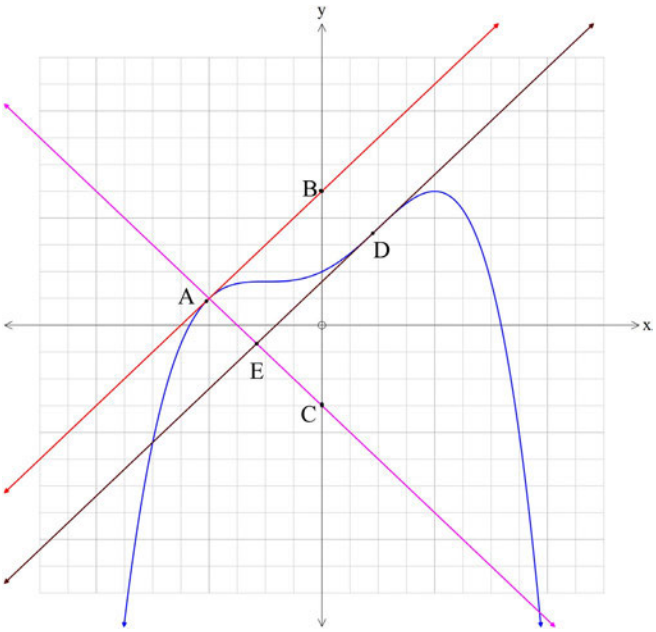
The tangent at D is parallel to the tangent at A. It intersects the line passing through A and C at E.



f. Find the co-ordinates of D. 2 marks

g. Find the length of the line ED. 2 marks

END OF PART TWO

END OF SAC