XAVIER MATHS

Mathematical Methods SAC 3 (2021)

PROBLEM SOLVING TASK – PROBABILITY AND STATISTICS
Wednesday 8th September

10 minutes reading time (1:30-:140pm)

120 minutes writing time (1:40-3:40pm)

Permissible items include: pens, pencils, highlighters, erasers, sharpeners, rulers.
• You are not permitted to use white out (liquid paper).
• You have 10 minutes reading and 120 minutes writing time for this task.
• Print out the task and complete in the spaces provide or (if this not possible) do the task on lined paper with each question and part clearly labelled AND EACH PAGE WITH YOUR NAME.
• Give answers in exact form unless told otherwise.
• You can use your CAS calculator and a bound set of notes to complete this task.
• Several questions are consequential in nature. You are advised to show all working, even for questions worth one mark. In questions worth more than 1 mark, working is required to gain full marks.
• You must work independently for the duration of the task and sign the declaration that this is the case.
• At the conclusion of the writing time, you have 10 minutes to submit your task to SEQTA as a single document in pdf form. Do not leave Teams until your teacher has given you permission.
• Submissions after the 10-minute deadline will not be graded. However, if you have any technical issues, you should contact HOF (Rob Hume) for consideration

COMPULSORY STUDENT DECLARATION

I, (*print your name neatly*) ____________________
acknowledge that I have read the SAC conditions and understand which items/materials I am permitted to use and have in my possession. I WILL NOT COMMUNICATE WITH ANYONE EITHER IN PERSON OR USING A DEVICE.

****If you have any doubts as to what is permitted, ask your teacher, DO NOT sign this declaration****

Student's Signature: ____________________

Student's Name: ____________________

Teacher's Name: ____________________

Mathematical Methods formulae

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A\|B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = \mathrm{E}(X)$	variance	$\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x-\mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$\mathrm{E}(\hat{P}) = p$
standard deviation	$\mathrm{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

Question One (10 marks)

A discrete random variable, J, representing the number of goals kicked by Jack Riewoldt in each game that he plays, has the following probability distribution, where $0 < k < \frac{5}{6}$

j	0	1	2	3	4	5
Pr (J = j)	$\frac{k}{5}$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{2k}{5}$	$\frac{1}{10}(5-6k)$

a) Show, with working and giving a reason, that this does fit the requirements of a discrete probability distribution.

(1 mark)

b) Show that the chance of Jack kicking 9 goals in total in two consecutive games is equal to

$$\frac{10k-12k^2}{25}.$$

(2 marks)

c) Show that the value of k that makes this probability a maximum is $\frac{5}{12}$ and find this maximum probability.

(2 marks)

The table below is available for optional use for the remainder of Question One.

J	0	1	2	3	4	5
Pr (J = j)		$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{4}$		

d) Using the value of k found in part **c)** find

i) the expected value of J and the standard deviation of J. Express your answers correct to three decimal places.

(3 marks)

ii) Pr $(\mu - \sigma \leq J \leq \mu + \sigma)$

(2 marks)

Question Two (7 marks)

The recruiting managers at Richmond have found that 1 in 15 players drafted to the club play at least 100 games at the club. Sixty randomly chosen players drafted to the club were surveyed with respect to the number of games played for the club.

a) If G is the random variable that defines the number of players in the survey who played at least 100 games for the club, find the expected value of G.

(1 mark)

b) Find the probability (correct to four decimal places) that fewer than six players from this sample played at least 100 games for the club.

(2 marks)

c) Find the probability (correct to four decimal places) that at least 3 players from this sample played at least 100 games for the club given that fewer than six players from the sample played at least 100 games for the club.

(2 marks)

d) Find the variance and the standard deviation of G (correct to two decimal places).

(2 marks)

Question Three (8 marks)

Dr. Turf, the grounds manager at Punt Road Oval, has just spread lawn seed on the centre square area. With constant watering and plenty of sunshine the time it takes for the lawn seed to germinate, *t* days after seeding can be determined by the probability density function:

$$f(t) = \begin{cases} 0, & t < 0 \\ k\,e^{-0.12t}, & t \geq 0 \end{cases}$$

a) Show that the value of *k* is 0.12.

(2 marks)

b) What is the expected time for germination of the lawn seed? Give your answer correct to the nearest day.

(2 marks)

c) Find the standard deviation of the germination time, Give your answer correct to two decimal places.

(2 marks)

d) Find the median for T. Give your answer correct to two decimal places.

(2 marks)

Question Four (13 marks)

Richmond player, Jayden Short, is training to try out as a kicker in NFL football in the USA.

The 'B standard' kicking distance is at least 67 metres.

The 'A standard' kicking distance is at least 73 metres.

If Jayden is to be selected in the initial squad, he must show that he can kick the ball at least to the 'A standard'. Jayden knows that the distance of his kicks is normally distributed with a mean distance of 68 metres and a standard deviation of 4.5 metres.

a) i) Find the probability that a random Jayden Short kick is of 'B standard' length, but is less than 'A standard' length. Give your answer correct to 4 decimal places.

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(1 mark)

ii) Complete the following table for one of Jayden's kicks, giving probabilities correct to four decimal places.

Distance Kicked	**Probability**
Less than 'B standard'	
'B standard' but less than 'A standard"	
'A standard'	

(2 marks)

b) 85% of Jayden's kicks travel at least x metres. Find the value of x, in metres, correct to one decimal place.

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(2 marks)

c) Jayden kicks a ball that does **not** reach ‘A standard’. What is the probability, correct to four decimal places, that it reaches ‘B standard’?

(2 marks)

d) Richmond sponsor, Jeep Australia, offer Jayden an incentive to perform at his best. The cash rewards for each kick, all tax free, are shown in the following table:

Distance Kicked	**Reward**
Less than ‘B standard’	$200
‘B standard’ but less than ‘A standard”	$1000
‘A standard’	$10000

Calculate Jayden’s expected reward, correct to the nearest dollar for each kick completed in the trial.

(2 marks)

e) In a particular trial Jayden completes seven kicks.

i) Find the total reward he would expect to receive, correct to the nearest ten dollars.

(1 mark)

ii) Find the probability, correct to two decimal places, that at least four of the kicks will be to at least 'B standard'.

(2 marks)

iii) Find the expected number of times his kicks will be at least 'B standard' giving your answer correct to two decimal places.

(1 mark)

Question Five (6 marks)

a) Dusty Martin, absolute superstar, is injured but still does his bit for the team. During training he is asked to go and grab 3 footballs from a bag which contains 4 red, 3 yellow and 2 pink footballs. Given he randomly chooses 3 footballs find the probability he chooses 3 of the same colour.

(3 marks)

b) Of course, you cannot do a drill without cones, so the coach reminds Dusty he needs 4 cones. Dusty goes to a bag containing 7 orange and 5 green cones. If Dusty randomly selects the 4 cones without replacement:

i) State the possible values of $\hat{p}$, the proportion of **green** cones in his selection.

(1 mark)

ii) Find $\Pr(\hat{P} \leq 0.6)$ where $\hat{P}$ is the proportion of **green** cones in the selection.

(2 marks)

Question Six (6 Marks)

Ruby, Henry and Tilly each spent a day collecting survey responses from Richmond Club Members. They each surveyed 200 people and asked them if they thought Richmond had been the best AFL team of the century. Ruby found that 84% said yes; Henry found that 56% said yes and Tilly found that 64% said yes. They want to obtain an estimate for the population proportion at an approximate 95% confidence interval. Ruby said they should each work out a confidence interval and then average them out to obtain the population proportion. Tilly said they should **combine** the data into one sample and then determine the population proportion and Henry said it will not matter.

a) i) Show your processes used to calculate the approximate 95% confidence interval for the population proportion based on Ruby's sample. Give the values correct to three decimal places.

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(1 mark)

ii) Work out the approximate 95% confidence intervals (correct to three decimal places) for each of the individual samples as well as the combined sample and fill in the table below.

Sample	**Confidence Interval**
Ruby	
Henry	
Tilly	
Combined (According to Tilly's suggestion)	

(3 marks)

b) They go to Angela to ask for advice on which model is likely to be the best model for calculating the population proportion. Which model should Angela advise and why? (Give details in your response with reference to significant features of modelling)

(2 marks)

Question Seven (10 marks)

The skinfolds of the players at Richmond Football Club are tested to measure percentage of body fat. At Richmond the results are normally distributed with 28% of the players having over 105 mm of body fat and 7% of the players having less than 48 mm of body fat.

a) Sketch this information on a normal distribution curve in the space below.

(1 mark)

b) Sketch this information on a z curve in the space below.

(2 marks)

c) Calculate the mean and the standard deviation (both correct to two decimal places) for the mm of body fat for the players at Richmond.

(2 marks)

d) A player defined to have a 'lot of work to do' is one with over 110 mm of body fat. Find the probability (correct to four decimal places) that a randomly selected is defined as having 'a lot of work to do'.

(1 mark)

e) If a randomly selected player has less than 115 mm of body fat, what is the probability (correct to four decimal places) that a player is defined as 'having a lot of work to do?

(1 mark)

f) Damian Hardwick, coach, decides to randomly select 8 players to go on a membership recruiting circuit. What is the probability (correct to four decimal places) that Damian chooses exactly two players defined as 'having a lot of work to do'?

(1 mark)

A sample of 50 Richmond Members is randomly selected from the entire population of Richmond Members to check for premiership tattoos. Remember Richmond has over 100,000 members. The sample proportion $\hat{p}$ of Richmond Members with premiership tattoos is 0.40.

g) Find an approximate 95% confidence interval (correct to four decimal places) for the proportion p of Richmond Members that have premiership tattoos based on this sample.

(1 mark)

h) Calculate the margin of error for this 95% confidence interval for this estimate, correct to the nearest percent.

(1 mark)

END OF SAC

ENSURE YOU INCLUDE EVERY PAGE (18) INCLUDING THE FIRST PAGE DECLARATION IN YOUR SCAN AND THEN UPLOAD TO SEQTA AS A SINGLE DOCUMENT