T03 Functions and Relations

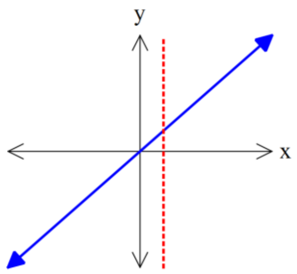
Ex 5D – One-to-one functions and implied domains

There of four types of relations:

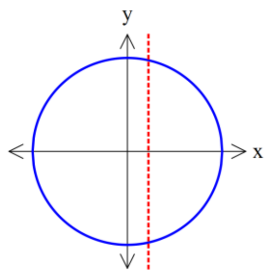
1. one to one
2. many to one
3. one to many
4. many to many

Functions have <u>unique</u> *y* values for every *x* value. This means <u>one to one</u> and <u>many to one</u> are relations that we consider to be **functions**.

To determine if a relation is a function we use the **vertical line test**.

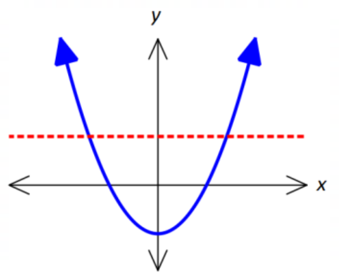


This is a function

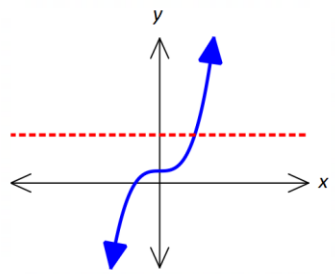


This is NOT a function

To determine if a function is <u>one to one</u> we use the **horizontal line test**.



This is NOT one-to-one



This is one-to-one

We can force a one-to-one function by **restricting** the domain.

When the domain of a function is not explicitly stated, we determine the implied (maximal) domain.

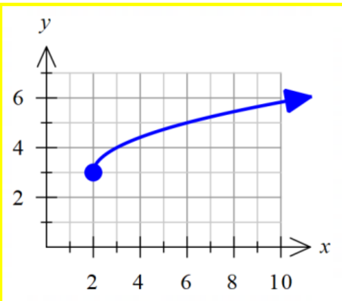
e.g. for $f(x)=x^2$ we assume $dom\ f=R$

note: to determine the RANGE of any function, you must sketch the graph.

Example: Find the maximal domain and range of the following.

a) $f(x)=\sqrt{x-2}+3$

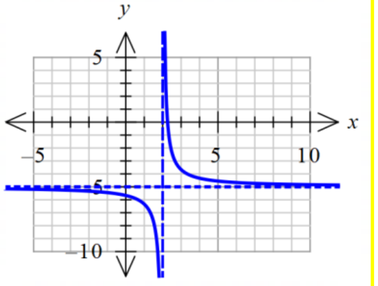
$x-2\geq 0$
$x\geq 2$



max domain $=[2,\infty)$
range $=[3,\infty)$

b) $f(x)=\dfrac{4}{3x-6}-5$

$3x-6\neq 0$
$3x\neq 6$
$x\neq 2$



max domain $=R\setminus\{2\}$
range $=R\setminus\{-5\}$

Ex 5F – Applying function notation

Remember function notation: $f:(-\infty,4]\rightarrow R,\ f(x)=(x-4)^2+1$

(co-domain: R; function name: f; "such that": $:$; domain: $(-\infty,4]$; "maps to": $\rightarrow$; rule: $(x-4)^2+1$)

When finding the rule of a function, always use the form of the function based on the information given.

For quadratic functions this means:

- General form: $y=ax^2+bx+c$ where the turning point occurs at $\left(-\dfrac{b}{2a},\ c-\dfrac{b^2}{4a}\right)$.
- Turning point form: $y=a(x-h)^2+k$ where the turning point occurs at (h,k).
- Intercept form: $y=a(x-d)(x-e)$ where the x-intercepts occur at $x=d$ and $x=e$.

Example: Find the quadratic function g such that $g(2)=g(-4)=0$ and $g(0)=32$

$g(2)$ *and* $g(-4)$ are x-ints
$\therefore g(x)=a(x-b)(x-c)$
$g(x)=a(x-2)(x+4)$
use $g(0)=32$ to find a
$32=a(0-2)(0+4)$
$32=-8a$
$a=-4$
$\therefore g(x)=-4(x-2)(x+4)$
In function notation:
$g:R\rightarrow R,\ g(x)=-4(x-2)(x+4)$

Do not expand unless you are asked to put your answer in a specific form.

Ex 5G – Inverse functions

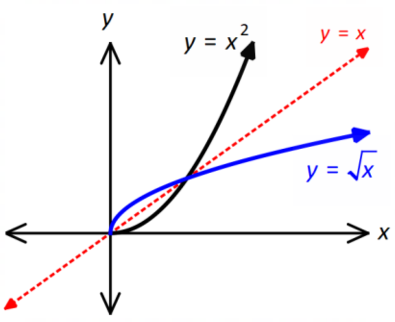
A relation can be represented by a set of ordered points.

The **inverse** of a relation is the set of ordered pairs obtained by interchanging the co-ordinates of each ordered pair. Inverse functions can only exist for a one to one function.

Consider: $f(x)=x^2$, where $x \geq 0$

The inverse, $f^{-1}(x)$, of $f(x)$ is reflected in the line $y=x$.

$dom f = ran f^{-1}$ $\qquad$ $ran f = dom f^{-1}$



To determine the equation for the inverse function we swap the *x* and *y* values and then solve to make *y* the subject.

e.g. $y=x^2$, where $x \geq 0$
swap $x \leftrightarrow y$
$x=y^2$
$y=\pm\sqrt{x}$
but $x \geq 0$
$\therefore y=\sqrt{x}$

don't forget that you must always put a $\pm$ in front the $\sqrt{\ }$ and then reject the side not needed.

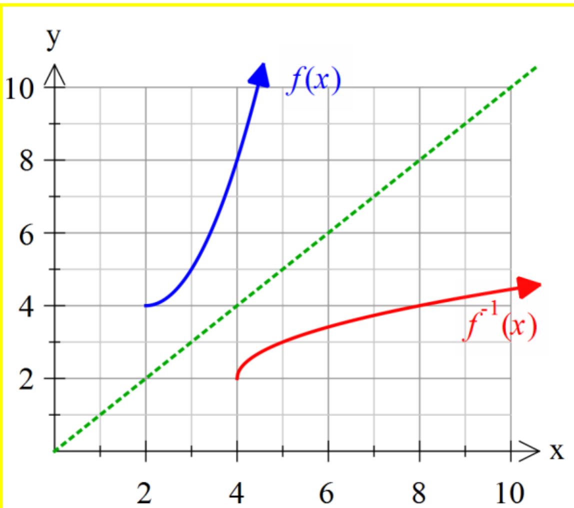
Example: Determine the inverse of $f:[2,\infty)\to R, f(x)=(x-2)^2+4$ and state the domain and range of the function.

remember:
to determine the RANGE you MUST sketch the graph over the given DOMAIN.

$y=(x-2)^2+4$
swap $x \leftrightarrow y$
$x=(y-2)^2+4$
$x-4=(y-2)^2$
$y-2=\pm\sqrt{x-4}$
$y=2\pm\sqrt{x-4}$

$dom\ f^{-1} = ran\ f = [4,\infty)$
$ran\ f^{-1} = dom\ f = [2,\infty)$

$\therefore f^{-1}:[4,\infty)\to R, f^{-1}(x)=2+\sqrt{x-4}$



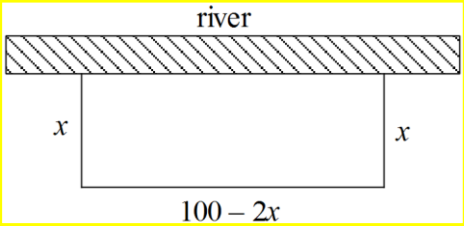
Ex 5H – Functions and Modelling exercises

Using function notation and transformations and applying these concepts to real-life application.

Example:

A farmer uses 100 metres of fencing to make a rectangular sheep pen. The straight bank of a river is used for a fourth side of the pen.

a) If x is the width of the sheep pen (the sides perpendicular to the river), draw a diagram of the sheep pen.



b) Express the area of the sheep pen as a function of the x.

$$\begin{aligned} Area &= x(100-2x) \\ &= -2x^2 + 100x \end{aligned}$$

c) What is the domain of this function?

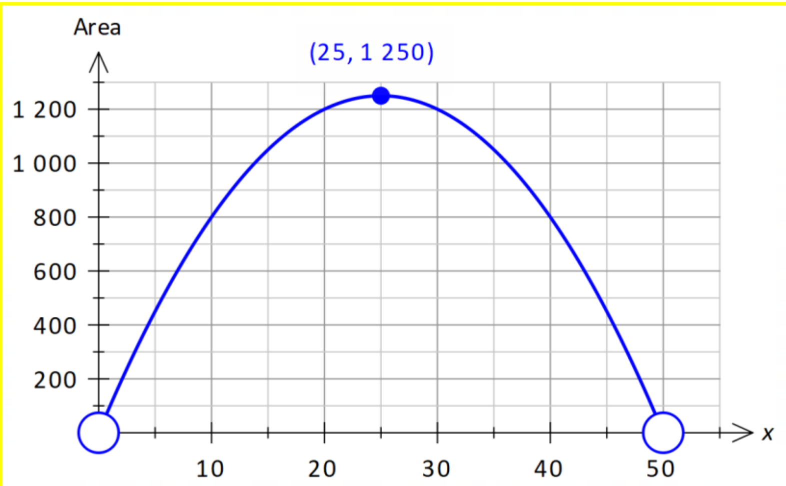
$Area > 0$ so find x-ints

$x = 0 \;\; and \;\; 100 - 2x = 0$

$x = 50$

$\therefore domain = (0, 50)$

d) Sketch the graph of this function over the above domain.



e) What is the maximum area of the sheep pen?

$1250\,m^2$