T03 Functions and Relations

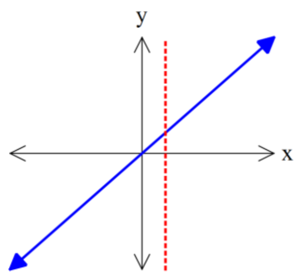
__Ex 5D – One-to-one functions and implied domains__

There of four types of relations:

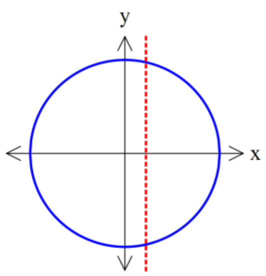
1. one to one
2. many to one
3. one to many
4. many to many

Functions have <u>unique</u> *y* values for every *x* value. This means <u>one to one</u> and <u>many to one</u> are relations that we consider to be **functions**.

To determine if a relation is a function we use the **vertical line test**.

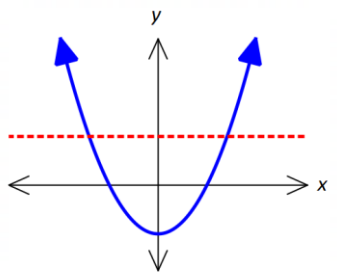


This is a function

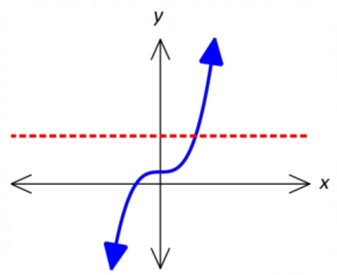


This is NOT a function

To determine if a function is <u>one to one</u> we use the **horizontal line test**.



This is NOT one-to-one



This is one-to-one

We can force a one-to-one function by **restricting** the domain.

When the domain of a function is not explicitly stated, we determine the implied (maximal) domain.

e.g. for $f(x) = x^2$ we assume $dom\ f = R$

note: to determine the RANGE of any function, you must sketch the graph.

Example: Find the maximal domain and range of the following.

a) $f(x) = \sqrt{x-2} + 3$

b) $f(x)=\dfrac{4}{3x-6}-5$

Ex 5F – Applying function notation

Remember function notation: $f:(-\infty,4]\to R,\ f(x)=(x-4)^2+1$

(co-domain: R; function name: f; "such that": $:$; domain: $(-\infty,4]$; "maps to": $\to$; rule: $(x-4)^2+1$)

When finding the rule of a function, always use the form of the function based on the information given.

For quadratic functions this means:

- General form: $y=ax^2+bx+c$ where the turning point occurs at $\left(-\dfrac{b}{2a},\ c-\dfrac{b^2}{4a}\right)$.
- Turning point form: $y=a(x-h)^2+k$ where the turning point occurs at (h,k).
- Intercept form: $y=a(x-d)(x-e)$ where the *x*-intercepts occur at $x=d$ and $x=e$.

Example: Find the quadratic function *g* such that $g(2)=g(-4)=0$ and $g(0)=32$

Ex 5G – Inverse functions

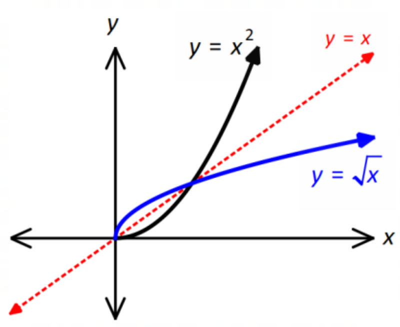
A relation can be represented by a set of ordered points.

The **inverse** of a relation is the set of ordered pairs obtained by interchanging the co-ordinates of each ordered pair. Inverse functions can only exist for a one to one function.

Consider: $f(x)=x^2$, where $x\geq 0$

The inverse, $f^{-1}(x)$, of $f(x)$ is reflected in the line $y=x$.

$domf = ranf^{-1}$ $\qquad$ $ranf = domf^{-1}$



To determine the equation for the inverse function we swap the *x* and *y* values and then solve to make *y* the subject.

e.g. $y=x^2$, where $x\geq 0$

swap $x\leftrightarrow y$

$x=y^2$

$y=\pm\sqrt{x}$

but $x\geq 0$

$\therefore y=\sqrt{x}$

don't forget that you must always put a $\pm$ in front the $\sqrt{\ }$ and then reject the side not needed.

Example: Determine the inverse of $f:[2,\infty)\rightarrow R, f(x)=(x-2)^2+4$ and state the domain and range of the function.

remember:
to determine the RANGE you MUST sketch the graph over the given DOMAIN.

Ex 5H – Functions and Modelling exercises

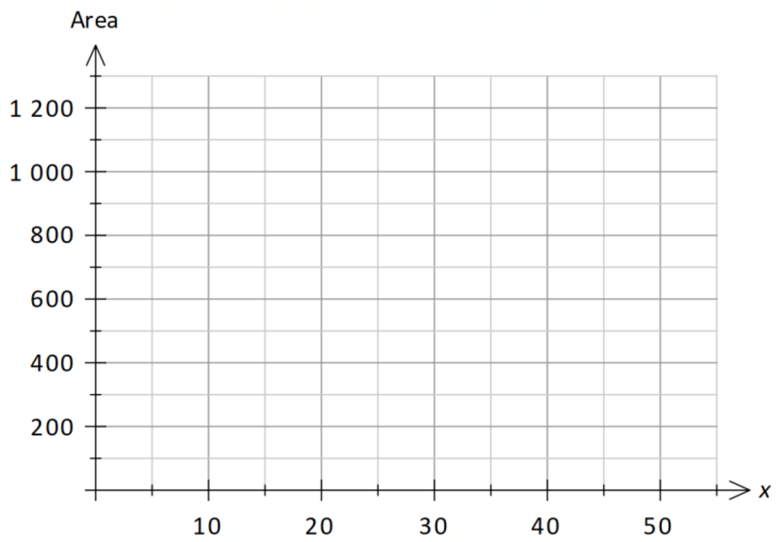
Using function notation and transformations and applying these concepts to real-life application.

Example:
A farmer uses 100 metres of fencing to make a rectangular sheep pen. The straight bank of a river is used for a fourth side of the pen.
a) If x is the width of the sheep pen (the sides perpendicular to the river), draw a diagram of the sheep pen.

b) Express the area of the sheep pen as a function of the x.

c) What is the domain of this function?

d) Sketch the graph of this function over the above domain.



e) What is the maximum area of the sheep pen?