<u>T05-T06 Mixed Application Questions – Part 1</u> **<u>SOLUTIONS – NO CAS</u>**

Instructions: These questions are to be completed **<u>without</u>** a CAS.

Question 1

Scientists conducting a research project into mice plagues, set up colonies of mice. Each colony lives under different environmental conditions and each colony starts with the same number of mice. The number of mice in each colony at time *t* years after the colonies are set up is given by *N* where

This means substitute ***t=0***

$$N(t) = 50 \times e^{\left(t^2 - qt + 1\right)} \quad \text{where } t \geq 0 \text{ and } q \geq 0.$$

a. Find the initial number of mice in each colony

$N(0) = 50e$

1 mark

b. By referring to the function *N(t)*, explain why the population of a colony can never equal zero.

e raised to any power is always > 0

$\therefore e^{t^2 - qt + 1} > 0$

$\therefore N(t) = 50e^{t^2 - qt + 1} > 0$ so $N(t) \neq 0$

1 mark

c. For a particular colony the number of mice after 3 year is $50e^5$, find the value of q for this colony.

$50e^{9-3q+1} = 50e^5$ M1

$e^{10-3q} = e^5$

$10 - 3q = 5$

$-3q = -5$ A1

2 marks

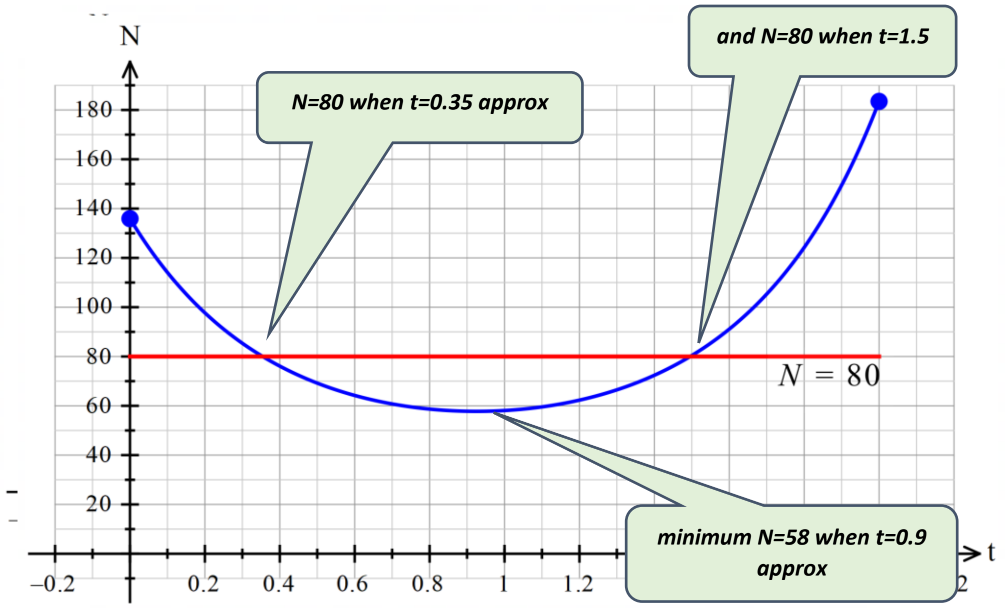
d. For a different colony the population is given by $N(t) = 50e^{t^2-t+1}$. For what value\s of t is the population of this colony $50e^7$ mice?

$50e^{t^2-t+1} = 50e^7$	M1
$e^{t^2-t+1} = e^7$	
$t^2 - t + 1 = 7$	
$t^2 - t - 6 = 0$	
$(t-3)(t+2) = 0$	M1
$t = 3, \cancel{-2}$ infeasible	
$\therefore t = 3$	A1

3 marks

e. For a particular colony $N(t) = 50e^{t^2-1.85t+1}$. The graph of N for this colony is shown below.

for $t \in [0, 2]$.



i. Estimate the minimum number of mice during the time period shown on the graph.

$Min\ N \approx 58$

1 mark

ii. Sketch an appropriate graph that could be used to estimate when $N = 80$.

Sketch $N = 80$

1 mark

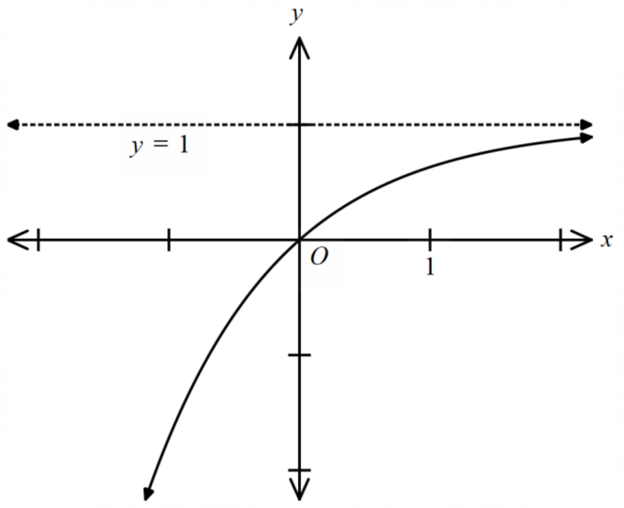
iii. Using your graph from e)ii) estimate the values of t for which the population of this colony was less than 80 during the first 2 years of the research project.

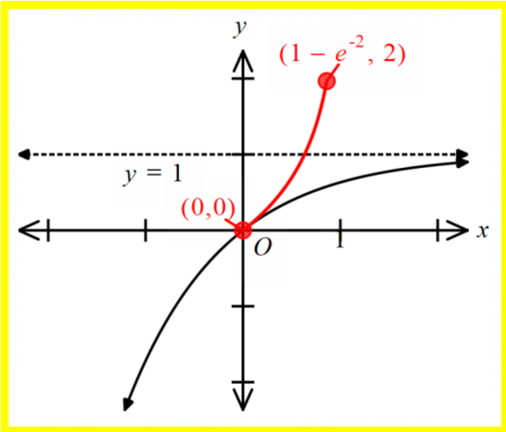
From graph: $N = 80$ when $t \approx 0.35, 1.5$	M1
$N < 80$ for $t \in (0.35, 1.5)$	A1

2 marks

Question 2

Part of the graph of the function $f: R \to R,\ f(x) = a + be^{-x}$ is shown below. The line with equation $y = 1$ is an asymptote and the graph passes through the origin.





a. Explain why $a = 1$ and $b = -1$

a is the horizontal asymptote
$\therefore$ if $y = 1$ then $a = 1$

$\therefore$ if $f(x) = 1 + be^{-x}$ and the graph passes through $(0, 0)$:
$0 = 1 + be^{-(0)}$
$0 = 1 + b$
$b = -1$
[A1]

1 mark

Let *h* be the function $h:[0, 2] \to R,\ h(x) = f(x)$

b. State the range of *h* using exact values.

$h(x) = f(x) = 1 - e^{-x}$
when $x = 0$
$y = 1 - e^{-(0)}$
$y = 1 - 1$
$y = 0$

when $x = 2$
$y = 1 - e^{-(2)}$
$y = 1 - e^{-2}$

$\therefore$ range of $h = \left[0,\ 1 - e^{-2}\right]$ ..[A1]

1 mark

c. **i.** Find the inverse function h^{-1}.

$y = 1 - e^{-x}$
swap $x \leftrightarrow y$
$x = 1 - e^{-y}$ [M1]
$x - 1 = -e^{-y}$
$1 - x = e^{-y}$
$-y = \log_e(1-x)$
$y = -\log_e(1-x)$

$\therefore h^{-1}: [0, 1-e^{-2}] \rightarrow R, h^{-1}(x) = -\log_e(1-x)$ [A1]

Note that when the function is asked for, the domain must be given.
IF only the rule is asked for, then the domain does not need to be stated.

2 marks

ii. Sketch and label the graph of the inverse function h^{-1} on the axes above.

See above.
Domain must be restricted properly. [A1]

1 mark