

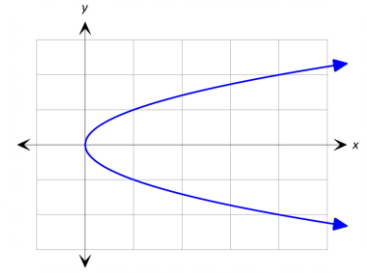
## T02: Graphs and Transformations

### Ex 4C – The graph of $y^2 = x$

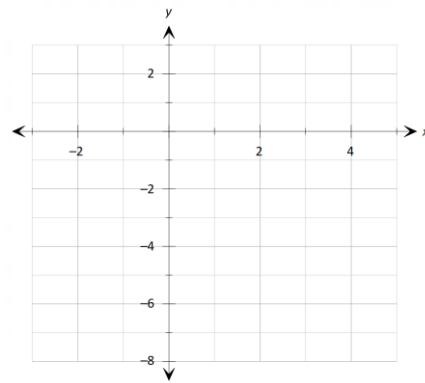
The graph of  $y^2 = x$  is a parabola. It can be obtained from the graph of  $y = x^2$  by a reflection in the line  $y = x$ .

The vertex is at  $(0, 0)$  and the axis of symmetry is the x-axis.

The general equation is given by  $(y - k)^2 = a^2(x - h)$ , where the value of 'h' and 'k' gives the vertex  $(h, k)$  and the axis of symmetry is  $y = k$ .



*Example:* Sketch the graph of  $y^2 + 6y - 2x + 5 = 0$ .



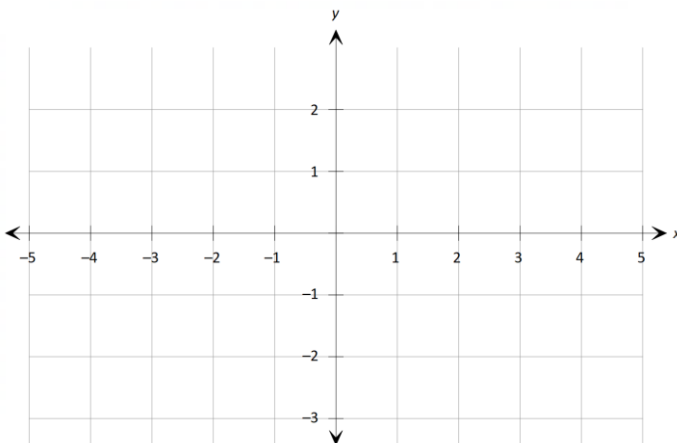
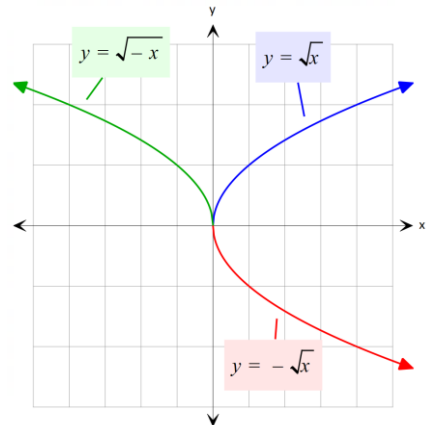
### WS1 – Square-root graph $y = \sqrt{x} = x^{\frac{1}{2}}$

Consider the graphs of  $y = \sqrt{x}$ ,  $y = -\sqrt{x}$  and  $y = \sqrt{-x}$ .

The graph starts at  $(0, 0)$  and has no asymptotes.

The general equation is given by:  $y = a\sqrt{x-h} + k$ , where the value of 'h' and 'k' gives the starting point  $(h, k)$ .

*Example:* Sketch the graph of  $y = -\sqrt{-(x-3)} + 2$ . State the domain and range.

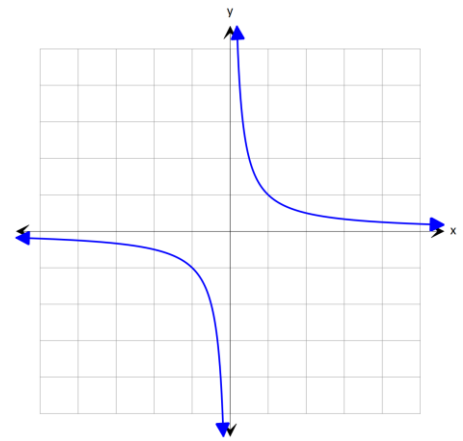


**Ex 4A / WS3 – Rectangular Hyperbolas**

A rectangular hyperbola has the equation:  $y = \frac{1}{x}$

The graph goes through the points (1,1) and (-1,-1).

By inspection we can see that the graph approaches the x and y axes but never quite crosses.



For a rectangular hyperbola, we say: as  $x \rightarrow +\infty, y \rightarrow +0$

$$x \rightarrow -\infty, y \rightarrow -0$$

$$x \rightarrow +0, y \rightarrow +\infty$$

$$x \rightarrow -0, y \rightarrow -\infty$$

This is explained by the presence of asymptotes. Asymptotes are straight lines that are never crossed. For  $y = \frac{1}{x}$  at  $x = 0$ ,  $y$  is undefined and vice versa.

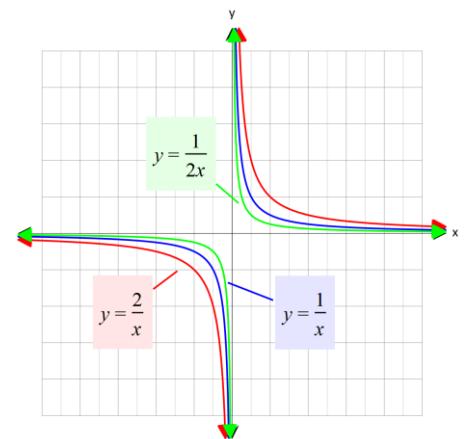
The general equation of a rectangular asymptote is given by:  $y = \frac{a}{x-h} + k$

Consider the graphs of  $y = \frac{1}{x}$ ,  $y = \frac{2}{x}$  and  $y = \frac{1}{2x}$ .

Here we are changing the 'dilation' factor of the equation, given the variable 'a'.

We say the dilated graph has been 'stretched', or dilated from the x-axis.

A reflection can also occur in the x or the y axes.



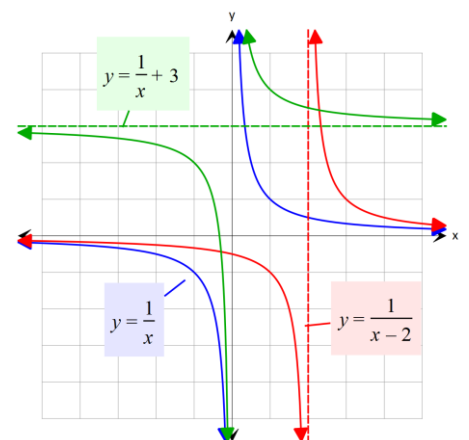
Consider the graphs of  $y = \frac{1}{x}$ ,  $y = \frac{1}{x-2}$  and  $y = \frac{1}{x} + 3$ .

Here we are changing the horizontal and vertical translation, given the variables 'h' and 'k' respectively

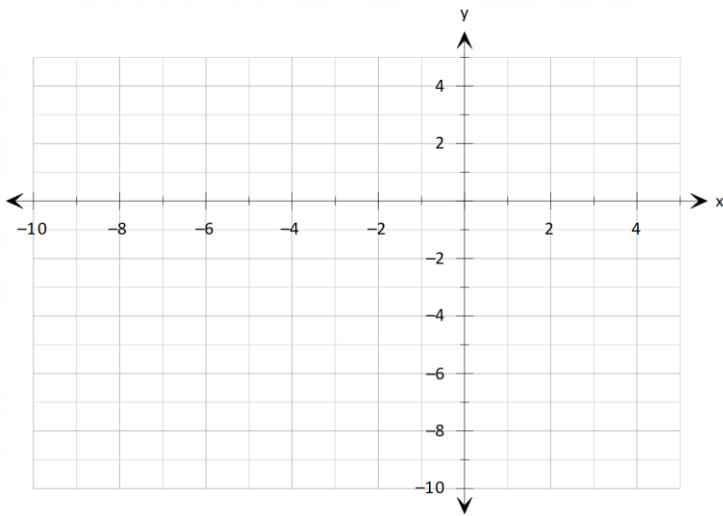
By changing the value of 'h' and 'k', we can change the equation of the vertical and horizontal asymptotes respectively.

Vertical asymptote:  $x = h$

Horizontal asymptote:  $y = k$



Example: Sketch the graph of  $y = -\frac{2}{x+2} - 3$ . State the domain and range.



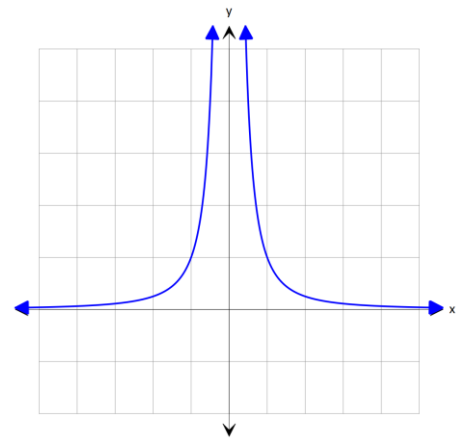
**Ex 4B / WS4 – The Truncus**

The truncus behaves in a similar fashion to the hyperbola, although has a slightly different shape.

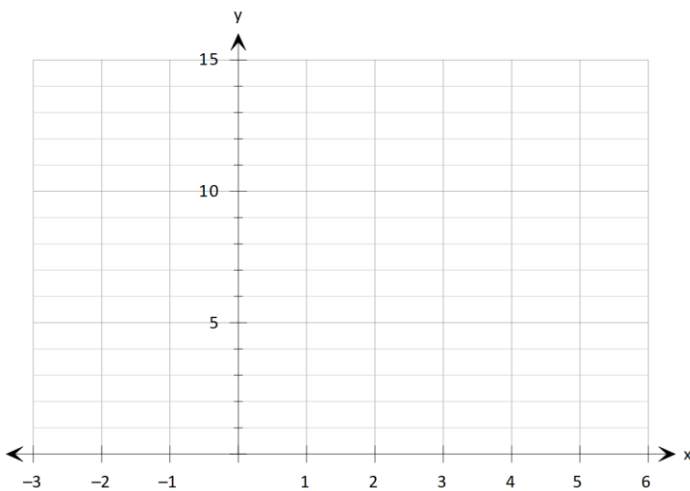
The equation for a truncus is  $y = \frac{1}{x^2}$

We write the general equation as:  $y = \frac{a}{(x-h)^2} + k$

The asymptotes are given by the equations:  $x = h$  and  $y = k$

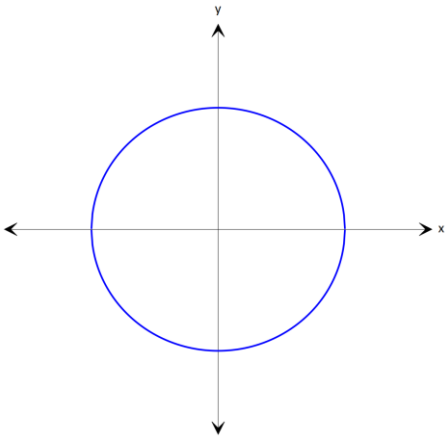


Example: Sketch the graph of  $y = \frac{1}{2(x-1)^2} + 4$ . State the domain and range.



### Ex 4E – Circles

Consider the equation  $x^2 + y^2 = 1$



This gives a circle with radius of 1 and origin at  $(0,0)$ .

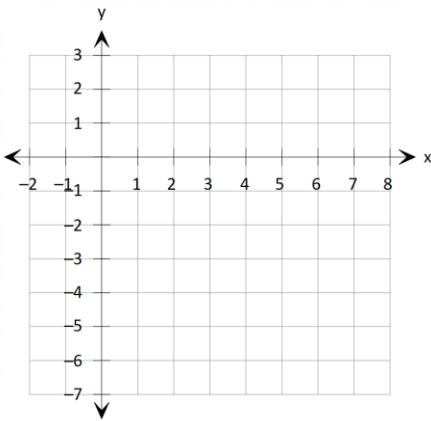
We can transform this graph in much the same way as with the other graphs we have looked at so far.

The general equation of a circle is:  $(x-h)^2 + (y-k)^2 = r^2$

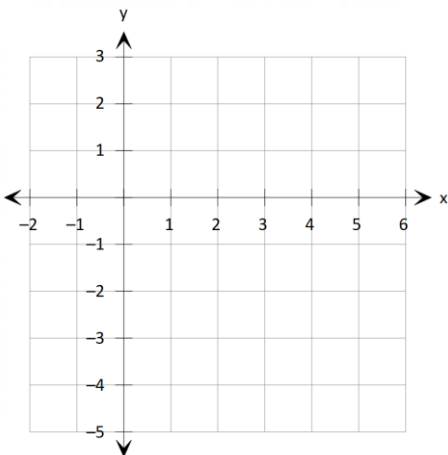
- 'h' and 'k' give the horizontal and vertical translation of the graph respectively, hence, origin of the graph can be found at  $(h,k)$ .
- 'r' changes the size of the circle through changing the length of the radius.

Examples:

1. Sketch the graph of  $(x-3)^2 + (y+2)^2 = 4^2$



2. Sketch the graph of  $x^2 + y^2 - 4x + 2y = 4$



$$x^2 + y^2 = r^2$$

$$y^2 = r^2 - x^2$$

$$y = \pm\sqrt{r^2 - x^2}$$

$$(y-k)^2 = r^2 - (x-h)^2$$

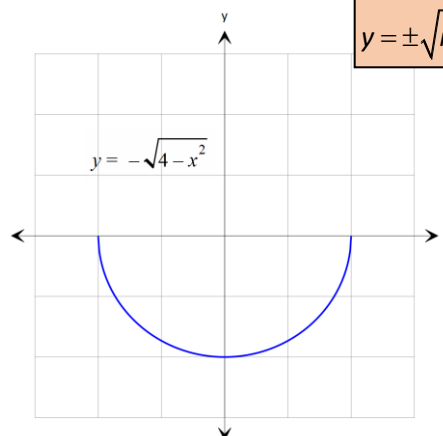
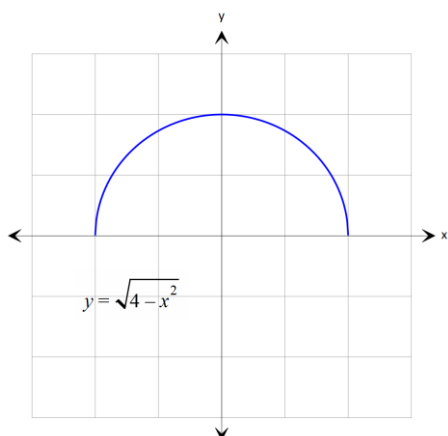
$$y-k = \pm\sqrt{r^2 - (x-h)^2}$$

$$y = \pm\sqrt{r^2 - (x-h)^2} + k$$

### Semi-circles

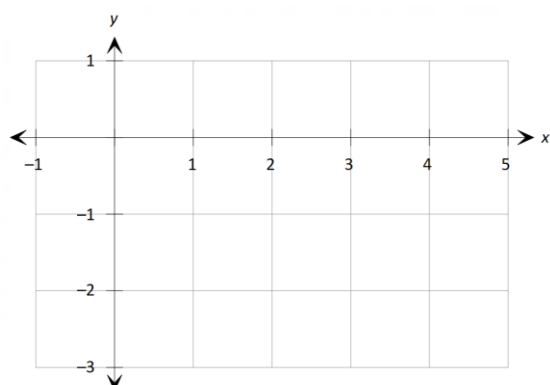
By transposing the equation of a circle we can find the equation of a semi-circle.

$$(x-h)^2 + (y-k)^2 = r^2$$



Semi-circles will follow transformations in the same way as circles.

*Example:* Sketch the graph of  $y = -\sqrt{4 - (x-2)^2}$



### Ex 4F – Determining Rules

*Example:*

The rectangular hyperbola  $y = \frac{a}{x} + k$  passes through the points  $(1, 8)$  and  $(-1, 7)$ . Find the values of  $a$  and  $k$ .

*Example:*

Find the equation of the circle whose centre is at the point  $(-2, 3)$  and which passes through the point  $(-3, 3)$ .

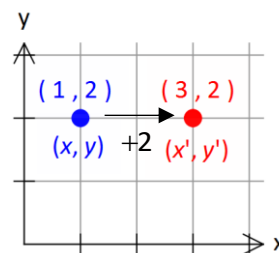
**Ex 7A/B/C – Combinations of transformations (image notation)**

For all transformations it is usual to let the original point be denoted by  $(x, y)$  and the *image* (after translation) by  $(x', y')$ .

A translation of  $h$  units in the positive  $x$  direction and  $k$  units in the positive  $y$  direction will translate the point  $(x, y)$  to  $(x+h, y+k)$ .

The image will therefore have the points:

$$x' = x + h \qquad y' = y + k$$



For graph with rule  $y = f(x)$ , we can find the rule for the translated graph by solving for  $x$  and  $y$  and substituting into the rule.

$$x = x' - h \qquad y = y' - k$$

In general for the equation:  $y = f(x)$

The translated function will be:  $y' - k = f(x' - h)$  or  $y' = f(x' - h) + k$

*Example:*

For the graph of  $y = \frac{1}{x}$ , write the new rule if it undergoes a translation of 2 units in the positive  $x$ -direction and 3 units in the negative  $y$ -direction.

Dilations and reflections on graphs can be determined in much the same way as translations.

A dilation *from* the  $x$ -axis of factor  $a$  affects the  $y$ -values.

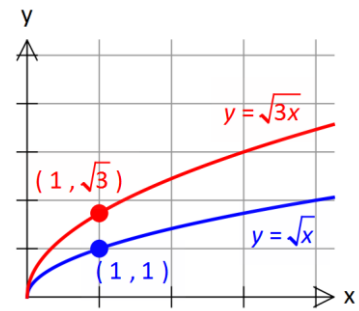
$$(x, y) \rightarrow (x, ay)$$

$$\therefore y = a \times f(x)$$

A dilation *from* the  $y$ -axis of factor  $b$  affects the  $x$ -values.

$$(x, y) \rightarrow (bx, y)$$

$$\therefore y = f\left(\frac{x}{b}\right)$$



A reflection in the  $x$ -axis affects the  $y$ -values.

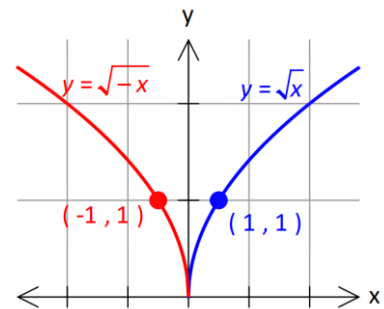
$$(x, y) \rightarrow (x, -y)$$

$$\therefore y = -f(x)$$

A reflection in the  $y$ -axis affects the  $x$ -values.

$$(x, y) \rightarrow (-x, y)$$

$$\therefore y = f(-x)$$



*Example:* Write down the rule obtained when  $y = \frac{1}{x}$  is:

a) Dilated by a factor of 2 from the  $x$  axis

b) Reflected about the  $y$ -axis, followed by a translation of 4 units in the negative  $x$ -direction.

### Ex 7D – Determining Transformations

To determine transformations, we can do mapping in reverse.

*Example:* Find the sequence of transformations which takes the graph of  $y = \sqrt{x}$  to  $y = -\sqrt{x-3} + 1$

*Example:* Find the sequence of transformation which takes the graph of  $y = (5x-1)^2 + 6$  to the graph of  $y = x^2$ .

### **Ex 7E – Transformations of graphs of functions**

*Example:*

A transformation is defined by the rule  $(x, y) \rightarrow (-2y-1, 3x+1)$ . Find the image of the straight line with equation  $y = 3x - 4$  under this transformation.

reflection in the line  $y = x$

*Example:*

Find a sequence of transformations that takes the graph of the quadratic function with rule  $f_1(x) = x^2 + 4x + 12$  to the graph of the quadratic function with rule  $f_2(x) = 3x^2 + 6x + 5$ .

transformations of quadratics