Checkpoints Chapter 19

Wave Properties of Matter

Multiple choice questions

Question 1

For the 'particle' to show wave behaviour when fired at a thin layer of the crystal, it must diffract as it passes through the atoms of the crystal. When it diffracts it will then show interference effects. It will only diffract if the ratio of $\frac{\lambda}{d}$ is approximately 1.

Partial diffraction occurs when $\frac{\lambda}{d}$ is between about 0.1 and 1.
So this question wants you to find the three different wavelengths and then find the ratio $\frac{\lambda}{d}$, or find the wavelength closest to the crystal spacing of 3×10^{-9} m.

A
60eV electron has a KE of $1.6 \times 10^{-19} \times 60$
$= 9.6 \times 10^{-18}$ J

$\therefore \frac{1}{2}mv^2 = 9.6 \times 10^{-18}$
$\therefore v^2 = 2 \times 9.6 \times 10^{-18} \div 9.1 \times 10^{-31}$
$\therefore v^2 = 2.1 \times 10^{13}$
$\therefore v = 4.6 \times 10^6$ m/s.

since $\lambda = \frac{h}{p}$ ((de Bröglie wavelength)

$\lambda = \frac{h}{mv}$

$= \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 4.6 \times 10^6}$
$= 1.56 \times 10^{-10}$ m.

B

X-rays travel at the speed of light, so $v = f\lambda$ becomes
$c = f\lambda$
$\therefore \lambda = c \div f$
$\therefore \lambda = 3 \times 10^8 \div 10^{19}$
$\therefore \lambda = 3 \times 10^{-11}$ m.

C

Since $\lambda = \frac{h}{p}$ (de Bröglie wavelength)

$\lambda = \frac{h}{mv}$

$= \frac{6.63 \times 10^{-34}}{1.0 \times 10^{-6} \times 0.01}$
$= 6.63 \times 10^{-26}$ m.

D
Gamma rays travel at the speed of light,

$\therefore E = \frac{hc}{\lambda}$

$\therefore \lambda = \frac{hc}{E}$

$\therefore \lambda = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{2.5 \times 10^6 \times 1.6 \times 10^{-19}}$
$\therefore \lambda = 5.0 \times 10^{-13}$ m

∴ A (ANS)

Question 2 (2010 Q8, 2m, 50%)

Increasing the accelerating voltage will increase the speed of the electrons. This will increase their momentum.

From $\lambda = \frac{h}{mv}$ increasing the momentum will decrease the deBroglie wavelength of the electrons.

The amount of diffraction (bending) is given by $\frac{\lambda}{w}$, so the smaller wavelength means smaller spacing between diffraction lines.

∴ A (ANS)

Question 3

Taylor found that photons would create a diffraction pattern with very low intensity, and Davisson and Germer showed that electrons behaves like waves.
∴ The diffraction pattern will form.

∴ B (ANS)

Question 4

The ratio $\frac{\lambda}{d}$ describes the amount of diffraction,

and $\lambda = \frac{h}{p}$.

Therefore both the size of the aperture and the momentum will control the spacing

$\therefore$ **D (ANS)**

Question 5

For diffraction to occur λ needs to be approximately the same size as d.

$\therefore \frac{h}{p} = d$

∴ **B (ANS)**

Question 6

Use $E = \frac{p^2}{2m}$, since the proton cannot travel at the speed of light.

Use $p = \frac{h}{\lambda}$.

$\therefore E = \frac{h^2}{2m\lambda^2}$

∴ **A (ANS)**

Question 7

Answers A and B are incorrect statements.

Use $\lambda = \frac{h}{mv}$ to show that as v increase λ will decrease, ∴ C is incorrect and D is correct.

∴ **D (ANS)**

Question 8

Use $p = \frac{h}{\lambda}$.to get h = λp.

∴ **B (ANS)**

(Note; Electrons cannot travel at the speed of light).

Question 9

Use $\Delta x \times \Delta p \geq \frac{h}{2\pi}$ to get

$\Delta x \times m\Delta v \geq \frac{h}{2\pi}$ to become

$\Delta x \times \Delta v \geq \frac{h}{2\pi}$

∴ **D (ANS)**

Question 10 (2017 Q16 1m, 66%)

Use $\Delta y \times \Delta p_Y = \frac{h}{4\pi}$.
(be careful with the directions).
The uncertainty of the position in the y direction is controlled by the slit width. As the slit width is small the uncertainty in Δy is small, so the uncertainty in Δp_Y is large.

∴ **B (ANS)**

Extended questions

Question 11a

The momentum of the x-rays is equal to the momentum of the electrons.

Momentum of the proton is given by $p = \frac{h}{\lambda}$

$\therefore h = p \times \lambda$

$= 7.94 \times 10^{-25} \times 8.35 \times 10^{-10}$

$= 6.63 \times 10^{-34}$

6.63×10^{-34} Js (ANS)

Question 11b

To produce the same diffraction pattern the ratio $\frac{\lambda}{d}$ must be the same. And since $\lambda = \frac{h}{p}$ (de Bröglie wavelength) then for λ to be the same, both must have the same momentum (p).

Question 12a

The momentum of the electron

$p = m \times v$

$= 9.1 \times 10^{-31} \times 1.78 \times 10^{7}$

$= 1.62 \times 10^{-23}$

Let $p = \frac{h}{\lambda}$

$\therefore \lambda = \frac{6.63 \times 10^{-34}}{1.62 \times 10^{-23}}$

∴ **4.1×10^{-11} m (ANS)**

Question 12b

120 pm = 1.2×10^{-10}m.
This makes the gap to be about 3 times the wavelength. The amount of diffraction is ¿ $\frac{\lambda}{w}$.

$\therefore \frac{\lambda}{w}$ ¿ 0.3.

There is complete diffraction when $\frac{\lambda}{w} \geq 1$.

There will be significant diffraction when $\frac{\lambda}{w} > 0.1$

$\therefore$ **Yes (ANS)**

Question 13

As the pattern remains the same, the momentum of the X-rays must be the same as the momentum of the electron.
The momentum of the X-rays is given by

$$p = \frac{h}{\lambda}$$

$$\therefore p = \frac{6.63 \times 10^{-34}}{56 \times 10^{-12}}$$

$\therefore p = 1.184 \times 10^{-23}$
Therefore the momentum of the electrons is the same.

$\therefore$ **p = 1.2×10^{-23} Ns (ANS)**

Question 14

The ratio $\frac{\lambda}{d}$ describes the amount of diffraction.

When $\frac{\lambda}{d}$ ~1, then there is complete diffraction, i.e. bending of 180^0 (half circle).

The smaller $\frac{\lambda}{d}$ is then the less diffraction.

A $\frac{\lambda}{d} = \frac{500 \times 10^{-9}}{0.05 \times 10^{-3}} = 0.01$

B The momentum of the electron
= m × v
= $9.1 \times 10^{-31} \times 5 \times 10^{6}$
= 4.55×10^{-24} Ns

$\therefore \lambda = \frac{h}{p}$

but $\frac{\lambda}{d} = \frac{h}{p.d}$

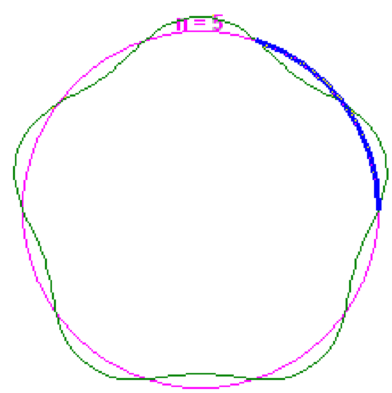
$$= \frac{6.63 \times 10^{-34}}{4.55 \times 10^{-24} \times 0.00015 \times 10^{-3}}$$

= 0.001.

$\therefore$ **A (ANS)**

Question 15 (2015 Q21, 5m, 40%)

De Bröglie suggested that electrons have wave properties such as wavelength, and that the orbits (energy levels) that could exist were those where the wavelength of the electron set up a stable standing wave. This is consistent with the quantisation of energy levels, because standing waves have quantised wavelengths.
De Bröglie said that, in a similar way, the wavelength of the electrons orbiting the nucleus must 'fit' into the circumference of the orbit exactly. This will only happen with particular wavelengths and, therefore, energies and explains why energy levels are quantised.
Electrons with wavelengths that do not set up standing waves destructively interfere with themselves and cancel out.
The standing wave is formed when the circumference of the orbit is a whole number of wavelengths, from $2\pi r = n\lambda$
The energy of the electron is linked to the wavelength, $\lambda = \frac{h}{\sqrt{2mKE}}$, so if only certain wavelengths exist, then only certain energies values are permissible.



Question 16 (2015 Q22, 2m, 65%)

Use $\lambda = \frac{h}{p}$, where p = mv.

$\therefore 1 \times 10^{-11} = \frac{6.63\times10^{-34}}{9.1\times10^{-31}\times v}$

$\therefore v = \frac{6.63\times10^{-34}}{9.1\times10^{-31}\times1\times10^{-11}}$

$\therefore$ v = 7.3 × 10^7 m s^{-1} (ANS)

Question 17a (2010 Q7, 2m, 65%)

Use $\lambda = \frac{h}{mv}$

$= \frac{6.63\times10^{-34}}{9.1\times10^{-31}\times1.5\times10^{7}}$

= 4.856 × 10^{-11}

= 4.9 × 10^{-2} × 10^{-9}

= 0.049 nm (ANS)

Question17b (2010 Q9, 2m, 60%)

The spacing of the pattern depends on the wavelength. Both the X-rays and the electrons will produce very similar diffraction patterns because they have very similar (or the same) wavelengths. This is one of those questions where less is better. A lot of students wrote very long answers and lost marks due to incorrect statements.

Question 17c (2010 Q10, 3m, 20%)

The momentum of the X-rays is the same as the momentum of the electrons. To find the momentum of the electrons use

$p = \sqrt{2mKE}$

$\therefore p = \sqrt{2\times9.1\times10^{-31}\times600\times1.6\times10^{-19}}$

= 1.322 × 10^{-23}

Then for the X-ray use E = pc,

$\therefore E = \frac{1.322\times10^{-23}\times3\times10^{8}}{1.6\times10^{-19}}$

= 2.48 × 10^4

So energy is 24.8 keV. (ANS)

Question 18a (2011 Q10, 2m, 75%)

For an X-ray use $E = \frac{hc}{\lambda}$,

$\therefore E = \frac{4.14\times10^{-15}\times3\times10^{8}}{0.2\times10^{-9}}$

= 6.210 × 10^3

= 6.21 × 10^3 eV (ANS)

Question 18b (2011 Q11, 2m, 25%)

If the diffraction pattern is similar for both the electrons and the photons, then the electrons need to have the same wavelength (hence momentum) as the photons.

Let $p = \frac{h}{\lambda}$

$\therefore p = \frac{6.63\times10^{-34}}{0.2\times10^{-9}}$

$\therefore$ p = 3.315 × 10^{-24}

Use $E = \frac{p^2}{2m}$

$\therefore E = \frac{(3.315\times10^{-24})^2}{2\times9.1\times10^{-31}}$

$\therefore$ E = 0.604 × 10^{17}

To convert into eV divide by 1.6 × 10^{-19}

$\therefore$ E = 0.377 × 10^2

$\therefore$ E = 38 eV (ANS)

Question 18c (2011 Q12, 2m, 65%)

Electrons behave exhibit wavelike properties and have a wavelength.
The spacing of the diffraction pattern is a property of the wavelength of the light/particles.
Since the X-rays have the same wavelength of the electrons they will both have the same diffraction pattern.

Question 19a (2012 Q3a, 2m, 60%)

KE = ½mv^2

= ½ × 9.1 × 10^{-31} × (1.5 × 10^5)^2

= 1.024 J

Convert into eV by dividing by 1.6 × 10^{-19}

$\therefore$ KE = 0.064 eV (ANS)

Question 19b (2012 Q3b, 2m, 60%)

As the pattern remains the same, the momentum of the photon must be the same as the momentum of the electron.

$p_{electron} = mv$

$= 9.1 \times 10^{-31} \times 1.5 \times 10^{5}$

$= 1.365 \times 10^{-25}$.

For the photon, E = pc

$\therefore E = 4.095 \times 10^{-17}$ J

Convert to eV by dividing by 1.6×10^{-19}

∴ E = 256 eV (ANS)

Question 20a (2013 Q23a, 2m, 36%)

Use E (in joules) = pc,

$\therefore p = \frac{E}{c}$

$\therefore p = \frac{80000 \times 1.6 \times 10^{-19}}{3 \times 10^{8}}$

∴ p = 4.3×10^{-23} kg m s^{-1} (ANS)

An alternative method is using $E = \frac{hc}{\lambda}$,

to get $\lambda = 1.55 \times 10^{-11}$ m,

then let $p = \frac{h}{\lambda}$

$\therefore p = \frac{6.63 \times 10^{-34}}{1.55 \times 10^{-11}}$

∴ p = 4.3×10^{-23} kg m s^{-1} (ANS)

Question 20b (2013 Q23b, 3m, 38%)

The first mark was allocated to stating that A was correct

To gain full marks you also needed to say:

(i) fringe spacing depends on wavelength

ii) wavelength depends on momentum.

Question 21 (2015 Q20 2m 30%)

Electron diffraction patterns. The light and dark bands resulting from constructive and destructive interference. Both electrons and X-rays give rise to similar diffraction patterns, if the wavelengths of both were similar.

Since interference and diffraction are wave properties, this is evidence that electrons can behave like waves.

Question 22

Classical physics does not allow particles to travel as waves. Therefore any particle that exhibits any wave behaviour, (typically diffraction), does not fit into classical physic models. Therefore electrons, protons, neutrons etc. demonstrating diffraction.

Question 23 (2014 Q21d, 3m, 33%)

The spacing of the lines in the diffraction pattern depends on the ratio $\frac{\lambda}{d}$.

Both beams are incident on the same aperture, so 'd' is constant. Therefore the pattern depends on λ.

If the patterns are different then the electrons and X-rays have different wavelengths.

Since $p = \frac{h}{\lambda}$, then they both have different momentums.

For an electron $E = \frac{p^2}{2m}$, and for a photon E = pc.

If they have the same energy, then they will have different momentums.

Question 24a

Use $E = \frac{p^2}{2m}$ to get

$\therefore 600 \times 1.6 \times 10^{-19} = \frac{p^2}{2 \times 9.1 \times 10^{-31}}$

$\therefore p^2 = 2 \times 9.1 \times 10^{-31} \times 600 \times 1.6 \times 10^{-19}$

$\therefore p^2 = 1.7472 \times 10^{-46}$

$\therefore p = 1.32 \times 10^{-23}$

Use $\lambda = \frac{h}{p}$, to get

$\therefore \lambda = \frac{6.63 \times 10^{-34}}{1.32 \times 10^{-23}}$

$\therefore \lambda = 5.02 \times 10^{-11}$

$\therefore w = 500 \times \lambda$

$\therefore w = 500 \times 5.02 \times 10^{-11}$

∴ w = 2.5×10^{-8} m (ANS)

Question 24b

Heisenberg's uncertainty principle can be written as $\Delta x \times \Delta p_x \geq \frac{h}{4\pi}$. If the position of the electron is uncertain (it is somewhere within the slit width) in the horizontal direction, then the horizontal momentum must have an uncertainty. This gives rise to the horizontal velocity of the electrons which creates the pattern.

Question 24c

If the slit was to be widened, then Δx (the uncertainty in the horizontal direction, because this is given by the width of the slit) will increase, therefore Δp_x must get smaller.
This answer is different to the one in the back of the book.

Question 25

Use $\Delta x \times \Delta p_x = \frac{h}{4\pi}$.

$\therefore \Delta x = \frac{h}{4\pi \Delta p_x}$

$\therefore \Delta x = \frac{6.63 \times 10^{-34}}{4\pi \times 660 \times 1}$

$\therefore \Delta x = 8 \times 10^{-38}$ m

The police officer is pretty certain of the car's position.
This answer is different to the one in the back of the book. Syd has used $\Delta x \times \Delta p_x = \frac{h}{2\pi}$.

Question 26

Electrons orbiting a nucleus can be modelled as circular standing waves, therefore the electron is exhibiting wave like properties.
The standing wave will exist only if the circumference of its orbit corresponds to a whole number of wavelengths.

i.e. $2\pi r = n\lambda$, where $\lambda = \frac{h}{mv}$,

Therefore $mvr = \frac{nh}{2\pi}$, where n is a whole number.
Therefore only specific values of wavelength are permitted. The momentum of the electron is related to its wavelength, and the energy of the electron is related to its momentum. Therefore the electron's energy is quantised, so only certain energy levels and orbits are possible.

Question 27

Classical physics has the electrons travelling in straight lines, because the electrons are considered as a very small discrete particle. This would give rise to a sharp 'shadow' of the gap, on the screen. The diffraction pattern can only be explained if the electrons are exhibiting wave-like properties. This is inconsistent with classical mechanics.

Question 28a (2016 Q20a 2m, 55%)

Use $\lambda = \frac{h}{mv}$

$\therefore \lambda = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times v}$

$\therefore v = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 0.36 \times 10^{-9}}$

$\therefore v = 2.0 \times 10^{6}$ m s^{-1} (ANS)

Question 28b (2016 Q20b 3m, 47%)

Since the two patterns are nearly identical, they must both have the same wavelength.

For an X-ray, $E = \frac{hc}{\lambda}$

$\therefore E = \frac{4.14 \times 10^{-15} \times 3 \times 10^{8}}{0.36 \times 10^{-9}}$

$\therefore E = 34.5 \times 10^{2}$

$\therefore E = 3.5 \times 10^{3}$ eV (ANS)

Question 28c (2016 Q20b 2m, 45%)

The shape of the diffraction patterns depends on the ratio of $\frac{\lambda}{w}$, where λ is the wavelength and w is the gap spacing. As the gap spacing was similar, if they both have the same wavelength then the patterns are similar.
