Checkpoints Chapter 3 Energy

Question 1

Use $\Delta PE = mg\Delta h$

$\therefore \Delta PE = 65 \times 9.8 \times (4300 - 1300)$

$\therefore \Delta PE = 1{,}911{,}000$

$\therefore \Delta PE = 1.91 \times 10^6$ J

∴ C (ANS)

Question 2

Use $\Delta PE = mg\Delta h$

$\therefore \Delta PE = 35 \times 9.8 \times \Delta h$

Where Δh is given by $150 \times \sin 15^0$

$\therefore \Delta h = 38.82$

$\therefore \Delta PE = 35 \times 9.8 \times 38.82$

$\therefore \Delta PE = 13{,}326$

∴ B (ANS)

Question 3

The thermal energy is the difference between her final kinetic energy and her loss in gravitational potential energy.

Final KE = $\frac{1}{2}mv^2$

$\therefore KE = 0.5 \times 35 \times 20^2$

$\therefore KE = 7{,}000$

$\therefore \Delta E = 13{,}326 - 7000$

$\therefore \Delta E = 6{,}326$

∴ B (ANS)

Question 4

There are several ways of looking at this problem. From a pure energy perspective, the total energy is constant throughout the motion. The total energy is the sum of the GPE and KE. Therefore if the GPE is a maximum, then the GPE must be a minimum.

Alternatively, we can consider velocity. The horizontal speed will remain constant, but the vertical speed will be zero, therefore the KE is a minimum, but not zero.

∴ A (ANS)

Question 5

Frankie starts from rest and is at rest at the end of the motion under consideration. Initially he had potential energy, which was changed into kinetic energy, which was then converted into stored elastic energy in the trampoline.

∴ D (ANS)

Question 6

The total energy of the system will remain constant. This is the sum of GPE, KE and elastic PE.

None, of answers A, B or D reflect this.

At the middle of its motion, the mass will have its maximum speed because from this point on the spring is opposing its motion and slowing it down.

∴ C (ANS)

Question 7

The mass will swing from its initial position through the equilibrium position and then an equal distance below this point.

At the equilibrium position use $mg = k\Delta x$

$\therefore 1.2 \times 9.8 = 20 \times \Delta x$

$\therefore \Delta x = 58.8$ cm

This means that the spring will be stretched by almost 60 cm by the mass. The spring length will go from 40 cm to about 160 cm. The amplitude of oscillation will be half of this.

∴ B (ANS)

Question 8

The mass will come to rest at the equilibrium point. Use $mg = k\Delta x$

$\therefore 1.2 \times 9.8 = 20 \times \Delta x$

$\therefore \Delta x = 58.8$ cm

∴ B (ANS)

Question 9

The mass has lost GPE, and the spring has gained elastic PE, the difference between these will be the energy dissipated to the environment.

Use $\Delta PE = mg\Delta h$

$\therefore \Delta PE = 1.2 \times 9.8 \times 0.59$

$\therefore \Delta PE = 6.94$ J

Elastic PE = $\frac{1}{2}kx^2$

$\therefore \Delta EPE = \frac{1}{2} \times 20 \times 0.59^2$

$\therefore \Delta EPE = 3.48$

$\therefore \Delta E_{system} = 6.94 - 3.48$

$= 3.46$

∴ A (ANS)

Question 10

The KE is given by $\frac{1}{2}mv^2$. Since both have the same mass and will both have the same speed (string does not stretch), they must both have the same KE

∴ A (ANS)

Question 11

The work done on the spring is given by the area under the graph. This is equal to the loss of KE of the model car.

$\therefore \frac{1}{2} \times 200 \times 0.5$

$\therefore$ KE = 50 J

$\therefore$ B (ANS)

Extended questions
Question 12a

You are given v = 0, u = 20 m/s, t = 4 secs, and you need to find 'x'.

Use $x = \frac{(u+v)}{2}t$

$\therefore x = \frac{(20+0)}{2} \times 4$

= **40 m (ANS)**

Question 12b

As soon as the question mentions 'rate' you need to use 'time'. In this case it takes 4 secs to stop.
The rate that thermal energy is being dissipated is the rate at which energy is being used. This is the rate at which KE is being lost.

Rate of losing KE = $\frac{\Delta KE}{\Delta t}$

$= \frac{\frac{1}{2} \times 900 \times 20^2}{4}$

= 45000

= **4.5×10^4 W (ANS)**

Question 13

Energy before collision

$\frac{1}{2} m_1 u^2 = \frac{1}{2} \times 500 \times 5^2$

= 6250 J

Energy after the collision

$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} \times 500 \times 1^2 + \frac{1}{2} \times 3000 \times 1^2$

= 1750 J

Therefore the final kinetic energy of the system is less that the initial kinetic energy of the system.

Question 14a

This is a force displacement graph. The area under the graph is the work done, or the energy stored. The equation of the line is F = kx, where k is known as the spring constant.

Note that the units on the vertical axis are 'kN' i.e. kilo-Newton and the horizontal axis have units of milli-metres.

The force constant (spring constant) is given by the gradient of the graph.

$\frac{\text{rise}}{\text{run}} = \frac{5 \times 10^3}{10 \times 10^{-3}}$

= **5×10^5 N/m (ANS)**

Question 14b

This is the area under the graph, make sure that you only calculate the area up to an 8 mm deformation.

$\frac{1}{2} \times 4 \times 10^3 \times 8 \times 10^{-3}$ = **16 J (ANS)**

Question 14c

The energy stored in the ball must be equal to the work done on it. This assumes that all the energy is being stored as PE.

$\therefore$ 16 J (ANS)

Question 14d

You now need to assume that all this energy is going to be converted into KE. This is reasonable if you ignore any energy lost to sound and heat, and assume that the ball is perfectly elastic.
So the PE (16 J) = KE

$\frac{1}{2} \times m \times v^2 = 16$

$\frac{1}{2} \times 0.05 \times v^2 = 16$

$\therefore$ v = 25 m s^{-1} (ANS)

Question 15a

In an elastic collision energy is conserved.
$\therefore$ the initial KE should equal to final KE.

$KE_i = \frac{1}{2} \times m \times 2^2 + \frac{1}{2} \times m \times 0^2 = 2m$ (J)

$KE_f = \frac{1}{2} \times m \times 0.5^2 + \frac{1}{2} \times m \times 1.5^2 = 1.25m$ (J)

$\therefore$ KE is lost, so the collision is not elastic.

Question 15b

$KE_i = 2m$ (J) $KE_f = 1.25m$ (J)

ratio = $\frac{2}{1.25}$

= **1.6 (ANS)**

Question 16a

The potential energy required to lift herself 7 m
= mgh
= 60 × 9.8 × 7
= 4116 J.
If she intends getting 90% from the pole, then the pole needs to supply
90% of 4116 = 3704.4
= 3.7 × 10³ J (ANS)
The working in the back of the book is inconsistent with the answer.

Question 16b

When the mat is compressed to its maximum, all her GPE will be stored as EPE in the mat. The EPE stored in the mat is the area under the graph.
The gradient of the graph is

$$'k' = \frac{3.6 \times 10^3}{3} = 1.2 \times 10^3 \text{ N m}^{-1}.$$

The area under the graph is given by $\frac{1}{2}k(\Delta x)^2$

$\therefore 4116 = \frac{1}{2} \times 1.2 \times 10^3 \times (\Delta x)^2$
$\therefore x^2 = 6.86$
$\therefore$ **x = 2.6 m (ANS)**
This answer is different to the back of the book.

Question 17a

In order to do this question you must find the difference between the initial KE and the final KE. Make sure that you convert the mass to kilograms.

$KE_i = \frac{1}{2} \times 0.45 \times 8^2 = 14.4$
$KE_f = \frac{1}{2} \times 0.45 \times 6^2 = 8.1$
$\Delta KE = KE_f - KE_i$
$\Delta KE = 14.4 - 8.1$
= **6.3 J (ANS)**

Question 17b

The energy will have gone into heat, deformation and sound energy. The ball gets warmer, deforms (during the actual collision) and makes a sound when it hits the ground.

Question 18

If a collision is elastic, then the final total kinetic energy is equal to the initial total kinetic energy.

Question 19a

The KE at the top of the hill is given by

$KE = \frac{1}{2} m v^2$
$= \frac{1}{2} \times 600 \times 10^2$
= 30 000 J
= **3.0 × 10⁴J (ANS)**

Question 19b

If there is no friction, this means that all the energy gained by the train, due to its loss in PE will be converted into KE. The easiest way to think about this is to consider the total energy at both points. Because there is no friction, the total energy at both points will be the same.
TE (at top) = KE + PE = 30 000 + mgh
= 30 000 + 600 × 9.8 × 20
= 147 600 J
TE (at bottom) = 147 600J = KE

$KE = \frac{1}{2} m v^2$
$= \frac{1}{2} \times 6\,00 \times v^2$
$\therefore v^2 = \frac{147600}{300}$
$\therefore v^2 = 492$
$\therefore$ **v = 22.2 m s⁻¹ (ANS)**

Question 19c

Since its speed was only 18 m/s, then the KE at the bottom was $\frac{1}{2} \times 600 \times 18^2 = 97\,200$ J.
The difference between the two energies is the amount of energy lost to heat.
$\therefore$ Energy lost = 147 600 – 97 200
= 50 400
= 5.0 × 10⁴ J (ANS)

Question 19d

If the train gained 4 MJ of energy, and this is equal to 20% of the energy supplied, then the train used 20 MJ of energy to get to the top of this hill.
$\therefore$ **20 MJ (ANS)**

Question 20a

A power rating of 35 kW means 3.5×10^4 Joules every second.
So in 1 minute (60 sec) it will generate
$3.5 \times 10^4 \times 60 = 2.1 \times 10^6$J
2.1 MJ (ANS)

Question 20b

In 10 minutes the petrol will supply 90 MJ of energy.
From the previous question, the car generates 2.1 MJ every minute, so 21 MJ in 10 minutes.

The efficiency is given by the ratio

$$\frac{\text{useful energy}}{\text{energy supplied}} = \frac{21}{90} \times 100\%$$

= 23% **(ANS)**

Question 21

Initial KE = $\frac{1}{2}$ m 5^2 + $\frac{1}{2}$ M 5^2
= 12.5 (M + m)

Final KE = $\frac{1}{2}$ m 1.5^2 + $\frac{1}{2}$ M 1.5^2
= 1.125 (M + m)

The amount lost is
(12.5 - 1.125) (M + m) = 11.375 (M + m)

The percentage lost is $\frac{11.375(M+m)}{12.5(M+m)} \times 100\%$

$= \frac{11.375}{12.5} \times 100\%$

= 91% **(ANS)**

Question 22a

The amount of energy that is needed to be absorbed is all the KE of the vaulter. This assumes that at the bottom the vaulter will actually come to rest, ie. will have zero KE. This KE is in the form of PE at the top of the vault, but you need to assume that at a point the vaulter is actually stationary at the top. This is not such a good approximation, because you would expect the pole-vaulter to have a reasonably constant horizontal velocity throughout the vault.

PE = mgh
= 80 × 9.8 × 7.0
= 5488
= **5.5 × 10^3 J** **(ANS)**

Question 22b

The force constant is the gradient of the Force vs compression graph.

Gradient = $\frac{22500}{2.0}$
= 11250 N/m
= 1.13 × 10^4 N m^{-1} **(ANS)**

Question 22c

The work done under compression is given by Energy stored, which is the area under the force-extension graph. E = $\frac{1}{2}$ × k × $(\Delta x)^2$

$\therefore$6000 = $\frac{1}{2}$ × 11250 × $(\Delta x)^2$

$\therefore$ x^2 = 1.067

$\therefore$ x = 1.03 m (ANS)

Question 23

The brakes are in good condition, so you can assume that the braking force remains constant. The stopping distance has two components, the distance travelled due to the reaction time the distance travelled whilst stopping.
When the speed doubles, the reaction distance will double (assuming same reaction time).
The distance travelled whilst slowing down is governed by WD = Fd = ΔKE
The ΔKE is given by Δ½mv^2. As the velocity has increased by a factor of 2, v^2 increases by a factor of 4.
$\therefore$$\Delta$KE is now 4 times as great.
On the assumption that the force is constant, then the distance will be ×4
Considering both these effects means that the stopping distance increases by a factor greater than 2

Question 24 (2010 Q14, 2m, 35%)

The initial energy stored in the spring is zero.
The initial GPE of the 2 kg mass can be said to be mgh = 2.0 × 9.8 × 0.4
= 7.84 J
The final energy stored in the spring is given by E = ½ kx^2
Use mg = fx to give k

$\therefore$ k = $\frac{2.0 \times 9.8}{0.4}$
= 49
$\therefore$ E = ½ × 49 × 0.4^2
$\therefore$ E = 3.92 J

Therefore the spring gained 3.92 J whilst the mass lost 7.84 J of GPE.
The difference is **3.9 J** **(ANS)**

Question 25a (2012 Q1a, 1m, 90%)

The block has a KE of 5.4 J.
$\therefore$ ½mv^2 = 5.4

$\therefore$ v^2 = $\frac{2 \times 5.4}{1.2}$

$\therefore$ v = 3 m s^{-1} **(ANS)**

Question 25b (2012 Q1b, 1m, 70%)

Work done by the spring on the block is the energy stored in the spring, which is given to the block as KE.

$\therefore$ 5.4 J **(ANS)**

Question 25c (2012 Q1c, 2m, 55%)

Energy stored in the spring is given by

$E = ½k(\Delta x)^2$

$\therefore 5.4 = ½ \times k \times 0.08^2$

(Make sure that you use Δx in metres)

$\therefore k = \dfrac{2 \times 5.4}{0.08^2}$

$\therefore k = 1688$

$\therefore$ **k = 1.7 × 10³ N m⁻¹ (ANS)**

Question 25d (2012 Q1d, 2m, 80%)

Impulse given to the block is equal to the change of momentum of the block.

$\therefore F \Delta t = m \Delta v$

$\therefore I = 1.2 \times (3 – 0)$

$\therefore$ **I = 3.6 N s (ANS)**

Question 26 (2013 Q5b, 3m, 43%)

Momentum is conserved.

$\therefore 2 \times 6 + 4 \times 0 = 6 \times v$

$\therefore 12 = 6v$

$\therefore$ final speed is 2 m s^{-1}

If collision is elastic, the $KE_{final} = KE_{initial}$

$KE_{initial} = ½ \times 2.0 \times 6^2 + ½ \times 4 \times 0^2$
$= 36$ J

$KE_{final} = ½ \times 6 \times 2^2$
$= 12$ J

The collision is inelastic as the final KE is less than the initial KE.

Question 27a (2013 Q6a, 1m, 81%)

From the graph, at Z, the GPE is zero, so the total energy is 20 J

$\therefore$ **20 J (ANS)**

Question 27b (2013 Q6b, 2m, 45%)

From the graph, at the point Y, the SPE = 5 J, the GPE = 10 J. Since the TE = 20 J, and remains constant, the KE must be 5 J.

$\therefore ½mv^2 = 5$ J

$\therefore ½ \times 1 \times v^2 = 5$

$\therefore v^2 = 10$

$\therefore$ **v = 3.2 m s⁻¹ (ANS)**

Question 27c (2013 Q6c, 3m, 17%)

The students have assumed that the SPE at Q = 0.

This is not correct because the spring has already been extended from its original length of 2.0 m.

$SPE_Q = ½k(\Delta x)^2$, where $\Delta x = 0.5$.

$\therefore SPE_Q = ½ \times 10 \times 0.5^2$
$= 1.25$ J

$\therefore TE_Q = 10 + 1.25 = 11.25$J

$SPE_P = ½k(\Delta x)^2$, where $\Delta x = 1.5$.

$\therefore SPE_P = ½ \times 10 \times 1.5^2$
$= 11.25$ J

$\therefore TE_P = 0 + 1.25 = 11.25$J

Question 28a (2014 Q2b, 3m, 20%)

At the lowest point the mass will be stationary, let GPE = 0, therefore the change in GPE will be stored in the spring as EPE, since KE = 0.

Use $mg\Delta x = \frac{1}{2} k\Delta x^2$

$\therefore 4 \times 0.050 \times 9.8 \times \Delta x = \frac{1}{2} \times 5.0 \times \Delta x^2$

$\therefore 1.96 = 2.5 \times \Delta x$

$\therefore$ **Δx = 0.78 m (ANS)**

This is on the assumption that the spring has not exceeded its elastic limit.

Question 38b (2014 Q2c, 2m, 35%)

The total energy is the sum of three forms of energy, spring potential energy, gravitational potential energy and Kinetic energy.

Since Jo is not including the kinetic energy, she is wrong, and the varying kinetic energy needs to be added to the other two to get a constant total energy at any point.

Question 28c (2014 Q2d, 4m, 13%)

The maximum speed will occur in the middle of the oscillation.

This is when $\Delta x = 0.4$. This is when a = 0, because $\sum F = 0$.

At the lowest point, when it is momentarily stationary, the KE = 0.

We also take this point to have GPE = 0.

Total $Energy_{(lowest\ point)}$ = KE + GPE + SPE
$= 0 + 0 + ½ \times 5 \times 0.8^2$
$= 2.5 \times 0.64$
$= 1.6$

At $\Delta x = 0.4$

TE = KE + GPE + SPE

$1.6 = KE + 0.2 \times 9.8 \times 0.4 + ½ \times 5 \times 0.4^2$

$\therefore KE = 1.6 – (0.784 + 0.4)$

$\therefore KE = 0.416$

$\therefore ½ \times 0.2 \times v^2 = 0.416$

$\therefore 0.1 \times v^2 = 0.416$

$\therefore v^2 = 4.16$

$\therefore$ **v = 2.0 m s⁻¹ (ANS)**

Question 29 (2014 Q1d, 3m, 70%)

From momentum conservation we get
$40\ 000 \times 4.0 = (40\ 000 + 4 \times 10\ 000) \times v_F$
$\therefore v_F = 2.0$ m s^{-1}
If the collision was elastic then KE would be conserved.
$\therefore KE_i = \frac{1}{2} m v^2$
$= \frac{1}{2} \times 40\ 000 \times 4^2$
$= 320\ 000$ J
$\therefore KE_f = \frac{1}{2} m v^2$
$= \frac{1}{2} \times 80\ 000 \times 2^2$
$= 160\ 000$ J
$\therefore$ Kinetic energy is lost, so the collision is **inelastic. (ANS)**

Question 30 (2015 Q1b, 2m, 65%)

For an elastic collision the final KE = Initial KE.

Initial KE = $\frac{1}{2}mv^2$
$= \frac{1}{2} \times 4 \times 8^2$
$= 128$ J
Final KE = $\frac{1}{2}mv^2 + \frac{1}{2}mv^2$
$= \frac{1}{2} \times 4 \times 2^2 + \frac{1}{2} \times 8 \times 5^2$
$= 8 + 100$
$= 108$ J
$\therefore$ **Inelastic, as KE is lost.**

Question 31a (2015 Q6a, 1m, 87%)

GPE = mgh
$\therefore 2.0 \times 9.8 \times 0.8$
$\therefore$ **15.7 J (ANS)**

Question 31b (2015 Q6b, 2m, 85%)

SPE = $\frac{1}{2} k(\Delta x)^2$
$\therefore \frac{1}{2} \times 49 \times (0.8)^2$
$\therefore 15.68$
$\therefore$ **15.7 J (ANS)**
The other way of thinking about this is:
At the bottom the KE = 0, the GPE = 0,
Therefore the energy stored in the spring must be the change in GPE of the mass.

Question 31c (2015 Q6c, 3m, 40%)

The total energy of the system = 15.68 J.
The maximum KE is at the midpoint.
$\therefore TE = \frac{1}{2} k(\Delta x)^2 + \frac{1}{2} mv^2 + mgh$
Where $\Delta x = 0.4$ m and the height above the lowest point is h = 0.4.
$\therefore 15.68 = \frac{1}{2}\times49\times0.4^2 + \frac{1}{2}\times2\times v^2 + 2\times9.8\times0.4$
$\therefore 15.68 = 3.92 + v^2 + 7.84$
$\therefore v^2 = 3.92$
$\therefore v = 1.98$
$\therefore$ **v = 2 m s^{-1} (ANS)**

Question 31d (2015 Q6d, 2m, 35%)

At the top, the net force is down, as the mass is about to move downwards, hence accelerate downwards. This has a negative value (since upwards is positive). The mass reaches its maximum speed at 0.4 m. Therefore the acceleration is zero.
At the bottom, the mass is about to move upwards, therefore the acceleration is up (positive).
$\therefore$ **C (ANS)**

Question 32 (2016 Q1c, 3m, 88%)

Momentum is conserved.
$\therefore p_i = p_f$
$\therefore 20\ 000 \times 3 + 10\ 000 \times 0 = 30\ 000 \times v$
$\therefore 60\ 000 = 30\ 000 \times v$
$\therefore$ **v = 2 m s^{-1} (ANS)**

Question 33 (2016 Q3b, 2m, 60%)

B
At the top the speed is zero, at the bottom the speed is zero.
The force on the mass is given by
$mg - k\,\Delta x$
$\therefore$ it varies with Δx,
$\therefore$ the acceleration isn't constant at any point, therefore the speed will not vary linearly.

Question 34a (2016 Q4a, 2m, 75%)

The energy stored in the spring is given by the area under the graph.
Area = $\frac{1}{2} \times 0.50 \times 72$
= 18 J (ANS)

Question 34b (2016 Q4b, 2m, 74%)

Use $\frac{1}{2}mv^2 = 18$
$\therefore 0.5 \times 4 \times v^2 = 18$
$\therefore v^2 = 9$
$\therefore$ **v = 3 m s^{-1} (ANS)**

Question 35 **(2017 Q8b, 2m)**

The total energy at the point P will be the total energy at Q.
At P, TE = KE + PE

$\therefore$ $TE_P = \frac{1}{2}mv^2 + mgh$

$\therefore$ $TE_P = \frac{1}{2} \times m \times 4^2 + m \times 9.8 \times 5$

$\therefore$ $TE_P = 57 \times m$

At Q, $57 \times m = \frac{1}{2}mv^2$

$\therefore$ $57 \times m = \frac{1}{2} \times m \times v^2$

$\therefore$ $v^2 = 114$

$\therefore$ $v = 10.67$

$\therefore$ **$v = 11$ m s^{-1}** **(ANS)**

Question 36a **(2017 Q13a, 3m)**

At the bottom the stored elastic energy will be equal to the loss in gravitational potential energy. This is because energy is conserved in the system and the KE at the top and bottom is zero.
This gives us the equation

$\frac{1}{2}k\Delta x^2 = mg\Delta x$

$\therefore$ $\frac{1}{2}k\Delta x = mg$

$\therefore$ $\Delta x = \frac{2mg}{k}$

$\therefore$ $\Delta x = \frac{2 \times 2.00 \times 9.8}{20.0}$

$\therefore$ **$\Delta x = 1.96$ m** **(ANS)**

Question 36b **(2017 Q13b, 4m)**

Total energy will remain constant.
From top to bottom.
GPE will go from a maximum value to a minimum value
SPE will go from a minimum value to a maximum value.
KE will go from zero to a maximum and then back to zero.