Checkpoints Chapter 5 Projectiles

Multiple Choice

Question 1

$\therefore$ **C (ANS)**

The others are all incorrect.

Question 2

There is a mathematical solution to this, but trial and error is much quicker.

$\therefore$ **C (ANS)**

For those that are interested, the sine function is symmetrical about 90^0 ($0 \le \theta \le 180$). $\therefore$ sin70 = sin(90 – 20)

= sin(90 + 20)

= sin 110.

$\therefore \theta = 110 \div 2$

$\therefore \theta = 55$.

The physics way of thinking about this is that if we ignore air resistance, then the maximum range is when θ is 45^0. The range is symmetrical about 45^0. So the two values will be 45 ± 10.

Question 3

The maximum value of the sine function is 1. This occurs when the angle is 90^0.

$\therefore \theta = 90 \div 2$

$\therefore \theta = 45$.

$\therefore$ **C (ANS)**

Question 4

The easiest way to solve this problem is to consider the vertical motion. At the midpoint, the vertical velocity will be zero.

Use initial vertical component to be 10 m s^{-1}. (From 20 $\sin 30^0$)

The speed at the top is zero,

Use v = u – gt,

$\therefore 0 = 10 - 9.8\ t$

$\therefore t = 1.02$ s

This is the time it takes to get to the top, so the time of flight is double that.

$\therefore$ **C (ANS)**

Question 5

If we take air resistance into consideration, there is now a force acting to oppose the motion. Combining this with the weight, (acting down), means that the best direction for the resultant is between these two.

$\therefore$ **F (ANS)**

Question 6

The only force acting in the horizontal direction is air resistance. Since the horizontal component of the velocity remains constant, then the air resistance must be small.

$\therefore$ **B (ANS)**

Question 7

Consider the vertical motion, assume $u_{vertical} = 0$,

Use $h = ut + \frac{1}{2}gt^2$, where g = 9.8 m s^{-2}

$\therefore 7 = 0 + 4.9 \times t^2$

$\therefore t^2 = 1.429$

$\therefore t = 1.2$ s

$\therefore$ **B (ANS)**

Question 8

Since we are to ignore air resistance, in the horizontal direction the distance travelled is given by

$d = u \times t$

$\therefore d = 10 \times 1.195$

$\therefore d = 12$ m

$\therefore$ **B (ANS)**

Question 9

She will have to fall the same distance, so she will take the same time to fall

$\therefore$ **B (ANS)**

Question 10

If the ball is in the air for 6 seconds, it takes 3 to get to the top, where its velocity in the vertical direction will be zero.

Use v -= u – gt

$\therefore 0 = u - 9.8 \times 3$

$\therefore u = 29.6$ m s^{-1}

$\therefore$ **B (ANS)**

Question 11

Use $h = ut + \frac{1}{2}gt^2$, where g = -9.8 m s^{-2}

$\therefore h = 29.6 \times 3 - 4.9 \times 3^2$

$\therefore h = 88.8 - 44.1$

$\therefore h = 44.7$ m

$\therefore$ **D (ANS)**

Extended questions

Question 12

The small part will fly off tangentially. It will go straight up therefore it will land directly below A, at close to S.

$\therefore$ **S (ANS)**

Question 13a

E_{tot} = constant = $KE_{bottom} = KE_{top} + PE_{top}$
E_{tot} = 1860 = 660 + mgh
$\therefore$ 1860 = 660 + 60 × 10 × h

$\therefore h = \frac{1860 - 660}{60 \times 10}$

$\therefore$**h = 2.0 m (ANS)**

Question 13b

Using KE_{top} = 660
= ½ × m × v^2
= ½ × 60 × v^2
$\therefore$ **v = 4.7 m s^{-1} (ANS)**

Question 14a

The distance is 1.5m, (from the diagram) you must take the displacement to be between the start point and the final resting place, $\therefore$ vertically down.

$\therefore$ **1.5 m, vertically down (ANS)**

Question 14b

The time taken will be the time to travel to the top of the flight and then down to the ground. Considering the upward motion, take up to be positive.
v = u + at becomes v = u – gt the acceleration due to gravity is down.
$\therefore$0 = 6.5sin30 – 10 × t
$\therefore$t = 0.325 s to go up
We must calculate the maximum height of the ball. This is given by
$v^2 = u^2 + 2ax$
$\therefore$ $0^2 = (6.5\sin 30)^2 - 2 \times 10 \times x$
$\therefore$x = 0.528 m
The total height of the ball above the ground is given by 1.5 + 0.528 = 2.028m
Considering the downward motion, take down to be positive.
x = ut + ½ at^2
$\therefore$2.028 = 0 + ½ × 10 × t^2
$\therefore$ t = 0.637 s to come down
the total travel time is the time to go up plus the time to go down which equals
0.325 + 0.637
= **0.96 s (ANS)**

Question 14c

The horizontal velocity is given by
vcos30 = 5.63 m/s
$\therefore$the horizontal distance travelled (the range) equals 5.63 × 0.96
= **5.4 m (ANS)**

Question 15a

The time the ball takes to fall 45 m is the same time as it takes to travel the 155 m.
Use x = ut + ½at^2
$\therefore$ 45 = 0 + ½ × 10 × t^2
$\therefore$ t^2 = 9
$\therefore$ t = 3 secs

In this time it travels 155 m.

$\therefore v = \frac{d}{t}$
$= \frac{155}{3}$
= **52 m s^{-1} (ANS)**

Question 15b

The initial angle of elevation is 30^0.
Then $v_{vertical} = v_0\sin 30^0$
= 50 × 0.5
= 25 m/s
Use $v^2 = u^2 - 2gh$
$\therefore$ At the top $v_{vertical}$ = 0,
$\therefore$ 0 = 25^2 - 2 × 10 × h
$\therefore$ h = 625 ÷ 20
$\therefore$ h = 31.25 = **31 m (ANS)**

Question 15c

Use v = u - gt
$\therefore$ 0 = 25 - 10 × t
$\therefore$ t = 2.5

This is how long it takes to reach it maximum height. It then needs to drop a total of
31.25 + 45 = 76.25m
Use h = ut + ½at^2
$\therefore$ 76.25 = 0 + ½ × 10 × t^2
$\therefore$ t^2 = 15.
$\therefore$ t = 3.905 secs
$\therefore$ total time = 3.905 + 2.5
= 6.405
= **6.4 sec (ANS)**

Question 15d

If the time of flight was 6.405 (**don't round off**) sec,
then the range = $v_{horizontal} \times$ time
= $50\cos 30^0 \times 6.405$
= 277.34
= **277 m** **(ANS)**

Question 16a
The vertical and horizontal components are equal, so the angle must be 45^0

Question 16b

Use Pythagoras,
$v^2 = 13.8^2 + 18.4^2$
$\therefore v^2 = 529$
$\therefore$ **v = 23 m s^{-1}** **(ANS)**

Question 17a

This is a question that is best solved using the range formula. I think that you should have this formula on your cheat sheet, but be VERY careful when using it.

$\therefore R = \frac{v^2 \sin 2\theta}{g}$

$\therefore 100 = \frac{v^2 \times \sin 60}{10}$

$\therefore v^2 = 1\,000 \div \sin 60^0$
$\therefore v^2 = 1154.7$
$\therefore$ v = **34 m s^{-1}** **(ANS)**

Question 17b

This is another question that is best solved using the range formula.

$R = \frac{v^2 \sin 2\theta}{g}$

$\therefore 100 = \frac{v^2 \times \sin 90}{9.8}$

$\therefore v^2 = 980 \div \sin 90^0$
$\therefore v^2 = 980$
$\therefore$ **v = 31.3 m s^{-1}** **(ANS)**

Question 18a

The vertical displacement
= 3.5 - 2.1
= **1.4 m (up)** **(ANS)**

Question 18b

The total displacement is the difference between the final and initial positions.
Vertically the difference is 1.4m
Horizontally the difference is 3.4m
Use Pythagoras to find the displacement.
$x^2 = 1.4^2 + 3.4^2$
$\therefore x^2 = 13.52$
$\therefore x = 3.677$
$\therefore$ **x = 3.7 m** **(ANS)**

Question 18c

In the horizontal direction, the ball travels 3.4 m in 1.1 secs

$\therefore v = \frac{d}{t}$

$= \frac{3.4}{1.1}$

= 3.09
$\therefore$ **v = 3.1 m s^{-1}** **(ANS)**

Question 18d

In the vertical direction the displacement
$y = ut - \frac{1}{2}gt^2$
$1.4 = u \times 1.1 - \frac{1}{2} \times 10 \times 1.1^2$
$1.4 + 6.05 = u \times 1.1$
$\therefore u = 6.77$
$\therefore$ **u = 6.8 m s^{-1}** **(ANS)**

Question 18e

Use Pythagoras to find the launch velocity.
(Don't use your rounded off figures)
$v^2 = 3.09^2 + 6.77^2$
$\therefore v^2 = 55.381$
$\therefore v = 7.44$
$\therefore$ **v = 7.4 m/s (ANS)**

Question 18f

Use tan $\theta = \frac{v_{vertical}}{v_{horizontal}}$

$= \frac{6.77}{3.09}$

$\therefore \theta = 65.47^0$
= **65^0** **(ANS)**

Question 19a

Initially the car is moving horizontally, so the initial vertical velocity must be zero

Question 19b

In the vertical direction the velocity
$v = u + gt$
$v = 0 + 10 \times 1.8$
v = 18 m s^{-1} **(ANS)**

Question 19c

Use $x = ut + ½at^2$, where u = 0.
∴ $x = ½ \times 9.8 \times 1.8^2$
∴ x = 15.9 m (ANS)

Question 20a

$KE = ½mv^2$
∴ $110 = ½ \times 0.550 \times v^2$
(Remember to convert the mass to kilograms)
∴ $v^2 = \frac{2 \times 110}{0.550}$
∴ $v = 20\ m\ s^{-1}$ (ANS)

Question 20b

At the ground level TE = KE + PE, (PE = 0)
∴ TE = 110J
At the top TE = 110J
= KE + PE
∴ $110 = 0.55 \times 9.8 \times 8 + KE$
∴ KE = 110 – 43.12
∴ KE_{top} = 66.9 J (ANS)

Question 20c

∴ $½mv^2 = 66.88$
∴ $v^2 = \frac{2 \times 66.88}{0.55}$
∴ $v^2 = 243.2$
∴ v = 15.49
∴ $v = 15.5\ m\ s^{-1}$ (ANS)

Question 20d

The height that the projectile reaches is given by $v^2 = u^2 - 2gh$
∴ $0^2 = (u\sin\theta)^2 - 2 \times 9.8 \times 8$
∴ $0 = (20\sin\theta)^2 – 156.8$
∴ $\sin^2\theta = 156.8 \div 400$
∴ $\sin\theta = \sqrt{0.392}$
∴ $\theta = 38.76^0$
∴ $\theta = 39^0$ (ANS)

Question 21a (2012 Q6a, 3m, 63%)

Using the velocities in the vertical direction, and $v^2 – u^2 = 2gx$, for the motion on the way up to the top of the flight.

∴ $0 - (u\sin60^0)^2 = 2 \times -10 \times 15$
∴ $-u^2 \times \left(\frac{\sqrt{3}}{2}\right)^2 = -300$
∴ $u^2 = 400$
∴ $u = 20\ m\ s^{-1}$(ANS)

Question 21b (2012 Q6b, 2m, 65%)

The time it takes to get to the top of the flight will be half the time of flight.
Use v = u – gt to get
$0 = 20\sin60^0 – 10t$
∴ 0 = 17.32 – 10t
∴ t = 1.7 sec
∴ Total time = 3.5 sec (ANS)

Question 22a (2013 Q8a, 3m, 50%)

The methodical way to complete this is to divide the problem into two parts, up and down.
Consider 'up'
Initial velocity is 10 m/s
Final velocity = o
∴ v = u – gt
∴ 0 = 10 – 10 t
∴ t = 1 sec.

Height at top
$x = ut – ½gt^2$
gives $x = 10 \times 1 – ½ \times 10 \times 1^2$
∴ x = 5 m
∴ height = 20 m.

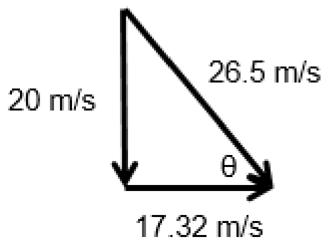
Consider 'down'
$x = ut + ½gt^2$
∴ $20 = 0 + 5t^2$
∴ $t^2 = 4$
∴ t = 2

∴ Total time = 1.0 + 2.0
= 3.0 sec (ANS)

Question 22b (2013 Q8b, 3m, 40%)

The horizontal component of the velocity will remain constant at
$v_H = 20\cos30^0$
= 17.32 m/s.
The vertical component will be
v = u + gt
on the way down, u = 0, g = 10, t = 2
∴ v = 20 m/s.
Use Pythagoras to find the magnitude of the velocity.
∴ $20^2 + 17.32^2 = v^2$
∴ $v^2 = 700$
∴ v = 26.5 m/s

To find the angle use



Use $\tan\theta = \frac{20}{17.32}$

$\therefore \theta = 49.1^0$

$\therefore$ v= 26.5 ms^{-1} at an angle of 49.1^0

$\therefore$ 26.5 m s^{-1} 49.1^0 (ANS)

Question 23a (2014 Q3a, 2m, 75%)

Use the initial vertical component of the velocity.

$v_{vertical} = 20 \times \sin 30^0$

$= 10$

In the vertical direction, use

$v^2 - u^2 = 2gx$

$\therefore 0^2 - 10^2 = 2 \times -10 \times x$

$\therefore 100 = 20 \times x$

$\therefore$ x = 5 m (ANS)

Question 23b (2014 Q3b, 3m, 53%)

To find the time that it takes for the ball to hit the advertising board, you need to know how long it takes to travel the 26 m in the horizontal.

Use d = v × t

$\therefore 26 = 20 \cos 30^0 \times t$

$\therefore t = 1.5$

In the vertical.

Then use $s = ut - ½ \times 10 \times 1.5^2$

$\therefore s = 20 \sin 30^0 \times 1.5 - 11.26$

$\therefore s = 10 \times 1.5 - 11.26$

$\therefore s = 3.73$ m

$\therefore$ 3.7 m (ANS)

Question 24a (2015 Q5a, 2m, 80%)

In the vertical direction the initial speed is

$v_v = 40 \times \sin 30^0$

$\therefore v_v = = 20.$

At the top of its flight the vertical component of the ball's velocity is zero.

Use $v^2 - u^2 = 2gh$.

$\therefore 0^2 - 20^2 = 2 \times (-10) \times h$

$\therefore 400 = 20 \times h$

$\therefore$ h = 20 m (ANS)

Question 24b (2015 Q5b, 3m, 50%)

Find the time taken to get to the point G, by using the initial horizontal speed.

In the horizontal direction

$d = v \times t$

$\therefore 173 = 40 \cos 30^0 \times t$

$\therefore t = 4.99$

$\therefore t = 5.$

Use $h = ut - ½ \times g \times t^2$ to get the vertical position of the ball at 5 seconds.

$\therefore h = 20 \times 5 - ½ \times 10 \times 5^2$

$\therefore h = - 25$ m

$\therefore$ 25 m (ANS)

Question 25a (2016 Q5a, 3m, 69%)

This question can be completed using the range formula.

$\therefore d = \frac{v^2 \sin 2\theta}{g}$

$\therefore d = \frac{40^2 \times \sin 60}{10}$

$\therefore d = 138.56$

$\therefore$ d = 139 m (ANS)

The other way of doing this is to consider the vertical direction and find the time it takes to get to the top.

Use v = u – gt

$\therefore 0 = 40 \times \sin 30 - 10\ t$

$\therefore 0 = 40 \times 0.5 - 10\ t$

$\therefore 10\ t = 20$

$\therefore$ t = 2 sec

$\therefore$ it takes 4 secs for the total flight.

In the horizontal direction

Use $d = v_{horizontal} \times t$

$\therefore d = 40 \cos 30 \times 4$

$\therefore d = 138.56$

$\therefore$ d = 139 m (ANS)

Question 25b (2016 Q5b, 2m, 43%)

The speed of the ball is a minimum at the top of the flight. (It is not zero, as it still has a horizontal component). The total energy (KE + GPE) of the ball will remain constant. The Gravitational PE will have the shape of the motion, so the KE must be A

$\therefore$ A (ANS)

Question 26 **(2017 Q9a, 3m)**

Use h = ut - $\frac{1}{2}$ gt^2 to find the height.

To find t, use (horizontal) v = $\frac{d}{t}$

$\therefore$ t = $\frac{d}{v}$

$\therefore$ t = $\frac{26}{20 \cos 30^0}$

$\therefore$ t = 1.50

$\therefore$ h = 20sin30 × 1.5 - $\frac{1}{2}$ × 9.8 × 1.5^2

$\therefore$ h = 3.975

$\therefore$ **h = 4.0 m** **(ANS)**